Course Overview

In this course students connect physical, verbal, and symbolic representations of the real number system; investigate properties including closure; estimate, compute, solve, and judge reasonableness of problems with real numbers including ratio, proportion, percent, integers, rational numbers, numbers expressed in scientific notation, and square roots of perfect and non-perfect squares. They generalize patterns and sequences and apply formulas to real-world problem situations. Students solve and graph linear equations and inequalities; compute polynomials; define functions; determine slope and intercepts; draw graphs of linear equations and inequalities; and explore simple quadratic equations. They graph solutions to equations; use coordinate geometry to analyze properties of two-dimensional figures and perform translations, reflections, rotations, and dilations; define basic trigonometric ratios in right triangles; and apply proportions to solve problems involving right triangle trigonometry. Students apply direct and indirect measurement techniques, tools, and derivation of formulas to determine perimeter, area, volume, and various attributes of plane and solid geometric figures. They use measures of center and spread to analyze data; evaluate the change of data and display it appropriately in graphs; make predictions based on samples representative of a larger population; use permutations and combinations to calculate the number of possible outcomes recognizing repetition and order; and compute the probability of compound events, independent events, and simple dependent events.

In some of the units, you will notice three links in the upper left corner of the page. The first link, "PDF File"*, is a link to a printable file that has all unit topics combined in one file. The second link, "Practice Problems", is a link to a PDF file* that provides several problems for practice. Once you open this file, click on the "Bookmarks" tab located on the upper left side of the file. You will be able to view the various problems provided in the review and to select the types of problems that you would like to practice before progressing to the "Questions" area of the unit. Answers to the practice problems are provided on the last page of the PDF file. The third link, "Practice Worksheet with Hints", is a PDF file* that provides the same problems that are given in the "Practice Worksheet" file; however, hints are provided to assist you with solving the problems.

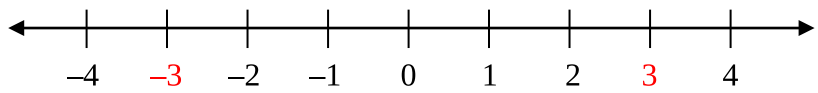
INTEGERS

Unit Overview

In this unit, you will learn about integers and the rules that apply to adding, subtracting, multiplying, and dividing these special numbers. You will also work with absolute value, a value that represents a number's distance from zero, and comparing integers. You will apply your knowledge of integers to solve problem scenarios.

Comparing Integers

The set of whole numbers consists of 0, 1, 2, 3 … and can be represented on a number line. We can match each whole number with another number that is the same distance from 0 but on the opposite side of 0.



If you take a look at the number line above, 3 or (positive 3) and –3 (negative 3) are on opposite sides of 0 but the same distance from 0. These numbers are called opposites and make up the set of **integers.** Integers are the set of positive and negative whole numbers.

Examples: Name the integer that is suggested by each situation.

a) The temperature is 5° below 0.

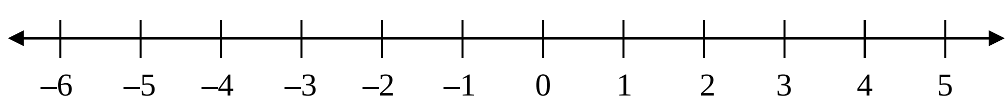
–5 Below 0 suggests a negative integer.

b) Emily's lemonade stand made a $24 profit.

24 A profit suggests a positive integer.

To compare integers, we will use the symbol "<" which means less than or the symbol ">" which means greater than. If you remember from the previous unit, these symbols are called **inequality** signs. An inequality can either be true or false. For example, the sentence 12 > 8 is true and the sentence 6 > 9 is false.

On a number line the numbers increase as you move from left to right. For any two numbers, the number that is farther to the right is the larger number and the number farther to the left is the lesser number. Let's take a look at comparing some integers using the number line below.



Write a true sentence using $<$ or $>$ in place of $\square$.

a) $3 \square 9 \quad \Rightarrow \quad 3 < 9$

Since 3 is to the left of 9 on the number line, 3 is less than 9.

b) $-5 \square 11 \quad \Rightarrow \quad -5 < 11$

Since –5 is to the left of 11 on the number line, –5 is less than 11.

c) $-3 \square -6 \quad \Rightarrow \quad -3 > -6$

Since –3 is to the right of –6 on the number line, –3 is greater than –6.

Hint: An easy way to remember the direction of the inequality sign is that it will always point to the smaller number.

If you study the number line above, you will notice that the integers 5 and –5 are the same distance from 0. This brings us to another algebraic concept called **absolute value**. The absolute value of a number is the distance it is from 0, which means that the absolute value will always be **positive**. Absolute value is symbolized by using two straight bars around a number.

Example 1: $|24|$ means the absolute value of 24, which is 24 because 24 is 24 units away from 0.

Example 2: $|-57|$ means the absolute value of –57, which is 57 because –57 is 57 units away from 0.

Absolute Value

Absolute value of an integer is the distance the integer is from zero.

5 units → ← *5 units*

{…–8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8...}

Absolute value is represented by two vertical bars around the number.

The absolute value of –5 is 5 because –5 is 5 units from 0. $|-5| = 5$

The absolute value of 5 is 5 because 5 is 5 units from 0. $|5| = 5$

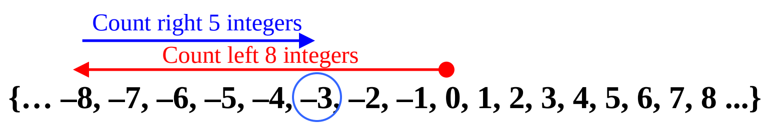
Introduction to Adding Integers

$$\{\ldots -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8 \ldots\}$$

Integers are used to show positive and negative quantities. When adding integers, start at 0 and count left for negative values or count right for positive values.

Example 1: Solve: –8 + 5

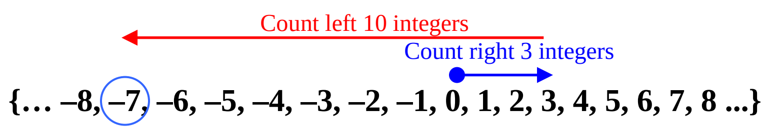
Start at 0 and count left for negative 8, and then count right for positive 5 from negative 8. The final answer is –3.



$$-8 + 5 = -3$$

Example 2: Solve: 3 + (–10)

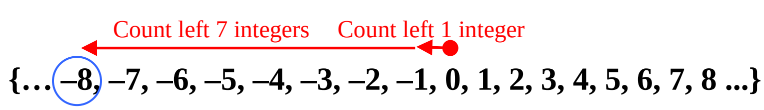
Start at 0 and count right for positive 3, and then count left for negative 10 from positive 3. The final answer is –7.



$$3 + (-10) = -7$$

Example 3: Solve: –1 + (–7)

Start at 0 and count left for negative 1, and then count left for negative 7. The final answer is –8.



$$-1 + (-7) = -8$$

Rule: When adding integers with the **same sign**, **add** and express the answer with the **same sign**.

$$6 + 4 = 10 \qquad -1 + (-7) = -8$$

Rule: When adding integers with **different signs**, **subtract** and express the answer with the **sign of the integer farthest from zero**.

$$-8 + 5 = (-3) \qquad 3 + (-10) = -7$$

Introduction to Subtracting Integers

{… –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8 ...}

Integers are used to show positive and negative quantities. For subtracting integers, let's look at the ***difference*** between two integers.

Example 1: Solve: 7 – (–4)

Locate the 7 and the –4. What is the *difference* between the two numbers? Counting right from –4 to 7, the difference is 11.

11

{… –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8 ...}

$$7 - (-4) = 11$$

Example 2: Solve: –8 – 5

Locate the –8 and the 5. What is the *difference* between the two numbers? Counting left from 5 to –8, the difference is –13.

-13

{… –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8 ...}

$$-8 - 5 = -13$$

Example 3: Solve: –4 – (–7)

Locate the –4 and the –7. What is the *difference* between the two numbers? Counting right from –7 to –4, the difference is 3.

3

{… –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8 ...}

$$-4 - (-7) = 3$$

A rule for subtracting integers is to "add the opposite".

The Problem	Switch to addition. Change the second number to its opposite.	Follow the rules for adding integers.	State the final answer.
7 – (–4) =	7 + 4 =	7 + 4 = 11	7 – (–4) = 11
–8 – 5 =	–8 + (–5) =	–8 + (–5) = –13	–8 – 5 = –13
–4 – (–7) =	–4 + 7 =	–4 + 7 = 3	–4 – (–7) = 3

Adding and Subtracting Integers

When adding and subtracting integers, it will be very helpful to understand the following rules:

Adding Integers

Adding

Same sign: If the numbers have the same sign, add the numbers and keep the sign given.

Example 1: (–5) + (–9) = –14 Both integers are negative, so the answer is negative.

Example 2: 6 + 15 = 21 Both integers are positive, so the answer is positive.

Different signs: If the numbers have different signs, subtract the absolute value of the numbers and use the sign of the larger absolute value.

Example 3:	(–9) + 3 = –6	Subtract 9 and 3 to get 6, and then determine that –9 has the larger absolute value; so, you will use the negative sign.
Example 4:	17 + (–4) = 13	Subtract 17 and 4 to get 13, and then determine that 17 has the larger absolute value; so, you will use the positive sign, or in this case no sign at all represents positive.

Subtracting Integers

Subtracting

Change the problem to addition and add the opposite of the second number. Then go to addition rules.

Example 5:	15 – (–8) =	
	15 + (–8) =	change the problem to addition
	15 + (+8) =	change (–8) to a (+8)
	23	Add the numbers because they now have the same sign.

Example 6:	18 – 26 =	
	18 + (26) =	change the problem to addition
	18 + (–26) =	change 26 to (–26)
	–8	Subtract the numbers because they now have different signs and use the sign of the larger absolute value.

Introduction to Multiplying Integers

To multiply integers, we will first look at a number pattern.

$$
\begin{aligned}
7 &\times 4 = 28\\
7 &\times 3 = 21\\
7 &\times 2 = 14\\
7 &\times 1 = 7\\
7 &\times 0 = 0\\
7 &\times (-1) = -7\\
7 &\times (-2) = -14\\
7 &\times (-3) = -21
\end{aligned}
$$

In the pattern, each time 7 is multiplied by one less number the resulting answer is 7 less.

At 7×0, 0 the product is 7 less than 7.
At $7\times(-1)$, –7 the product is 7 less than 0.
At $7\times(-2)$, –14 the product is 7 less than –7.

…and the pattern continues on.

When multiplying integers with different signs, multiply the numbers and the answer will be negative.

$7 \times (-3) = -21$ $\qquad$ $-2 \times 7 = -14$

Now we will examine a pattern that shows how a "negative number times a negative number" becomes a positive number.

$$
\begin{aligned}
-3 &\times 7 = -21\\
-3 &\times 6 = -18\\
-3 &\times 5 = -15\\
-3 &\times 4 = -12\\
-3 &\times 3 = -9\\
-3 &\times 2 = -6\\
-3 &\times 1 = -3\\
-3 &\times 0 = 0\\
-3 &\times (-1) = 3\\
-3 &\times (-2) = 6\\
-3 &\times (-3) = 9\\
-3 &\times (-4) = 12\\
-3 &\times (-5) = 15
\end{aligned}
$$

In the pattern, each time 3 is multiplied by one less number the resulting answer is 3 more.

At -3×0, 0 the product is 3 more than –3.
At $-3\times(-1)$, 3 the product is 3 more than 0.
At $-3\times(-2)$, 6 the product is 3 more than 3.
At $-3\times(-3)$, 9 the product is 3 more than 6.

…and the pattern continues on.

When multiplying integers with same signs, multiply and the answer will be positive.

$-3 \times -5 = 15$ $\qquad$ $4 \times 3 = 12$

Introduction to Dividing Integers

To divide integers, we will look at the multiplication, and then write a **related** division statement.

$$-7 \times 8 = -56, \text{ therefore } -56 \div 8 = -7$$
$$-7 \times -8 = 56, \text{ therefore } 56 \div (-8) = -7$$

When dividing integers with different signs, divide and the answer will be negative.

$$48 \div (-6) = -8 \qquad -72 \div 9 = -8$$

$$7 \times 8 = 56, \text{ therefore } 56 \div 8 = 7$$
$$7 \times -8 = -56, \text{ therefore } -56 \div (-8) = 7$$

When dividing integers with same signs, divide and the answer will be positive.

$$-64 \div (-8) = 8 \qquad 70 \div 10 = 7$$

Multiplying and Dividing Integers

The rules for multiplying and dividing integers are a little easier to remember because there are only two rules.

Multiplying & Dividing

Same sign: If the numbers have the same sign, the answer will be **positive**.

$(-4)(-5) = 20$ $\quad$ $(-35) \div (-7) = 5$

$(6)(3) = 18$ $\quad$ $(60) \div (10) = 6$

Different signs: If the numbers have different signs, the answer will be **negative**.

$(-4)(8) = -32$ $\quad$ $(-99) \div (9) = -11$

*When multiplying more than one number, it may be helpful to remember the following rule in determining the sign of your answer:

Even number of negative signs results in a **positive** answer.

Odd number of negative signs results in a **negative** answer.

Example 1: $(-4)(-3)(5)(-2)$

Since there are 3 negative signs, the answer to this will be negative.

Let's check it out by multiplying from left to right.

$(-4)(-3)(5)(-2)$

$(12)(5)(-2)$

$(60)(-2)$

-120

Example 2: (–2)(–4)(–3)(–2)

Since there are 4 negative signs, the answer to this will be positive.

Let’s check it out by multiplying from left to right.

(–2)(–4)(–3)(–2)

(8)(–3)(–2)

(–24)(–2)

48