PROPERTIES OF TRIANGLES

In this unit, you will begin with a review of triangles and their properties. You will look at classifying triangles by both angles and sides. You will then examine the special relationship about the sum of the three angles in a triangle. You will explore congruency and similarity between triangles by examining their corresponding angles and sides.

Classifying Triangles

Classifying Triangles by Angles

triangle – A triangle is a three-sided polygon.

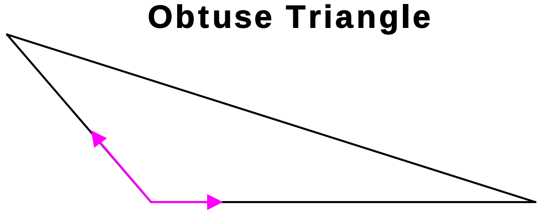
right triangle – A right triangle is a triangle that has one right angle.

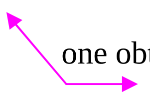
obtuse triangle – An obtuse triangle is a triangle that has one obtuse angle.

acute triangle – An acute triangle is a triangle in which all the angles are acute.

equiangular triangle – An equiangular triangle is a triangle in which all three angles are equal in measure.

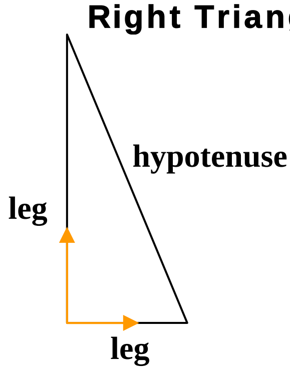
Obtuse Triangle

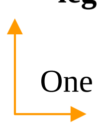




one obtuse angle (greater than 90°)

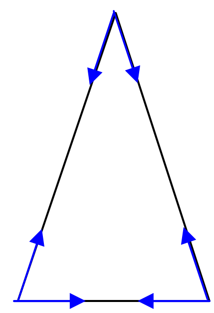
Right Triangle

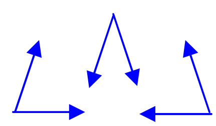




One right angle (90°)

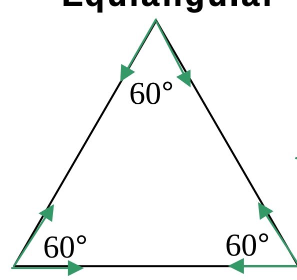
Acute Triangle

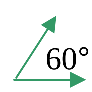




All are acute angles (angles that measure less than 90°).

Equiangular





All angles measure the same(60°).

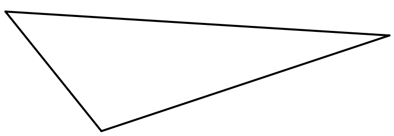
Classifying Triangles by Sides

scalene triangle – A scalene triangle is a triangle that has all sides measuring different lengths.

isosceles triangle – An isosceles triangle is a triangle that has two congruent sides.

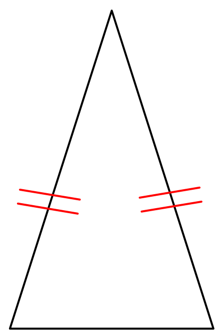
equilateral triangle – An equilateral triangle is a triangle that has all sides congruent.

Scalene Triangle



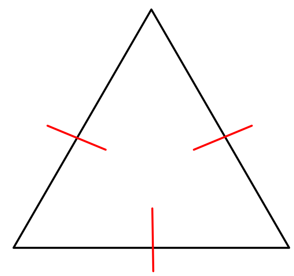
All sides are different lengths.

Isosceles Triangle



Two sides are the same lengths.

Equilateral Triangle

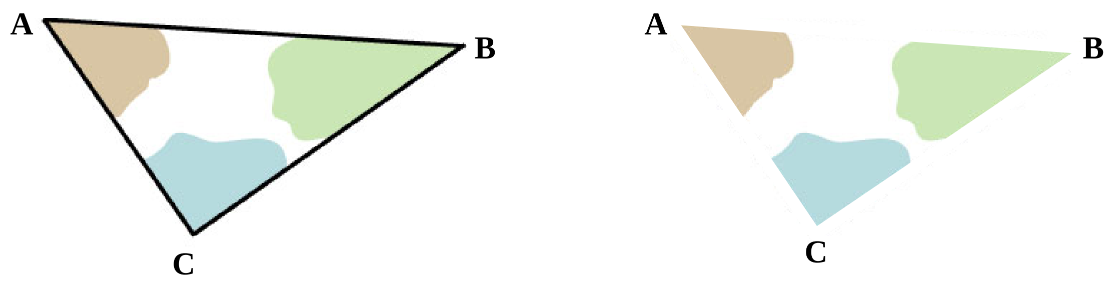


All three sides are the same length.

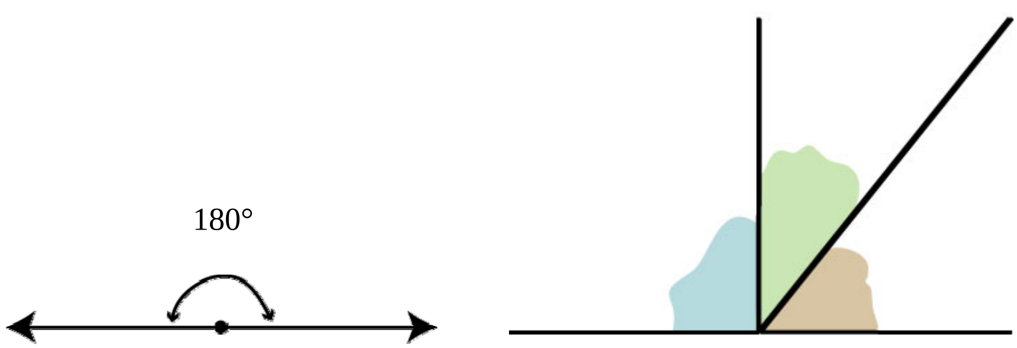
Angles of Triangles

The sum of the measures of the angles of a triangle is 180 degrees.

Activity: Look at the angles in the triangle below. Draw a triangle similar to the one shown below. It does not have to be the exact same size. Cut the angles away from the triangle.



Place the angles side by side at one vertex point, so that there are no gaps between the angles as shown below.



The three angles of the triangle total 180°.

* $m\angle A$ is read "the measurement of angle A".
$\triangle ABC$ is read "triangle ABC".

Example 1: In $\triangle ABC$, what is the $m\angle A$ if the $m\angle C = 95°$ and the $m\angle B = 40°$?

$m\angle A + m\angle B + m\angle C = 180$	A triangle's angles total 180°.
$m\angle A + 40 + 95 = 180$	Substitute
$m\angle A + 135 = 180$	Simplify
$m\angle A = 45$	Subtract

Congruent Triangles

We will now examine congruent triangles through transformations. The first transformation is a slide.

Slide

$\triangle ABC \cong \triangle XYZ$

Slide $\triangle ABC$ to the right and down until it fits exactly on top of $\triangle XYZ$. These two triangles are indeed congruent.

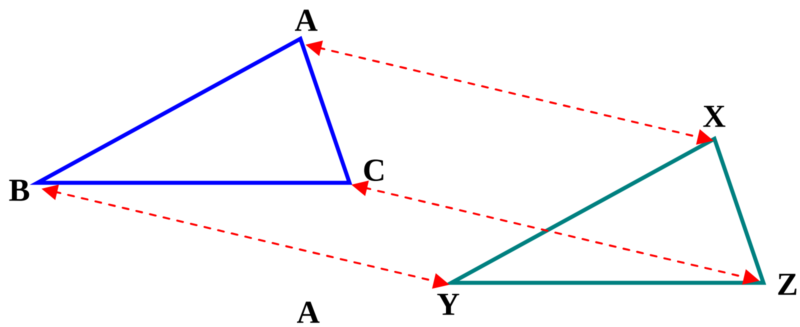
When two geometric figures are congruent, they have congruent corresponding parts.

$\angle$A corresponds to $\angle$X.   The symbol $\leftrightarrow$ means "corresponds to".

$\angle A \leftrightarrow \angle X \qquad \therefore \angle A \cong \angle X$

$\angle C \leftrightarrow \angle Z \qquad \therefore \angle C \cong \angle Z$

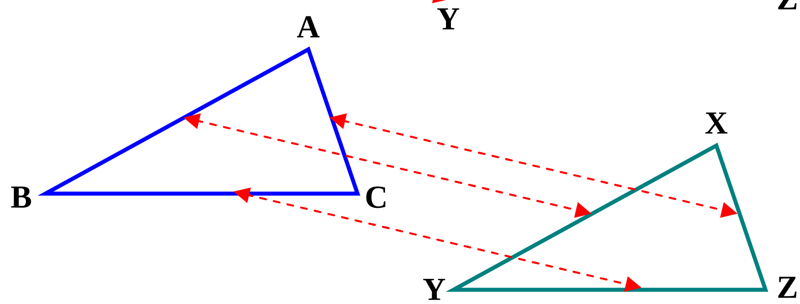
$\angle B \leftrightarrow \angle Y \qquad \therefore \angle B \cong \angle Y$



$\overline{AC} \leftrightarrow \overline{XZ} \qquad \therefore \overline{AC} \cong \overline{XZ}$

$\overline{BC} \leftrightarrow \overline{YZ} \qquad \therefore \overline{BC} \cong \overline{YZ}$

$\overline{AB} \leftrightarrow \overline{XY} \qquad \therefore \overline{AB} \cong \overline{XY}$



Definition of Congruent Triangles
(CPCTC)

Two triangles are congruent if and only if their corresponding parts are congruent.

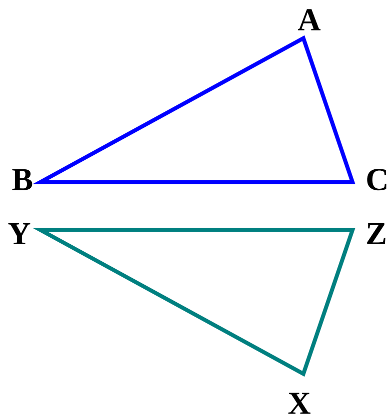
This definition is one that you will use many times in proofs about triangles. Be sure to memorize it!

Corresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent!

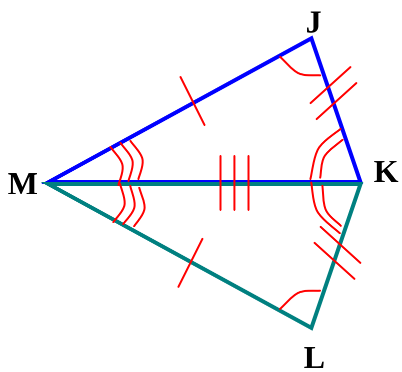
Flip (Reflection)

These figures show congruent triangles that are a reflection of each other.

$\triangle ABC \cong \triangle XYZ$



$\triangle JKM \cong \triangle LKM$



$\angle J \leftrightarrow \angle L$ $\quad \therefore \angle J \cong \angle L$

$\angle JKM \leftrightarrow \angle LKM$ $\quad \therefore \angle JKM \cong \angle LKM$

$\angle JMK \leftrightarrow \angle LMK$ $\quad \therefore \angle JMK \cong \angle LMK$

$\overline{JK} \leftrightarrow \overline{LK}$ $\quad \therefore \overline{JK} \cong \overline{LK}$

$\overline{JM} \leftrightarrow \overline{LM}$ $\quad \therefore \overline{JM} \cong \overline{LM}$

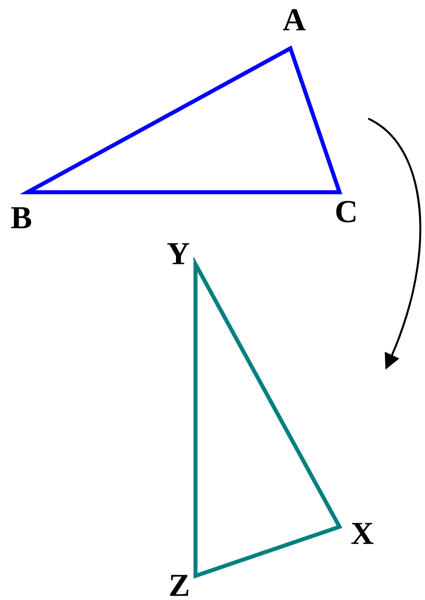
$\overline{MK} \leftrightarrow \overline{MK}$ $\quad \therefore \overline{MK} \cong \overline{MK}$

*Note: In this figure, $\overline{MK}$ corresponds to itself.

Rotation

These figures show congruent triangles that are rotated. The second figure is a 90 degree rotation of the first figure.

$$\triangle ABC \cong \triangle XYZ$$



$\angle A \leftrightarrow \angle X \qquad \therefore \angle A \cong \angle X$

$\angle C \leftrightarrow \angle Z \qquad \therefore \angle C \cong \angle Z$

$\angle B \leftrightarrow \angle Y \qquad \therefore \angle B \cong \angle Y$

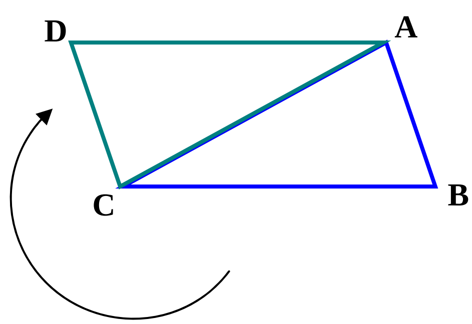
$\overline{AC} \leftrightarrow \overline{XZ} \qquad \therefore \overline{AC} \cong \overline{XZ}$

$\overline{BC} \leftrightarrow \overline{YZ} \qquad \therefore \overline{BC} \cong \overline{YZ}$

$\overline{AB} \leftrightarrow \overline{XY} \qquad \therefore \overline{AB} \cong \overline{XY}$

In this figure, $\triangle ACD$ is a 180 degree rotation of $\triangle ACB$ and vice versa.

$$\triangle ABC \cong \triangle ADC$$



Example 1: Complete each correspondence and congruence statement.

$\angle BAC \leftrightarrow$ ____ $\quad \angle BAC \cong$ ____

$\angle B \leftrightarrow$ ____ $\quad \angle B \cong$ ____

$\angle BCA \leftrightarrow$ ____ $\quad \angle BCA \cong$ ____

$\overline{AB} \leftrightarrow$ ____ $\quad \overline{AB} \cong$ ____

$\overline{BC} \leftrightarrow$ ____ $\quad \overline{BC} \cong$ ____

$\overline{AC} \leftrightarrow$ ____ $\quad \overline{AC} \cong$ ____

Solution:

$\angle BAC \leftrightarrow \angle ACD \quad \angle BAC \cong \angle ACD \quad \overline{AB} \leftrightarrow \overline{CD} \quad \overline{AB} \cong \overline{CD}$

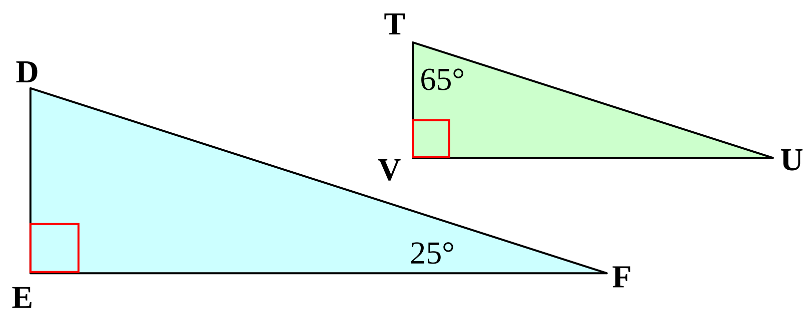
$\angle B \leftrightarrow \angle D \quad \angle B \cong \angle D \quad \overline{BC} \leftrightarrow \overline{DA} \quad \overline{BC} \cong \overline{DA}$

$\angle BCA \leftrightarrow \angle DAC \quad \angle BCA \cong \angle DAC \quad \overline{AC} \leftrightarrow \overline{AC} \quad \overline{AC} \cong \overline{AC}$

Similar Triangles

If two triangles are similar, then their corresponding angles are congruent.

$\triangle DEF \sim \triangle TVU$ Find the missing angles.



*The red square in the angle designates that the angle is a right angle.

In similar triangles, the corresponding angles are congruent.

$\angle D \cong \angle T$, $\angle E \cong \angle V$, $\angle F \cong \angle U$

$m\angle D \cong m\angle T = 65°$

$m\angle E \cong m\angle V = 90°$

$m\angle F \cong m\angle U = 25°$

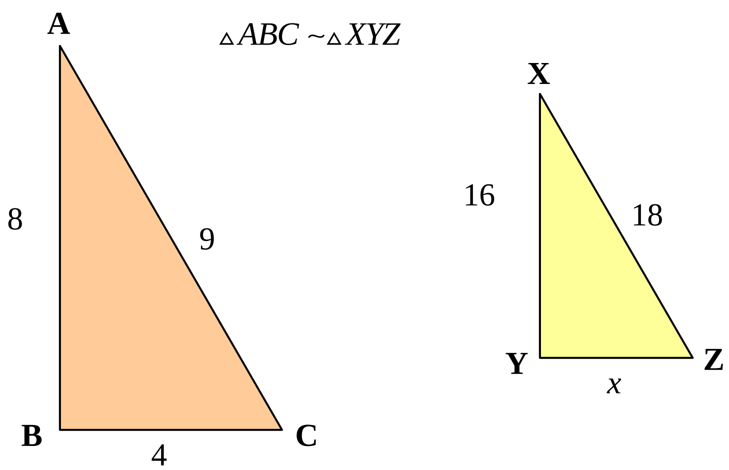
Therefore, the missing angles and their measures are:

$m\angle D = 65°$

$m\angle E = 90°$

$m\angle U = 25°$

If two triangles are similar, then their corresponding sides are proportional.



$\triangle ABC \sim \triangle XYZ$ is read "triangle ABC is similar to traingle XYZ".

To find x, write equivalent ratios since the triangles are similar. Also, since the scale factor is the same between the corresponding sides, use any pair of corresponding sides to write the proportion.

scale factor - The scale factor of similar triangles is a ratio between a pair of corresponding sides in a similar figure.

The scale factor for the two triangles scan be determined any pair of corresponding sides.

For the triangles shown above, there are two pairs of corresponding sides with given values; thus, either pair of corresponding sides may be used to determine the scale factor.

$\overline{AC} \rightarrow \overline{XZ}$ $\qquad$ $\overline{AB} \rightarrow \overline{XY}$

$\frac{9}{18}$ simplifies to $\frac{1}{2}$ $\qquad$ $\frac{8}{16}$ simplifies to $\frac{1}{2}$

Therefore, the scale factor between the two similar triangles is 1 : 2.

The relationship of all three pairs of corresponding sides can be summarized as shown below.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

To solve for *x*, set up a proportion based on these ratios. Use one pair of corresponding sides with the values given and another pair of corresponding sides that includes the unknown and one given value.

$\frac{AB}{XY} = \frac{BC}{YZ}$	Corresponding Sides of Similar Triangles
$\frac{8}{16} = \frac{4}{x}$	Substitute (We could have used the reduced scale factor, 1/2.
$8x = 64$	Cross Multiply
$x = 8$	Divide.

The length of YZ equals 8.

Now, let's take a look at the solution if we would have used the other given pair of corresponding sides.

$\frac{BC}{YZ} = \frac{AC}{XZ}$	Corresponding Sides of Similar Triangles
$\frac{4}{x} = \frac{9}{18}$	Substitute.
$9x = 72$	Cross Multiply
$x = 8$	Divide.

Again, we find that the length of YZ equals 8.

Since the scale factor is the same between any two pairs of corresponding sides of similar triangles, both solutions work!