

POINTS, LINES, ANGLES, AND CIRCLES

In this unit, you will review the language of geometry. You will make some geometric constructions, draw circle graphs, and examine angle relationships and parallel lines.

Points, Lines, and Planes

Constructions: Line Segment and Perpendicular Bisector

Angles

Constructions: Angle Bisector

Angle Relationships

Lines Cut by a Transversal

Parallel Lines Cut by a Transversal

Constructing a Circle Graph

Points, Lines, and Planes

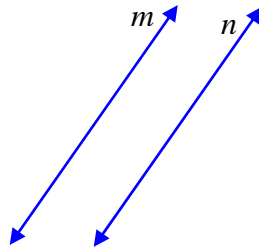
Point - A point is a location on a line. It has no dimensions but is represented by a dot.

Line - A line is a straight length that extends indefinitely into space. Lines have no width or thickness but are represented by straight edge marks.

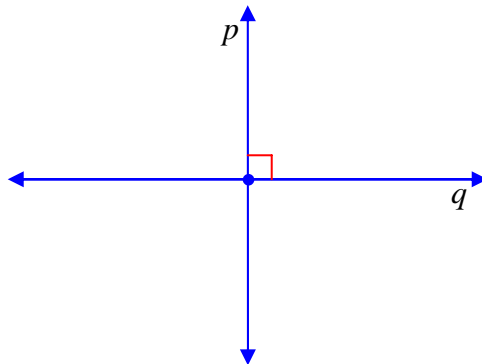
Line segment - A line segment is a part of a line usually named with its endpoints.

Intersection - Intersection is the point or line where two shapes meet. When two lines cross each other, there is one point at the place where they cross called the point of intersection. Two planes meet at and share a line of intersection.

Parallel lines - Parallel lines are lines that lie in the same plane, are equidistant apart, and never meet.



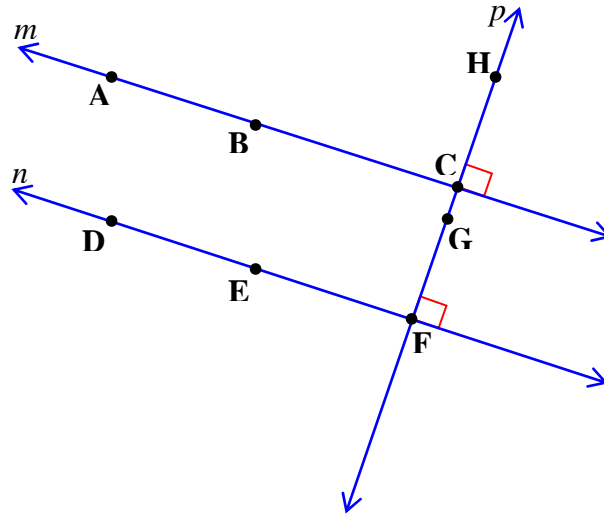
Perpendicular lines - Perpendicular lines are lines that intersect and make right angles at the point of intersection. Right angles are denoted by a square shape as shown in the diagram below.



Plane - A plane is a flat surface. A plane extends forever in all directions. Flat tables, floors, ceilings, and walls are examples of parts of planes.

Coplanar points - Coplanar points are points that lie on the same plane.

Example 1: Refer to the given diagram to answer the questions.



(a) Name a point.

Point A – Other points are B, C, D, E, F, G, H. Generally in this geometry course, points will be named with capital letters.

(b) Name a line.

Line m is one line in the diagram. It can be named another way by using any two points named on the line. Have you ever heard the expression “the shortest distance between two points is a straight line”? Well, we can name a line by using any two points on it. So line m can also be named as \overline{AB} , read Line AB. Other names for this line are \overline{AC} , \overline{BC} , \overline{BA} , \overline{CA} , and \overline{CB} .

*** When writing the name of a line on paper, you draw a mini-line above the two letters as shown in the previous paragraph. When referring to a line online, just type in “line AB” via the keyboard.**

(c) Name a line segment.

One line segment (part of a line) starts at Point A and ends at Point B. Its name is \overline{AB} , read segment AB. Some other segments are \overline{BC} , \overline{GH} , and \overline{EF} . There are many more segments in the diagram.

*** When writing the name of a line segment on paper, draw a mini-segment above the two letters as shown in the previous paragraph. When referring to a line segment online, just type in “segment AB” via the keyboard.**

(d) Name a point of intersection.

Point C is a point of intersection. It is the point where line m intersects with line p .

(e) Name a pair of parallel lines.

Lines m and n are parallel because they are equidistant apart. The lines may also be called \overline{AB} and \overline{DF} . Another way to state this answer is $m \parallel n$ or $\overline{AB} \parallel \overline{DF}$.

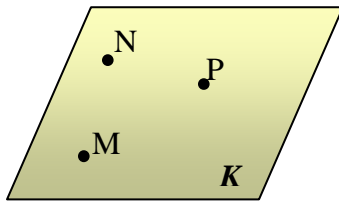
*** When referring to parallel lines on paper, draw mini-parallel lines between the names for the lines as shown in the previous paragraph. When referring to parallel lines online, just type in “line m is parallel to line n” or “line AB is parallel to line DF”.**

(f) Name a pair of perpendicular lines.

Line m is perpendicular to line p since the two lines intersect to make right angles. Another way to state this answer is $m \perp p$.

*** When referring to perpendicular lines on paper, draw mini-perpendicular lines between the names for the lines as shown in the previous paragraph. When referring to perpendicular lines online, just type in “line m is perpendicular to line n”.**

Example 2: Refer to the diagram name the plane.

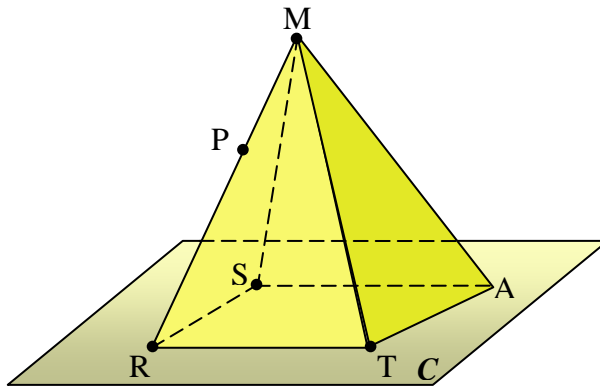


You can name the plane several ways:

K is the single letter designated as the plane's name.

Also any three letters in a plane can be used to name the plane such as plane MNP, plane MPN, plane PMN, plane PNM, plane NPM, and plane NMP.

Example 3: Refer to the 3-dimensional diagram to answer the questions.



a) Name a plane that contains point S.

plane C or plane SRT .

b) Name a plane that does not contain point S.

plane MAT or plane MTR

c) Name three coplanar points.

Points R , S , and A are coplanar because they all lie in plane C . Another example of coplanar points are points M , T , and A which lie in a different plane, a side of the pyramid.

d) Name a point that would not be coplanar with point A .

Point P would not be coplanar with A because they do not fall on the same planes. Point P lies in plane MRT while point A lies in planes C and MAT .

e) True or False. The bottom of the pyramid is part of plane C .

True. The bottom of the pyramid and the portion of plane C that is displayed are both parts of the same plane that extends on forever. Another name for plane C is plane RSA .

Constructions: Line Segment and Perpendicular Bisector

Line Segment

To make geometric constructions, you will need a compass and a straight edge such as a ruler.

Compass – A compass is a measurement tool used to draw arcs and circles.

Arc – An arc is a portion of a circle.

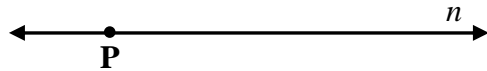
Congruent figures – Congruent figures are geometric figures that have the same size and shape.

Example 1: On line n draw segment PQ so that it is congruent to segment XY .

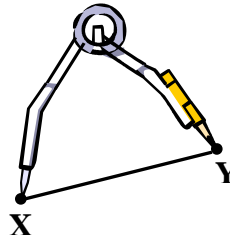
Step 1: Draw segment XY ,



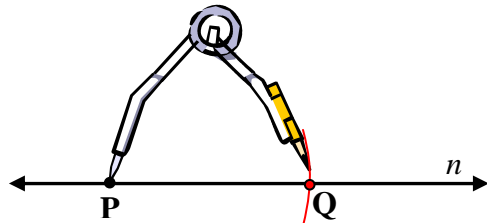
Step 2: Draw line n and label point P on the line.



Step 3: Place the metal point of the compass on segment XY and adjust the pencil point to touch point Y .



Step 4: Move the compass to line n and without changing the setting of the compass, place the metal point at P and draw an arc on line n . Label the point Q as the point where the arc crosses the line.



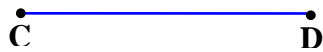
Segment PQ is congruent to segment XY .

Perpendicular Bisector

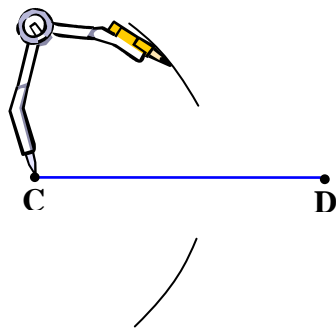
Perpendicular bisector – A perpendicular bisector is a line or segment that intersects another line segment forming 90-degree angles at the point of intersection and divides the segment into two congruent segments.

Example 2: Draw line PQ through segment CD so that it line PQ is a perpendicular bisector of segment CD.

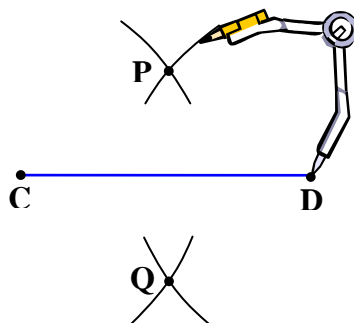
Step 1: Start with \overline{CD} .



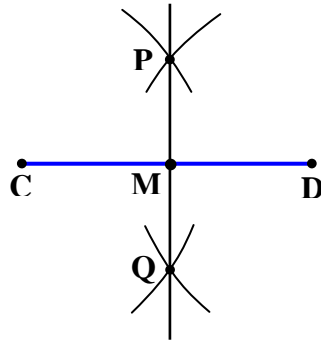
Step 2: Place the metal point of a compass on one of the endpoints of the segment. Adjust the setting of the compass so that it is greater than halfway across the segment. Draw an arc above and below the segment.



Step 3: Place the metal point of the compass on the other end point. Without changing the setting of the compass, draw an arc above and below the segment so that the arcs intersect. Mark the points of intersection as points P and Q.



Step 4: Draw a line that passes through the two points of intersection, points P and Q .



$$\overline{PQ} \perp \overline{CD}$$

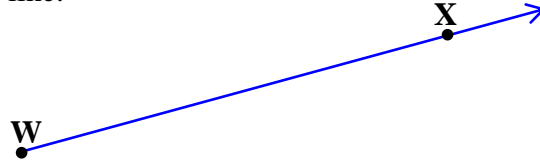
Line PQ is perpendicular to segment CD . In this construction, line PQ bisects segment CD .

Therefore, \overline{PQ} is a perpendicular bisector of \overline{CD} .

Angles

Features of Angles

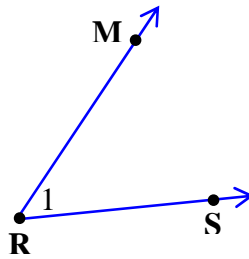
Ray – A ray is a half line.



The name of this ray is \overrightarrow{WX} or Ray WX. Since the ray starts at point W, W must be the first letter of its name.

Vertex – A vertex is the point where two rays meet.

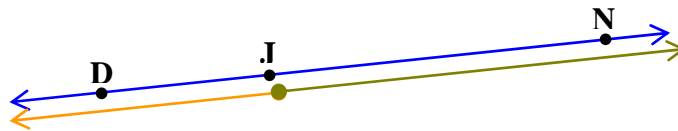
Angle – An angle is formed when two rays meet at a common point. The measure of an angle is the amount of circular rotation about a point starting with a ray and ending with a second ray.



Angle 1, also denoted as $\angle 1$, may be named $\angle MRS$ (Angle MRS). Point R is the vertex, \overrightarrow{RS} and \overrightarrow{RM} are the rays.

* When using three letters to name an angle, be sure to make the vertex letter the center letter of the name.

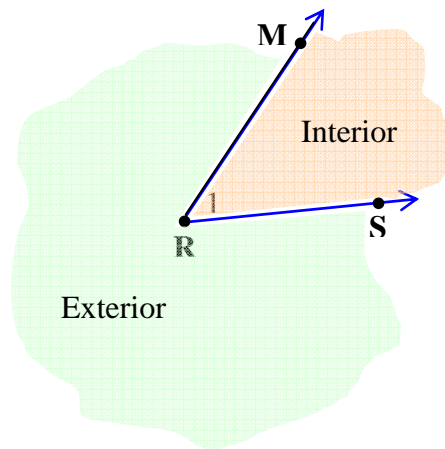
Opposite Rays – Opposite rays are two half lines that are formed at a point on a line.



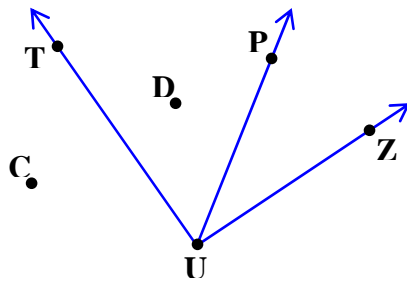
The opposite rays in this diagram are \overrightarrow{JD} and \overrightarrow{JN} .

Interior – The interior of an angle is the area within the two rays.

Exterior – The exterior of an angle is the area outside the two rays.



Example 1: Refer to the diagram to answer the questions.



a) Name a vertex.

Point U is the vertex.

b) Name a ray.

There are three rays. They are \overrightarrow{UT} , \overrightarrow{UP} , \overrightarrow{UZ} .

c) Name three angles.

They are $\angle TUP$, $\angle PUZ$, $\angle TUZ$.

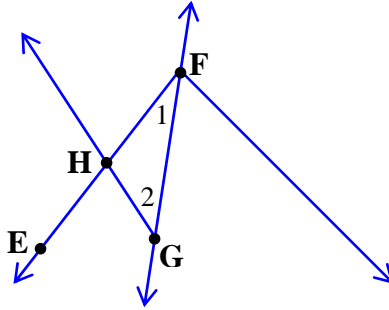
d) Name a point that lies in the interior of $\angle TUZ$.

Point D or Point P

e) Name a point that lies in the exterior of $\angle TUZ$.

Point C

Example 2: Refer to the diagram to answer the questions.



a) What are other names for $\angle 1$?

$\angle HFG$ and $\angle GFH$ (The vertex letter may be used to name an angle but should not be used when it can also be used to name other angles, so $\angle F$ would not be a good choice.)

b) What is the name of the vertex for $\angle FHG$?

Point H

c) What is the name of the vertex for $\angle 2$?

Point G

d) Name two different rays on \overleftrightarrow{FG} .

\overrightarrow{FG} starting at Point F extending downward and \overrightarrow{GF} starting at Point G and extending upward.

Measuring Angles

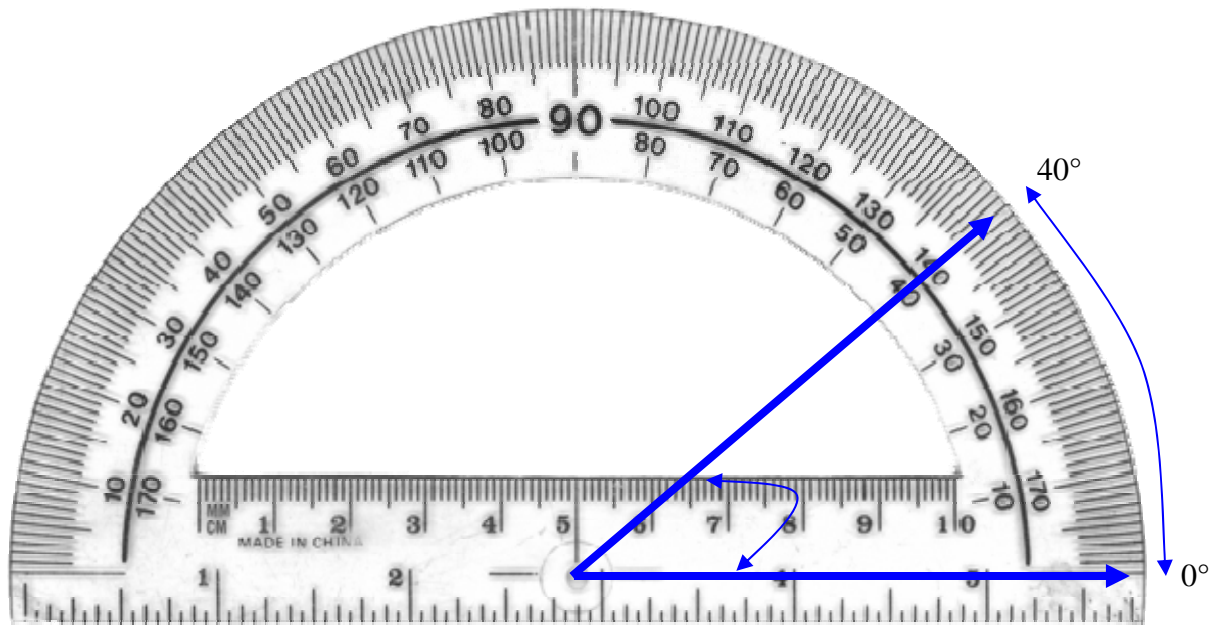
Measurement of an angle – Angles are measured in degrees.

Degree – A degree is a unit of rotation around a point that may be used to measure

angles. One degree is $\frac{1}{360}$ th of a rotation around a point.

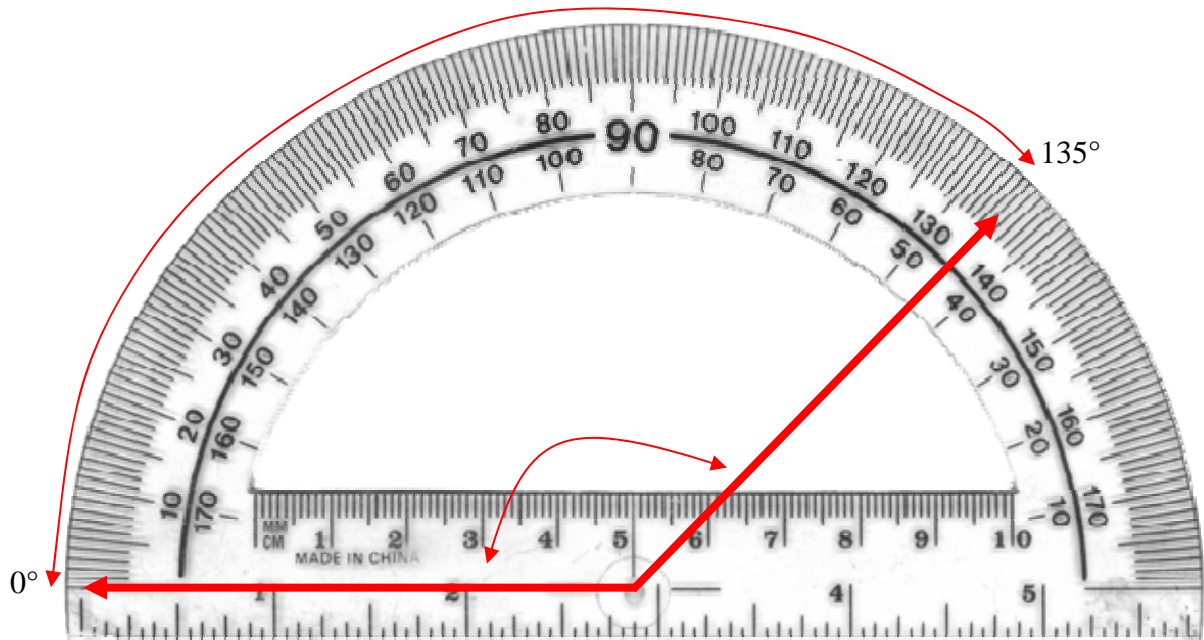
Protractor – A protractor is a measurement tool used to measure angles in degrees.

Let's examine a forty-degree angle (40°).



The starting ray, in this case, the bottom ray, is at 0° . Read the other ray. The ray is passing through both 40 and 140. You must decide which number makes sense. Think about a right angle – it measures 90° . This angle is not as open as a right angle; thus, you would read the smaller number. This angle measures 40° . Notice the numbers near the bottom ray, the lower set of numbers start at 0, then 10, 20, etc. That is the set of numbers that is used to read this angle. The starting ray starts at 0° . This is an **acute angle** because it measures more than 0° and less than 90° .

Now let's examine a 135-degree angle (135°).



The starting ray, in this case, the bottom ray, is at 0° . Read the other ray. The ray is passing through both 45 and 135. You must decide which number makes sense. Think about a right angle – it measures 90° . This angle is open wider than a right angle; thus, you would read the larger number. This angle measures 135° . Notice the numbers near the bottom ray, the upper set of numbers start at 0, then 10, 20, etc. That is the set of numbers that is used to read this angle. The starting ray starts at 0° . This is an **obtuse angle** because it measures more than 90° and less than 180° .

Definition of Right, Acute and Obtuse Angles

$\angle A$ is a right angle if $m\angle A$ is 90.

$\angle A$ is an acute angle if $m\angle A$ is less than 90.

$\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180.

Constructions: Angle Bisector

Compass – A compass is a measurement tool used to draw arcs and circles.

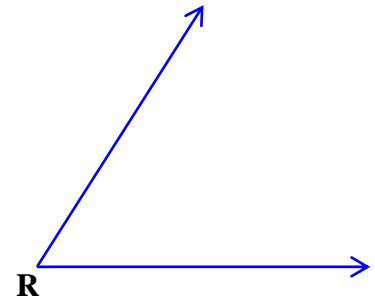
Arc – An arc is a portion of a circle.

Congruent figures – Congruent figures are geometric figures that have the same size and shape.

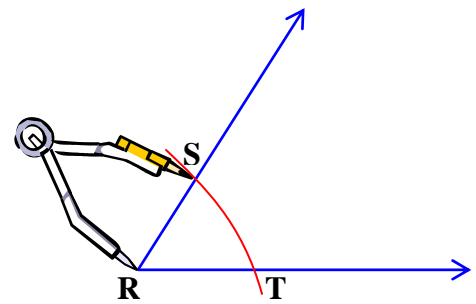
Angle bisector – An angle bisector is a ray that divides an angle into two congruent angles.

Example: Draw ray RU so that it bisects angle R.

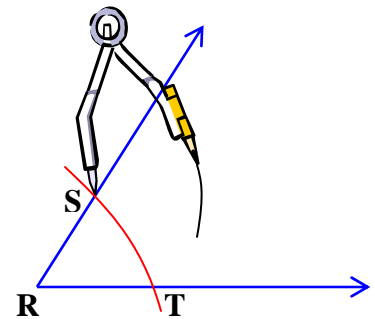
Step 1: Draw $\angle R$.



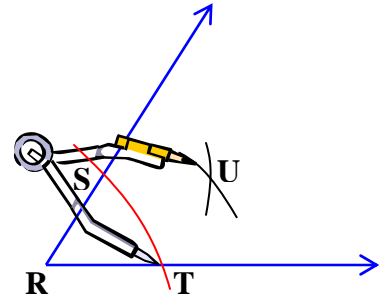
Step 2: Place the metal point of the compass at point R and draw an arc through the angle rays, naming the points of intersection, S and T.



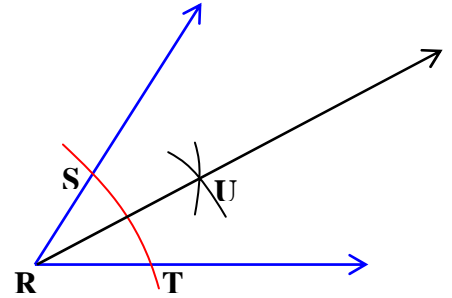
Step 3: Adjust the compass settings a little wider and place the metal point of the compass at point S. Draw an arc in the interior of the angle.



Step 4: Keep the compass setting the same and place the metal point of the compass at point T. Draw a second arc in the interior of the angle letting it cross the other arc.



Step 5: Draw \overline{RU} so that it starts at the vertex R and extends through the intersection of the two arcs, point U.



\overline{RU} is the angle bisector of $\angle SRT$.

Therefore, $m\angle SRU = m\angle URT$ and $\angle SRU \cong \angle URT$.

Angle Relationships

Right Angle - A right angle measures 90° .

Straight Angle – A straight angle measures 180° .

Supplementary angles – Supplementary angles are angles that total 180° .

Complementary angles – Complementary angles form a right angle.

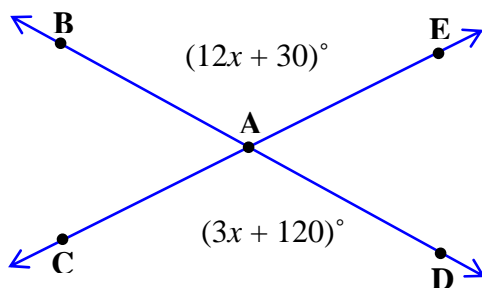
Linear pair – A linear pair is a pair of adjacent angles whose sum forms a straight angle.

Vertical angles – Vertical angles are the opposite congruent angles formed when two lines intersect.

Vertical angles are congruent.

Example 1: Find $m\angle BAE$.

Since $\angle BAE$ and $\angle CAD$ are vertical angles, they are congruent and their measures are equal.



$$12x + 30 = 3x + 120$$

$$9x = 90$$

$$x = 10$$

$$m\angle BAE = 12x + 30$$

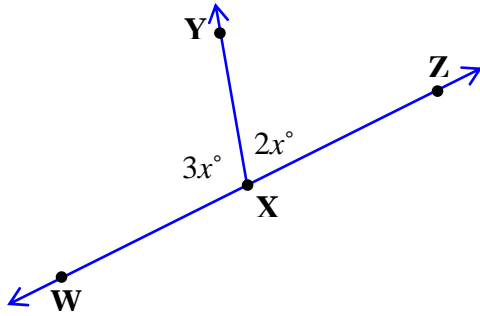
$$m\angle BAE = 12(10) + 30$$

$$m\angle BAE = 150^\circ$$

The sum of the measures of the angles in a linear pair is 180° .

Example 2: Find the measure of $\angle WXY$ and $\angle ZXY$.

Since $\angle WXY$ and $\angle ZXY$ are a linear pair, the sum of their measures is 180 degrees.



$$m\angle WXY + m\angle ZXY = 180$$

$$3x + 2x = 180$$

$$5x = 180$$

$$x = 36$$

$$m\angle WXY = 3x$$

$$m\angle ZXY = 2x$$

$$m\angle WXY = 3(36)$$

$$m\angle ZXY = 2(36)$$

$$m\angle WXY = 108^\circ$$

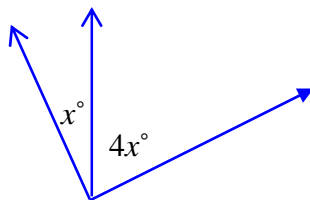
$$m\angle ZXY = 72^\circ$$

Check: Linear pairs total 180° . $72^\circ + 108^\circ = 180^\circ$

The sum of the measures of complementary angles is 90° .

Example 3: A pair of angles is complementary. One of the angles is 4 times larger than the other angle. How large are each of the angles?

Draw a picture.



Let x represent the smaller angle.

$$x$$

Let $4x$ represent the larger angle.

$$4x$$

Complementary angles total 90.

$$\text{first angle} + \text{second angle} = 90$$

Write an equation and solve.

$$x + 4x = 90$$

$$5x = 90$$

$$x = 18$$

The first angle (x) is 18° ; the second angle ($4x$) equals $4(18)$ which is 72° .

Check: Complementary angles total 90° . $18^\circ + 72^\circ = 90^\circ$

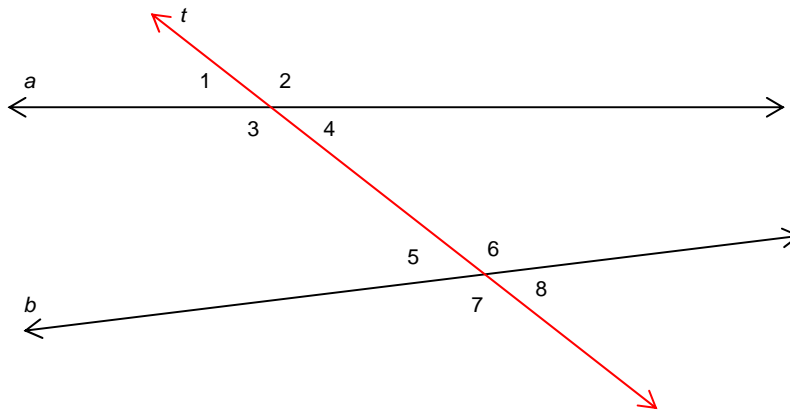
Lines Cut by a Transversal

In the given drawing two lines, a and b , are cut by a third line, t , called a transversal.

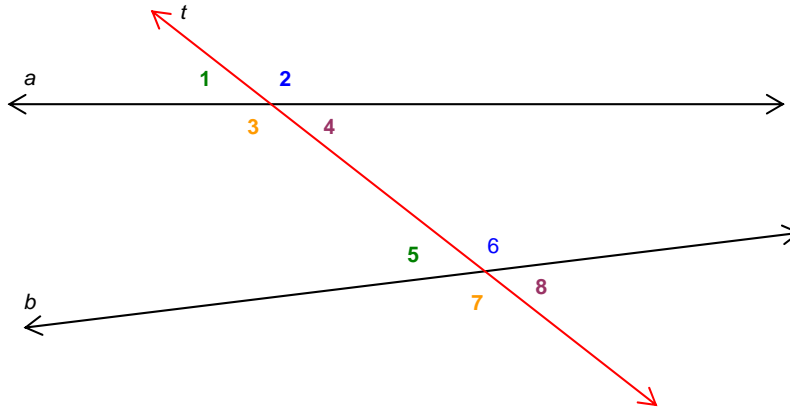
transversal – A transversal is a line that crosses two or more lines at different points.

Many angles are formed when a transversal crosses over two lines. Some of these angles are given special names and/or definitions based on their positions relative to the transversal and the lines.

The definitions in this section relate two lines cut by a transversal.



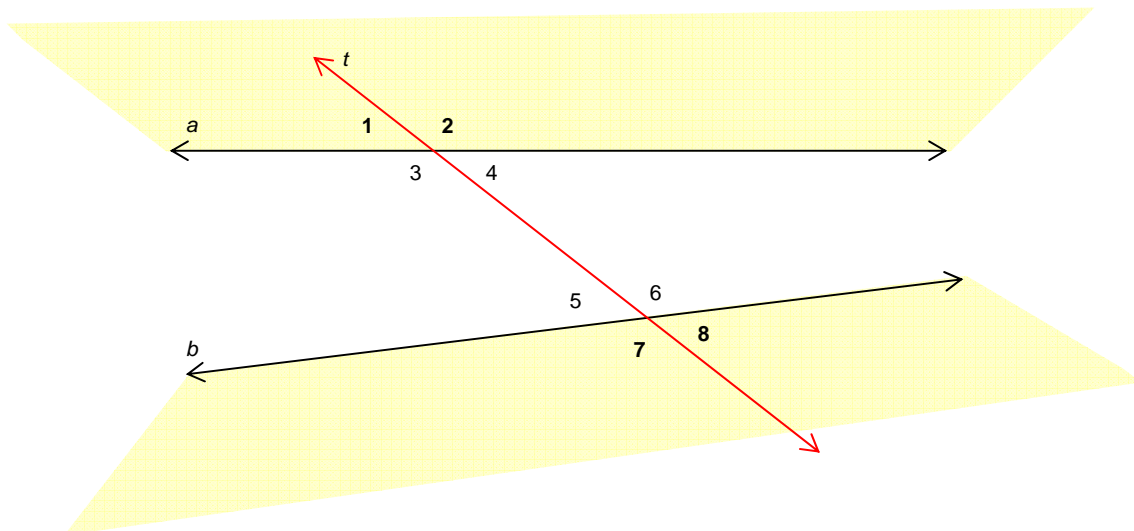
corresponding angles – When two lines are cut by a transversal, the corresponding angles are the angles that are located on the same side of the transversal and their position is corresponding relative to the lines.



Corresponding Angles:

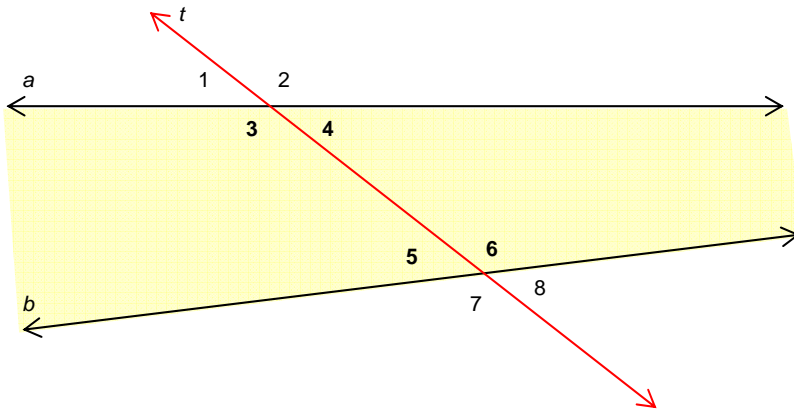
- $\angle 1$ and $\angle 5$ Location: left of the transversal; above the lines
- $\angle 2$ and $\angle 6$ Location: right of the transversal; above the lines
- $\angle 3$ and $\angle 7$ Location: left of the transversal; below the lines
- $\angle 4$ and $\angle 8$ Location: right of the transversal; below the lines

exterior angles – When two lines are cut by a transversal, the exterior angles are the angles that are located within the exterior of the lines.



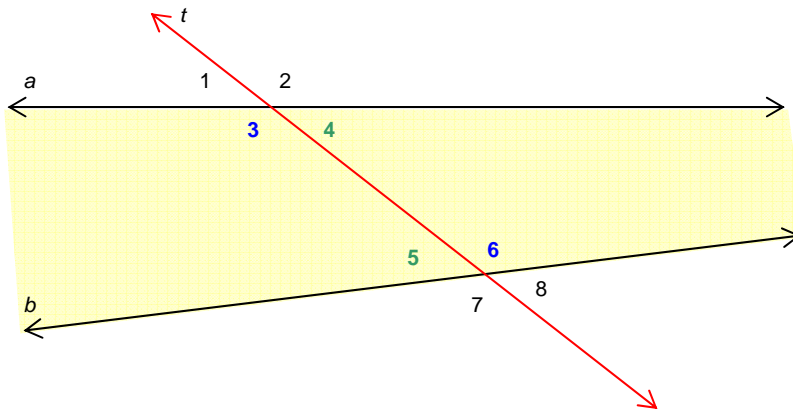
Exterior Angles: $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$

interior angles – When two lines are cut by a transversal, the interior angles are the angles that are located within the interior of the lines.



Interior Angles: $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$

alternate interior angles – When two lines are cut by a transversal, the alternate interior angles are the angles that are located within the interior of the lines and on opposite sides of the transversal.

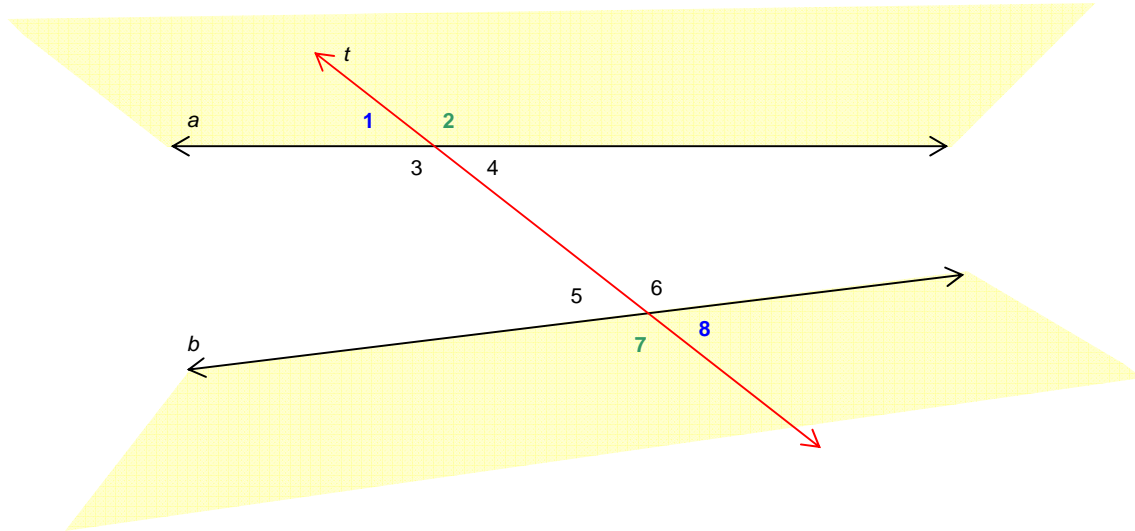


Alternate Interior Angles:

$\angle 3$ & $\angle 6$ Location: interior; opposite sides of transversal

$\angle 4$ & $\angle 5$ Location: interior; opposite sides of transversal

alternate exterior angles – When two lines are cut by a transversal, the alternate exterior angles are the angles that are located within the exterior of the lines and on opposite sides of the transversals.

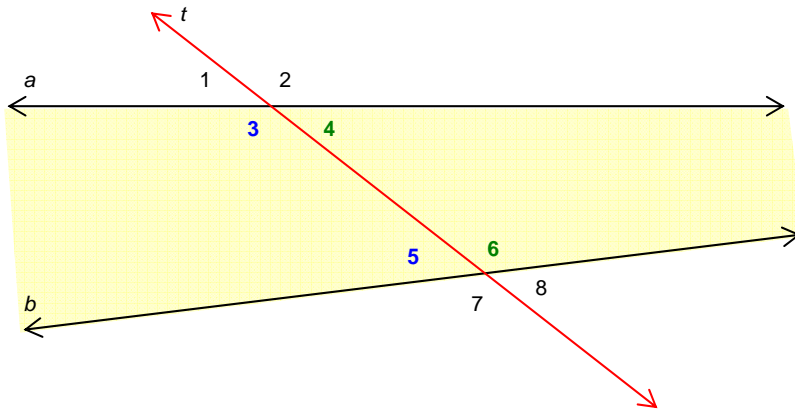


Alternate Exterior Angles:

$\angle 1$ & $\angle 8$ Location: exterior; opposite sides of transversal

$\angle 2$ & $\angle 7$ Location: exterior; opposite sides of transversal

consecutive interior angles – When two lines are cut by a transversal, the consecutive interior angles are the angles that are located within the interior and on the same side of the transversal.



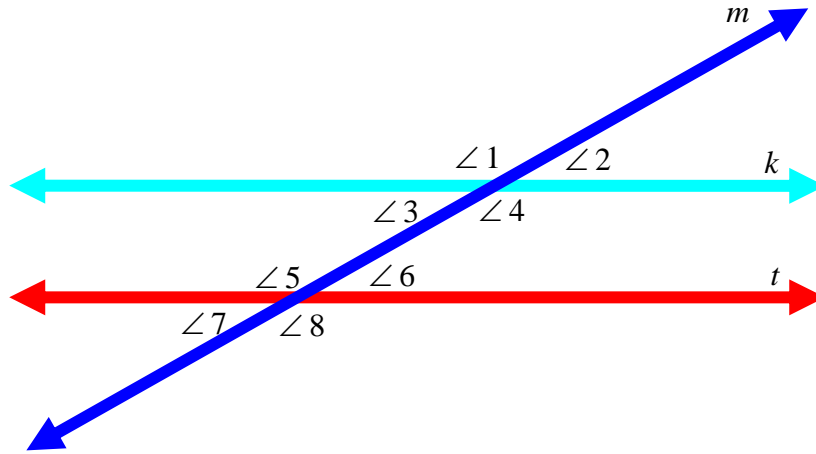
Consecutive Interior Angles:

$\angle 3$ & $\angle 5$ Location: interior; same side of transversal

$\angle 4$ & $\angle 6$ Location: interior; same side of transversal

Parallel Lines Cut by a Transversal

This picture is a classic example of two parallel lines (k and t) cut by a transversal (m).



Notice the position of the eight angles formed. There are many types of angles formed and given special names. We will discuss these various angles below.

Vertical Angles

Angles 1 and 4 are vertical angles. Vertical angles are congruent (equal in measure) and are the opposite angles formed when two lines intersect.

$$\angle 1 \cong \angle 4$$

Other vertical angles formed are:

$$\angle 2 \cong \angle 3$$

$$\angle 5 \cong \angle 8$$

$$\angle 6 \cong \angle 7$$

Corresponding Angles

Now take a look at angle 1 and compare it to angle 5.

These angles are in the same position along the transversal line. Their positions are corresponding, so we call them corresponding angles. Notice too, if they are corresponding, then they can also be shown to be congruent.

The corresponding angles are:

- $\angle 1 \cong \angle 5$ Location: **Left** of the transversal, **above** the parallel lines.
- $\angle 2 \cong \angle 6$ Location: **Right** of the transversal, **above** the parallel lines.
- $\angle 3 \cong \angle 7$ Location: **Left** of the transversal, **below** the parallel lines.
- $\angle 4 \cong \angle 8$ Location: **Right** of the transversal, **below** the parallel lines.

Alternate Interior Angles

Now consider angles 3 and 6. These angles are called alternate interior angles and you guessed it! They can also be shown to be congruent. They are named by their position; they are located on opposite sides of the transversal and within the interior of the parallel lines.

The alternate interior angles are:

- $\angle 3 \cong \angle 6$
- $\angle 4 \cong \angle 5$

Supplementary Angles

Recall that supplementary angles are angles that together form a straight line (or total 180°).

There are lots of supplementary angles formed when parallel lines are cut by a transversal.

Some supplementary angles formed:

*Note: " $m\angle 1$ " is read "the measurement of angle 1".

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$m\angle 1 + m\angle 3 = 180^\circ$$

$$m\angle 3 + m\angle 5 = 180^\circ$$

$$m\angle 6 + m\angle 8 = 180^\circ$$

You can check the sum of angles 3 and 5 by using thin paper and tracing the angles. After tracing the angles, slide them together to see if they form a straight line with their outer rays.

This ends the discussion of parallel lines cut by a transversal. You may be able to spot this classic geometric setting in the real world around you. Look around to see!


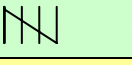


Constructing a Circle Graph

In a survey Madison determined her classmates' favorite colors based on the following choices: blue, green, yellow, red. She decided to display the results in a **circle graph**.

First she made a **tally** chart to record the responses.

She then made another chart to **organize** her calculations for drawing the graph.

Madison made fractions based on 20 (total responses) and changed them to percent. She then multiplied the percent by 360 since there are 360° in a circle to determine the central angle to represent each color.

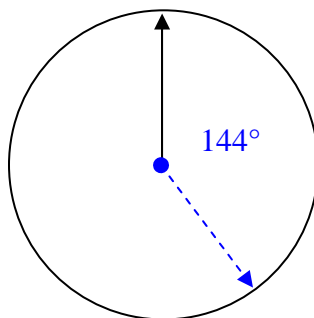
Tally of Favorite Color		
Blue		8
Green		5
Yellow		1
Red		6

Favorite Color		
Blue	$\frac{8}{20} = 40\%$	$40\% \times 360 = 144^\circ$
Green	$\frac{5}{20} = 25\%$	$25\% \times 360 = 90^\circ$
Yellow	$\frac{1}{20} = 5\%$	$5\% \times 360 = 18^\circ$
Red	$\frac{6}{20} = 30\%$	$30\% \times 360 = 108^\circ$
Check	$\frac{20}{20} = 100\%$	360°

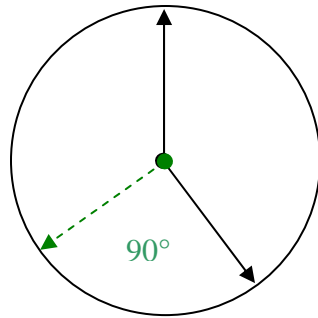
To check the calculations, Madison added the results. The fractions total $\frac{20}{20}$, the percents total 100% , and the degrees total 360° which makes **1 whole circle**.

Now Madison is ready to draw the circle graph.

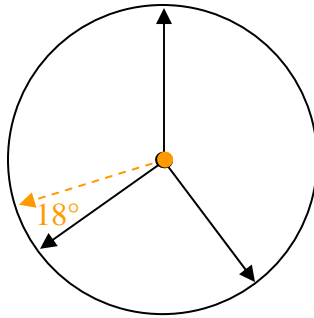
Step 1: Draw a circle with one radius. Place a protractor on the radius, using the center of the circle as the vertex, and draw the first angle (144° - blue).



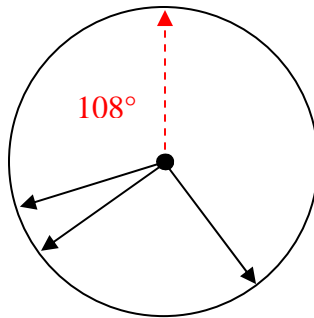
Step 2: Move the protractor to rest on the ray just drawn and proceed to make the second angle (90° -green).



Step 3: Then move the protractor to rest on the ray she drew and proceed to make the third angle (18° - yellow).



Step 4: Madison will not have to draw the last angle, but will measure it. It should measure 108° (red).



Now the circle has been divided proportionally to the percent for each category.
Complete the graph by adding color, category labels, and a title.

