# DECIMALS AND SCIENTIFIC NOTATION

In this unit, you will begin with a review of computations in decimals. You will also write decimals as fractions and vice versa. You will then examine numbers expressed in scientific notation by writing and comparing numbers expressed in scientific notation, and also computing with numbers in scientific notation.

## Adding and Subtracting Decimals

To add or subtract decimals, be sure to **line up the decimal points** so that the place values will also line up – tenths with tenths, hundredths with hundredths, and so on.

The answer to an **addition** problem is called the **sum**.

The answer to a **subtraction** problem is called the **difference**.

*Example 1*: Find the **sum**.

8.3 + 17.82 + 15 = ?

$$\begin{array}{r} 8.3\phantom{0} \\ 17.82 \\ +15.\phantom{00} \\ \hline 41.12 \end{array}$$

*Note:* 15 is a whole number so the decimal point is placed after the 15.

*Example 2*: Find the **difference**.

5.308 – 3.746 = ?

$$\begin{array}{r} \scriptsize{4\ \ 12\ 10} \\ \not{5}.\,\not{3}\,\not{0}\,8 \\ -\ 3.\,7\,4\,6 \\ \hline 1.\,5\,6\,2 \end{array}$$

*Example 3*: Find the **difference**.

12 – 5.3524 = ?

$$\begin{array}{r} \scriptsize{0\ 11\ \ 9\ 9\ 9\ 10} \\ \not{1}\not{2}.\,\not{0}\not{0}\not{0}\not{0} \\ -\ 5.\,3\,5\,2\,4 \\ \hline 6.\,6\,4\,7\,6 \end{array}$$

*Note:* 12 is a whole number so it is written 12.0000 (fill in the empty places with zeros).

*Example 4:* Find the **difference**.

8.6 – 2.953 = ?

$$\begin{array}{r} \scriptsize{7\ 15\ 9\ 10} \\ \not{8}.\,\not{6}\,\not{0}\,\not{0} \\ -\ 2.\,9\,5\,3 \\ \hline 5.\,6\,4\,7 \end{array}$$

Sometimes, especially when making computations with calculators, it is a very good idea to estimate to make sure the answers are reasonable.

*Check with estimation by rounding each decimal to the nearest whole number.*

| | | |
|---|---|---|
| 1. 8 + 18 + 15 = 41 | Actual Answer is 41.12. | Estimate is very close. |
| 2. 5 – 4 = 1 | Actual Answer is 1.562. | Estimate is close. |
| 3. 12 – 5 = 7 | Actual Answer is 6.6476. | Estimate is close. |
| 4. 9 – 3 = 6 | Actual Answer is 5.657. | Estimate is close. |

## Multiplying Decimals

### Multiplying Decimals Less Than One

The answer to a multiplication problem is called the product.

To place the decimal point when multiplying decimals, count the places to the right of the decimal point in each factor and total them. This total is the number of decimal places that will be in the product.

*Note: All of these problems involve decimals less than one; thus, a zero is used as a placeholder for the whole number.

**Multiply 0.7 × 0.9**

*Estimate: 1 × 1 = 1*

one decimal place → **0.7**

one decimal place → **× 0.9**

*total* – two decimal places → **0.63**

**Why two places?**
*Write both as fractions and multiply.*

$$\frac{7}{10} \times \frac{9}{10} = \frac{63}{100} = 0.63$$

**Answer: 0.63 = 63 hundredths**

*Check: The estimate 1 is close to 0.63.*

**Multiply 0.12 × 0.36**

*Estimate: 0 × 0 = 0*

two decimal places → **0.12**

two decimal places → **× 0.36**

**72**

**36**

*total* – four decimal places → **0.0432**

**Why four places?**
*Write both as mixed fractions and multiply.*

$$\frac{12}{100} \times \frac{36}{100} = \frac{432}{10000} = 0.0432$$

*Use 0 as a placeholder before the numerals to give 4 decimal places.*

**Answer: 0.0432 = 432 ten thousandths**

*Check: The estimate 0 is close to 0.0432.*

## Multiplying Decimals with Whole numbers

**Multiply 5.23 × 7.9**

*Estimate: 5 × 8 = 40*

| | |
|---|---|
| two decimal places → | 5.23 |
| one decimal place → | × 7.9 |
| | 4707 |
| | 3661 |
| *total* – three decimal places → | 41.317 |

**Answer: 41.317 = 41 and 317 thousandths**

*Check: The estimate 40 is close to 41.317.*

**Why three places?**
*Write both as mixed fractions and multiply.*

$$5\frac{23}{100} \times 7\frac{9}{10} =$$

$$\frac{523}{100} \times \frac{79}{10} = \frac{41317}{1000} =$$

$$41\frac{317}{1000} = 41.317$$

**Multiply 46 × 2.8**

*Estimate: 50 × 3 = 150*

| | |
|---|---|
| zero decimal places → | 46 |
| one decimal place → | × 2.8 |
| | 368 |
| | 92 |
| *total* – one decimal place → | 128.8 |

**Answer: 128.8 = 128 and 8 tenths**

*Check: The estimate 150 is close to 128.8.*

**Why one decimal place?**
*Write both as mixed fractions and multiply.*

$$\frac{46}{1} \times 2\frac{8}{10} =$$

$$\frac{46}{1} \times \frac{28}{10} = \frac{1288}{10} = 128.8$$

## Multiplying Mixed Decimals

**Multiply 5.23 × 3.79**

*Estimate: 5 × 4 = 20*

```
two decimal places  →        5.23
two decimal places  →    ×   3.79
                             4707
                            3661
                           1569
total – four decimal places →  19.8217
```

**Why four places?**

*Write both as mixed fractions and multiply.*

$$5\frac{23}{100} \times 3\frac{79}{100} =$$

$$\frac{523}{100} \times \frac{379}{100} = \frac{198217}{10000} =$$

$$19\frac{8217}{10000} = 19.8217$$

**Answer: 19.8217 = 19 and 8217 ten thousandths**

*Check: The estimate 20 is close to 19.8217.*

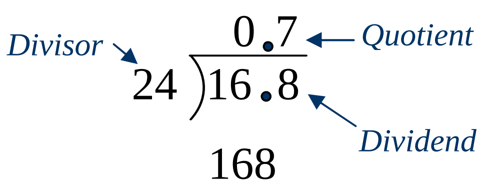
# Dividing Decimals

## Dividing Decimals by Whole Numbers

To divide a decimal by a whole number, **place the decimal point** in the ***quotient*** right **above the decimal** in the ***dividend***.

**Divide 16.8 by 24**

*Estimate: 20 ÷ 20 = 1*




*Check: The estimate 1 is close to 0.7.*

## Dividing By Decimals in Tenths

To divide a decimal by a decimal number, make the divisor a whole number by multiplying it by the power of ten needed to **move the decimal to the right of all of the digits in the divisor**. Then multiply the dividend by the same power of ten.

**Divide 29.24 by 3.4**

*Estimate: 30 ÷ 3 = 10*

*The shortcut to multiply by ten is to move the decimal point right one place.*

*Place the decimal point first*

$3.4\overline{)29.24}$

*5 Steps for Division*

**Divide** 34 into 292 to get 8
**Multiply** 8 × 34 to get 272
**Subtract** 272 from 292 to get 20
**Compare** 20 with 34
(20 must be smaller than 34)
**Bring Down** 4

*Divide, Multiply, Subtract, Compare*

```
        8.
 34.)292.4
    272
     20
```

*Bring down and start over*

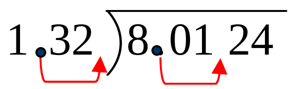
```
        8.6
 34.)292.4
    272
     20 4
     20 4
```

*Check: The estimate 10 is close to 8.6.*

## Dividing By Decimals in Hundredths

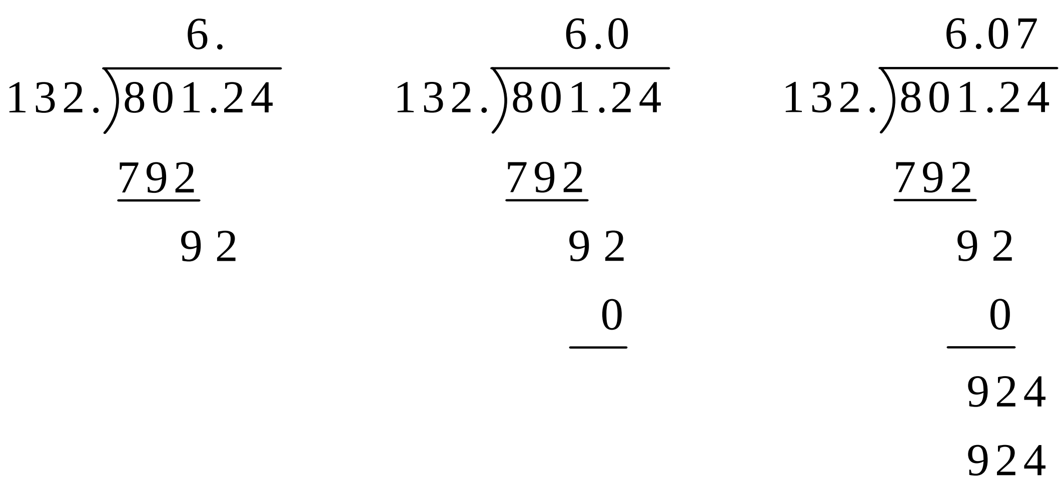
To divide a decimal by a decimal number, make the divisor a whole number by multiplying it by the power of ten needed to **move the decimal to the right of all of the digits in the divisor**. Then multiply the dividend by the same power of ten.

**Divide 8.0124 by 1.32**




$$\frac{8.0124}{1.32} \times \frac{100}{100} = \frac{801.24}{132}$$

Zero is needed as a placeholder since 92 is not large enough to be divided by 132.

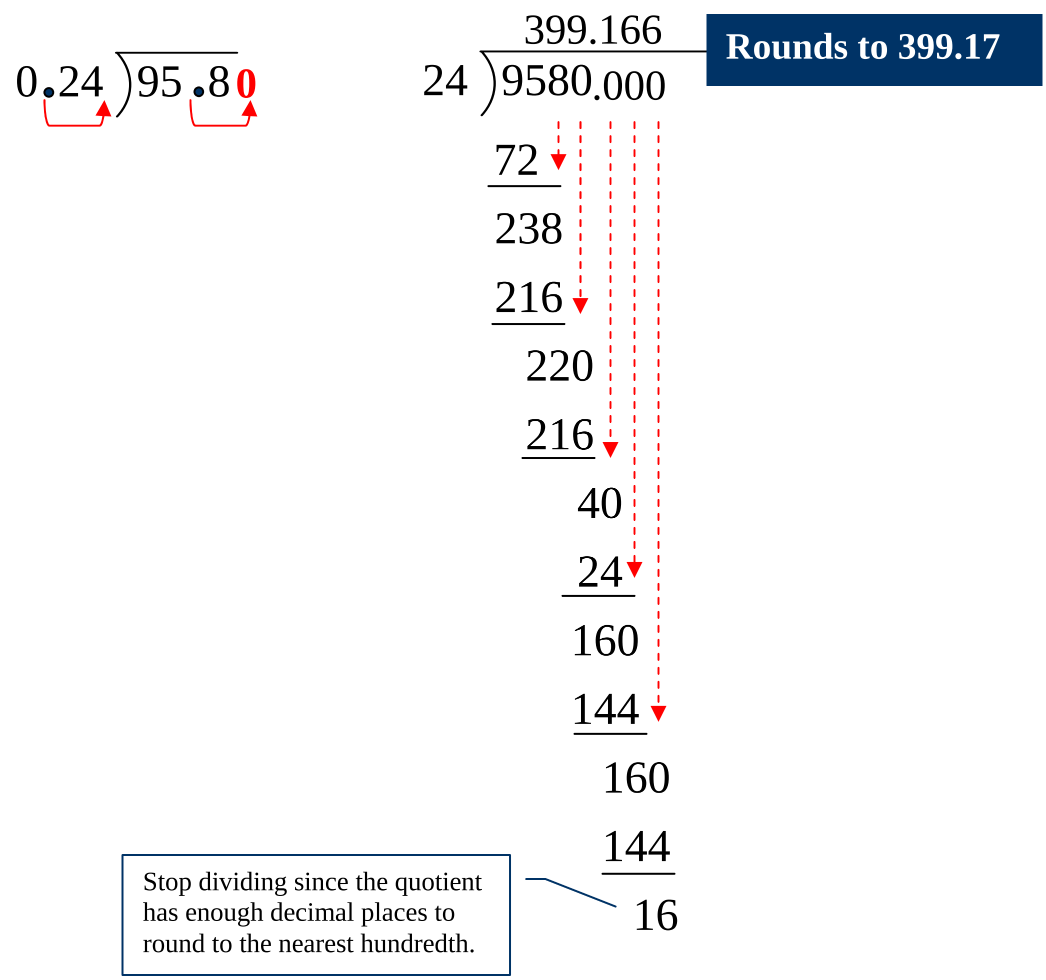


## Dividing Decimals and Rounding Quotients

In division, sometimes the answer does not come out even. That's when we divide until the quotient has one extra decimal place, then round to the given place.

**Divide 95.8 by 0.24**

***Round the quotient to nearest hundredth.***



## Decimals to Fractions and Vice Versa

### Decimals to Fractions

Decimals may be written as fractions and simplified, if necessary.

*Example 1*: Express 4.53 as a mixed fraction and simplify, if necessary.

4.53 is read as 4 and 53 hundredths and can be written as $4\frac{53}{100}$.

Thus, $4.53 = 4\frac{53}{100}$

**Note*: Two decimal places gives two zeros in the denominator of the fraction.

*Example 2*: Express 7.5 as a mixed fraction and simplify, if necessary.

7.5 is read as 7 and 5 tenths and can be written as $7\frac{5}{10}$.

*Note: One decimal place gives one zero in the denominator of the fraction.

$7\frac{5}{10} \div \frac{5}{5} = 7\frac{1}{2}$ Simplify the fraction.

Thus, $7.5 = 7\frac{1}{2}$

*Example 3*: Express 6.250 as a mixed fraction and simplify, if necessary.

6.250 is read as 6 and 250 thousandths and can be written as $6\frac{250}{1000}$ or $6\frac{1}{4}$.

*Note: Three decimal places gives three zeros in the denominator of the fraction.

$6\frac{250}{1000} \div \frac{10}{10} = 6\frac{25}{100} \div \frac{25}{25} = 6\frac{1}{4}$ Simplify the fraction.

Thus, $6.250 = 6\frac{1}{4}$

## Fractions to Decimals

To write a fraction as a decimal, divide to find the decimal.

*Example 1*: Find the terminating decimal for 3/4.

To find the decimal for $\frac{3}{4}$, divide the denominator into the numerator, and then add a decimal point and zeros to divide until it comes out even (terminates).

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \end{array}$$

$$\frac{3}{4} = 0.75$$

Some fractions do not produce decimals that come out even; but, instead continue on forever in a repeating pattern. For this type of fraction, divide and round to the given place.

*Example 2*: Find the decimal for 2/3 and round to the nearest hundredth.

To find the decimal for $\frac{2}{3}$, divide to get one extra decimal place for rounding. In this case, since the directions are to round to the nearest hundredth, divide up through thousandths, and then round.

$$\begin{array}{r} .666 \\ 3\overline{)2.000} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$$\frac{2}{3} \approx 0.67$$

Another way to represent repeating decimals is to write the repeating pattern with a bar above it.

Two - thirds may be written as $0.\overline{6}$.

An answer such as 0.18181818... may be written as $0.\overline{18}$.

## Scientific Notation

### Standard Form for Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:

- a number that is less than 10, but greater than or equal to 1
- a power of ten

**An example of a number in scientific notation is $5.6 \times 10^4$.**

$$\mathbf{5.6 \times 10^4 = 56{,}000}$$

Let's express a number given in scientific notation as a standard number.

*Example 1*: Find the standard notation for $7 \times 10^4$.

*Solution*: $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

*Therefore*, $7 \times 10{,}000 = 70{,}000$

The standard notation for $7 \times 10^4$ is 70,000.

*Example 2*: Find the standard notation for $8.4 \times 10^5$.

*Solution*: $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$

*Therefore*, $8.4 \times 100{,}000 = 840{,}000$

The standard notation for $8.4 \times 10^5$ is 840,000.

A shortcut to multiply by a power of ten is to start at the decimal point's location and move it **right** as many places as the given power. Add zeros as needed.

$8.4 \times 10^5 \rightarrow 840000 \rightarrow 840{,}000.$

## Expressing Standard Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:

- a number that is less than 10, but greater than or equal to 1
- a power of ten

**An example of a number in scientific notation is $6 \times 10^4$.**

$$60{,}000 = 6 \times 10^4$$

*Example 3*: Find the scientific notation of 8,000,000.

Look at the first nonzero digit(s) of the number and write it as a **number between 1 and 10**. Place a decimal point after the first digit.

*In this example a zero is used as a placeholder since there is only one nonzero digit.*

$8.0 \times 1{,}000{,}000$

$8.0 \times 10^6$

Multiply the number by a **power of ten** that would equal the standard number.

Express the power of ten in exponential form.

A shortcut for finding the **exponent** is to **count** the decimal places from **after the first digit** to the **end of the number**.

$8{,}000{,}000 \rightarrow 8\underbrace{000000} \rightarrow 8.0 \times 10^6$

$$8{,}000{,}000 = 8.0 \times 10^6$$

**Another example of a number in scientific notation is $2.5 \times 10^4$.**

$$25{,}000 = 2.5 \times 10^4$$

*Example 4*: Find the scientific notation for 673,000.

Look at the first nonzero digit(s) of the number and write them as a **number between 1 and 10**. Place a decimal point after the first digit.

$6.73 \times 100{,}000$

$6.73 \times 10^5$

Multiply the number by a **power of ten** that would equal the standard number.

Express the power of ten in exponential form.

A shortcut for finding the **exponent** is to **count** the decimal places from **after the first digit** to the **end (right) of the number**; thus, a positive exponent is used.

$673{,}000 \rightarrow 6\underline{73000} \rightarrow 6.73 \times 10^5$

$$\mathbf{673{,}000 = 6.73 \times 10^5}$$

**Another example of a number in scientific notation is $3.47 \times 10^{-3}$.**

**$0.00347 = $ is $3.47 \times 10^{-3}$**

*Example 5*: Find the scientific notation for 0.0000295.

Look at the first nonzero digit(s) of the number and write them as a **number between 1 and 10.** Place a decimal point after the first digit.

**2.95** $\times$ **0.00001**

**2.95** $\times$ $10^{-5}$

Multiply the number by a **power of ten** that would equal the standard number.

Express the power of ten in exponential form.

A shortcut for finding the **exponent** is to **count** the decimal places from **after the first nonzero digit** back (left) to the original placement of the decimal; thus, a negative exponent is used.

$$0.0000295 \rightarrow 0.0000295 \rightarrow 2.95 \times 10^{-5}$$

**$0.0000295 = 2.95 \times 10^{-5}$**

# Computing in Scientific Notation

## Scientific Notation

Please recall that numbers in scientific notation have two parts:

a.) One part is a value between 1 and 10.

b.) The other part is a factor of ten.

When the two parts are multiplied, the result is equal to the original number.

Consider the following number which is expressed in scientific notation: $2.334 \times 10^5$.

The first part, 2.334, is a number between 1 and 10. $\left[2\frac{334}{1000}\right]$

The second part is $10^5$, a factor of 10.

The number could be written 233,400.

When we write a number in scientific notation, we must be consistent to keep one digit (not zero) in the ones place. This allows us to compare the place value by looking at the exponent (power of ten) only.

A number like $4.34 \times 10^5$ will be larger than $4.34 \times 10^4$ because the five places the number in a larger place value.

A number like $8.98 \times 10^4$ will be smaller than $7.9 \times 10^6$ because the six places the number in a larger place value.

## Using Scientific Notation to Multiply

Using scientific notation will also make some calculations easier (especially multiplication and division).

*Example 1*: Consider 4,250,000,000 times 3,000.

To enter this into a normal handheld calculator would be impossible. So, we'll try it this way.

*Step 1*: Write each number in scientific notation.

4,250,000,000 times 3,000 = $(4.25 \times 10^9)(3 \times 10^3)$

*Step 2*: Multiply the values (first parts).

$4.25 \times 3 = 12.75$

*Step 3*: Multiply the powers of ten.

$10^9 \cdot 10^3 = 10^{12}$

*Note: Since this is exponent multiplication and the base is the same (10), we only have to add the exponents. Nine factors of ten and 3 factors of ten make 12 factors of ten or $10^{12}$.

*Step 4*: Put the two parts together.

$12.75 \times 10^{12}$

The answer is correct, but not in scientific notation (The first part must fall between one and ten.)

*Step 5*: We must move the decimal…

$12.75 \rightarrow 1.275$ smaller by one place value (ie. divide by 10)

*Step 6*: We will adjust the 10's…

$10^{12} \rightarrow 10^{13}$ which is larger by one place value (ie. multiply by 10)

Multiplying by 10 in *Step 6* balances dividing by 10 in *Step 5*; and thus, the value of the answer remains the same.

By adjusting one part of the number up and one part down, we have balanced and kept the answer the same while being correct with our scientific notation rules.

*Step 7*: Write the answer in scientific notation.

$1.275 \times 10^{13}$

Summarizing our work:

| | | |
|---|---|---|
| 4,250,000,000 times 3,000 | = | $(4.25\times10^{9})(3\times10^{3})$ |
| 4,250,000,000 times 3,000 | = | $12.75\times10^{12}$ |
| 4,250,000,000 times 3,000 | = | $1.275\times10^{13}$ |
| 4,250,000,000 times 3,000 | = | 12,750,000,000,000 |

*Example 2*: Consider $(4.5\times10^{5})\,(7.3\times10^{3})$.

*Step 1*: Numbers… $4.5 \times 7.3 = 32.85$

*Step 2*: Powers… $10^{5}\times10^{3} = 10^{8}$

*Step 3*: Answer… $32.85\times10^{8}$

*Step 4*: Adjust power and decimal for scientific notation… $3.285\times10^{9}$

$(4.5\times10^{5})\,(7.3\times10^{3}) = 3,285,000,000$

## Using Scientific Notation to Add

When we try addition (or subtraction), it is important to note the powers of ten should match. This way the short cut will work.

*Example 3*: Consider $4.1\times10^{4} + 5.3\times10^{4}$.

*Note: the powers are the same and do not need adjusted.

*Step 1*: Numbers first… $4.1 + 5.3 = 9.4$

*Step 2*: Powers next, but since we are adding, the place value will be the same, so the power gets copied… $10^{4}$

*Step 3*: Put the two parts together… $4.1\times10^{4} + 5.3\times10^{4} = 9.4\times10^{4}$

*Note: This answer does not need adjusted as it is in scientific notation.

$4.1\times10^{4} + 5.3\times10^{4} = 94,000$

## Using Scientific Notation to Subtract

*Example 4*: Consider $6.8\times10^{7} - 2.6\times10^{7}$.

*Note: the powers are the same and do not need adjusted.

*Step 1*: Numbers first… $6.8 - 2.6 = 4.2$

*Step 2*: Powers next, but since we are subtracting, the place value will be the same, so the power gets copied… $10^{7}$

*Step 3*: Put the two parts together… $6.8\times10^{7} - 2.6\times10^{7} = 4.2\times10^{7}$

*Note: This answer does not need adjusted as it is in scientific notation.

$6.8\times10^{7} - 2.6\times10^{7} = 42,000,000$

## Using Scientific Notation to Divide

*Example 5*: Consider $\frac{9.42\times10^{16}}{3\times10^{12}}$ which may also be written as $9.42\times10^{16} \div 3\times10^{12}$.

*Step 1*: Numbers… $9.42 \div 3 = 3.14$

*Step 2*: Powers… $10^{16} \div 10^{12} = 10^{4}$

*Note: Since this is exponent division and the base is the same (10), we only have to subtract the exponents.

$$\frac{\cancel{10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10}\times10\times10\times10\times10}{\cancel{10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10\times10}} = \frac{10\times10\times10\times10}{1} = 10^{4}$$

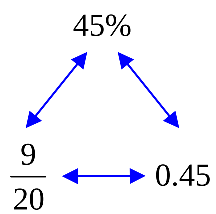
*Step 3*: Answer… $3.14\times10^{4}$

*Step 4*: Putting it all together… $\frac{9.42\times10^{16}}{3\times10^{12}} = 3.14\times10^{4}$

*Note: This answer does not need adjusted as it is in scientific notation.

## Comparing Real Numbers

Percents, fractions, and decimals may all be used to represent the same quantity.  Let's take a look at how to apply this connection.




$$45\% = 0.45 = \frac{9}{20}$$

*Example 1*:  Rearrange the given numbers in order from least to greatest.

$\frac{3}{4}$, 15 out of 16, 0.075, 79%

To solve, express each number as a decimal and then compare.

| | |
|---|---|
| $\frac{3}{4}$ = | 0.75 |
| 15 out of 16 = 15 ÷ 16 = | 0.9375 |
| 0.075 | No change |
| 79% = | 0.79 |

List the decimals and add enough zeros to compare.

0.7500
0.9375
0.0750
0.7900

Least to greatest as decimals:  0.0750, 0.7500, 0.7900, 0.9375

Least to greatest as original numbers:  0.075, $\frac{3}{4}$,  79%,  15 out of 16

Now, let's look at another example that includes negative numbers.

*Example 2*: List the numbers shown below in order from greatest to least:

$$\sqrt{19},\ 65\%,\ -4\frac{2}{3},\ -7.5\times10^{-1}$$

To make a comparison, change each number to a decimal and then line up the decimal points.

| | | |
|---|---|---|
| $\sqrt{19}$ | = | 4.3588989435... |
| 65% | = | 0.65 (65% means $\frac{65}{100}$) |
| $-4\frac{2}{3}$ | = | –4.6666... |
| $-7.5\times10^{-1}$ | = | –0.75 |

1) There are two negative numbers. Negative numbers are smaller than positive numbers.

2) –4.666… is further left on the number line than –0.75; thus, –4.666 is smaller than –0.75.

$$-4.666... < -0.75 \text{ or } -0.75 > -4.666...$$

3) For the two positive numbers, look at each whole number before the decimal. You can see that 4 is greater than 0; therefore 4.35… is greater than 0.65.

$$4.35... > 0.65$$

4) Next, use the decimal numbers and line them up from greatest to least:

$$4.35,\ 0.65,\ -0.75,\ -4.666...$$

5) Now, refer back to the original numbers to state the final answer.

$$\sqrt{19},\ \ 0.65,\ \ -7.5\times10^{-1},\ -4\frac{2}{3}$$

6) Look at the number line below to find the location of each number. As you go right on the number line, the numbers get larger; and as you go left on the number line, the numbers get smaller.