APPLYING PERCENT

This unit is about applying the knowledge of percent to various types of everyday percent problems. When purchasing items, the price for some items increases because sales tax is added, based on the percent assessed for a particular locality. Shopping for items on sale is fun for a wise customer because discounts, based on a percent off, are subtracted from the original price. Savers and investors are interested in interest earned over time, based on varying percent rates provided through the bank or investment brokers, because they want their invested money to grow quickly. Purchasing large items over time is also a concern for buyers as they to find good interest rates (low percents) so that they can repay the amount borrowed in a reasonable amount of time. Some types of jobs pay on commission; that is, the more merchandise sold, the higher the take-home pay will be, based on a commission rate (percent).

Sales Tax

Income Tax

Commission

Discounts and Sale Prices

Simple Interest

Compound Interest

Sales Tax

When you make a purchase at the store, **sales tax** may be added to the total amount of the purchase. A sales tax is a percentage of the total sales and is collected on behalf of the state, county, or local government. The percentage of sales tax varies between states.

To find the amount of sales tax on your purchase, you will multiply the amount of your purchase by the decimal value of the percent. If you are trying to find the total amount of the purchase, then you want to **add** the sales tax to your purchase amount.

Example 1: Greg purchased a television set for $296.50. How much sales tax did he have to pay if the tax rate was 5%?

Think: 5% = 0.05

Purchase price × Rate = Sales tax

296.50 × 0.05 = 14.83

Greg will have to pay $14.83 sales tax for his purchase.

Example 2: Amy purchased a computer totaling $598.50 before tax was added. In her state, sales tax is 7%. Determine her final cost.

Step 1: Find the sales tax. (Round to the nearest cent.)

Think: 7% = 0.07

Purchase price × Rate = Sales tax

598.50 × 0.07 = 41.90

(41.8950 rounds to 41.90)

Step 2: Find the final cost of the purchase by adding the amount of sales tax to the purchase price.

598.50 + 41.90 = $640.40

Amy’s total price for her computer is $640.40.

Income Tax

Tax brackets are used to determine the amount of income tax a person in the United States will pay. The tax is determined by the following formula:

$$\text{tax} = \text{base tax} + (\text{tax rate} \times \text{amount over})$$

Example: Darren’s taxable income for 2005 was $46,500. He is single. Refer to the table below to determine the amount of income tax he owes.

*Note: The tax table shown below is a simulated model for single people only. There are other tax tables for married, head of household, and other possible situations.

Darren’s tax rate is highlighted since his income falls between $29,700 and $71,950.

2005 IRS Income Tax Brackets (Single)		
If taxable income is over--	**But not over--**	**The tax is:**
$0	$7,300	10% of the amount over $0
$7,300	$29,700	$730 plus 15% of the amount over 7,300
$29,700	$71,950	$4,090.00 plus 25% of the amount over 29,700
$71,950	$150,150	$14,652.50 plus 28% of the amount over 71,950
$150,150	$326,450	$36,548.50 plus 33% of the amount over 150,150
$326,450	no limit	$94,727.50 plus 35% of the amount over 326,450

Follow these steps to solve the problem.

- What was Darren’s taxable income? $46,500
- Look up his taxable income in the table and determine how he will be taxed:

  $4,090.00 plus 25% of the amount over 29,700

- What is Darren’s base tax? $4090

- What is the "amount over"?

$$\begin{array}{r} 46{,}500 \\ \underline{-29{,}700} \\ \$16{,}800 \end{array}$$

- What is Darren's tax for the "amount over"?

$$\begin{array}{l} 25\% \text{ of the amount over } 29{,}700 \\ 0.25 \quad \times \quad 16{,}800 \quad = \quad \$4{,}200 \end{array}$$

- What is Darren's total income tax due?

$4,090.00 plus 25% of the amount over 29,700

$$4{,}090 \qquad + \qquad 4{,}200 \qquad = \qquad \$8{,}290$$

To summarize the steps:

$$\begin{array}{l} \$4{,}090.00 \text{ plus } 25\% \text{ of the amount over } 29{,}700 \\ 4090 + 0.25(46{,}500 - 29{,}700) \\ 4090 + 0.25(16{,}800) \\ 4090 + 4200 \\ \$8290 \end{array}$$

Darren's income tax for 2005 was $8290.

*Note: This example shows how the income tax is calculated. In some instances all of these calculations are completed for the tax payer and provided in a table.

Commission

In some jobs, an employee’s pay depends on the amount of goods or services the person sells. The salesperson receives a **commission,** or specified amount of money for sales made during a pay period. Commission is usually expressed as a percentage of sales and has the purpose of encouraging salespeople to sell more goods or services. The percent of total sales paid as commission is the **commission rate**.

Example: Mr. Green sold $25,000 worth of computer equipment. His rate of commission is 1.4%. What is his commission on the sale?

Commission = Sales × Commission Rate

Step 1: Change the percent to a decimal.

1.4% = 0.014

Step 2: Multiply the sales by 0.014.

25,000 × 0.014 = 350.00

Mr. Green’s commission on his sales is $350.00.

Discounts and Sale Prices

A **discount**, or **markdown**, is the amount of money that you save by buying an item at a discounted price, or sale price. To find the discount or markdown when the percent of reduction is given, first express the percent as a decimal and multiply it by the regular price. The result will be the discount amount that will be subtracted from the original price.

Example 1: Annie purchased a sweater at Gaylord's. The regular price was $39.95. The markdown rate was 20%. What was her discount and sale price?

Step 1: Change 20% into **0.20**.

Step 2: Multiply the regular price by 0.20.

$$0.20 \times 39.95 = 7.99$$

Annie received a discount of $7.99 off the original price.

Step 3: Subtract 7.99 from the original price of 39.95.

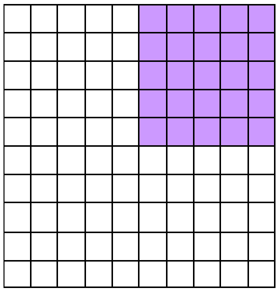
$$39.95 - 7.99 = \$31.96$$

Annie paid a sale price of $31.96 for the sweater.

Example 2: Lyle went shopping with his mom and found a jacket that was on sale and marked **25% off**. The cost of the jacket was approximately $40.00. He wanted to figure the savings quickly.

Here is how Lyle figured his discount.

From math class, he remembered that 25% can be written as $\frac{25}{100}$, and then simplified to $\frac{1}{4}$.



He multiplied $\$40 \times \frac{1}{4}$ and figured that the discount was $10.

He subtracted the discount from the original price to get the sale price.

$$\frac{1}{4} \text{ of } 40 =$$
$$\frac{1}{4} \times \frac{40}{1} =$$
$$\frac{1}{\cancel{4}_1} \times \frac{\cancel{40}^{10}}{1} = \frac{10}{1} = 10$$

$$\$40 - \$10 = \$30$$

Lyle told his mom that the price of the jacket was $30 on sale.

Example 3:   Lyle’s mother had a calculator in her purse and decided to check his math. She found the discount another way. She thought of 25% as the decimal, 0.25, and then multiplied by $40.00. She also figured $10 for the discount. Since$10 was a good discount on the jacket and the final price of $30 was reasonable, Lyle took home a new jacket!

$$\begin{array}{rl} 40 & \\ \times\underline{0.25} & \quad \$40 \\ 200 & \quad \underline{-10} \\ \underline{80} & \quad \$30 \checkmark \\ \$10.00 & \end{array}$$

Simple Interest

Calculating interest is a very important application of percent. Interest is used when saving money through a financial institution. Interest is also used when making a car loan or a house mortgage from a bank.

We will look at the simple interest formula that is the basis for more complicated types of interest like compound interest or interest on car loans.

The simple interest formula is ***I* = *p r t***, where ***I*** represents ***i**nterest*, ***p*** represents ***p**rincipal*, ***r*** represents ***r**ate*, and ***t*** represents ***t**ime*.

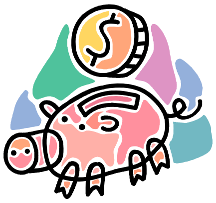
Example: Find the interest on \$2,500 at an annual interest rate of $6\frac{1}{2}\%$ for 18 months.

When we calculate the interest, rate and time must agree over time. In this problem, since the interest rate is an annual yearly rate, the time must also be expressed in years.

Rate: $6\frac{1}{2}\% = 6.5\% = 0.065$ (Express as a decimal.)

Time: 18 months $= \frac{18}{12} = 1\frac{6}{12} = 1\frac{1}{2} = 1.5$

years



$I = p \cdot r \cdot t$

$I = 2500 \cdot 0.065 \cdot 1.5$

$I = \$243.75$

The interest on \$2500 for 18 months at $6\frac{1}{2}\%$ is \$243.75.

Compound Interest

If money is deposited in a savings account, the money will earn interest. It will most likely be *compound interest*. **Compound interest** is interest earned on both the principal and any interest that has been earned previously.

Compound interest is computed on the principal plus any interest already earned in a previous period.

Based on the lending institution's saving plans, the interest may be compounded:

- annually (once a year)
- semiannually (twice a year)
- quarterly (4 times in a year)
- monthly (12 times in a year)
- daily, or continuously

Example 1: Theresa's savings account pays 5% annual interest compounded quarterly. At the end of one year, find the savings total and the amount of interest Theresa's account will have earned if Theresa deposited $1,500.

The table below organizes and shows how the interest compounds over the year by being calculated every three months using the simple interest formula.

The original principal (p) is $1500, but it increases every quarter because the interest earned that quarter is added to it.

The rate (r) is 5% which equals the decimal 0.05.

Compounded quarterly means compounded every quarter (1/4) of a year. So, every three months (1/4 of a year), Theresa's account's earns some interest which is added to the principal.

The time (t) is 1/4 of a year.

Qtr	**Principal (P)**	**Interest (I)** **P • r • t = I**	**Total at End of Quarter (T)** **T = P + I**
1	1,500	$1500\times 0.05\times\frac{1}{4}=$ $1500\times 0.05\times 0.25=$ **18.75**	1500.00 $\underline{18.75}$ 1518.75
2	**1,518.75**	$1518.75\times 0.05\times\frac{1}{4}=$ $1518.75\times 0.05\times 0.25=$ **18.984375**	1518.75 $\underline{18.984375}$ 1537.734375
3	**1,537.734375**	$1{,}537.734375\ \times 0.05\times\frac{1}{4}=$ $1{,}537.734375\ \times 0.05\times 0.25=$ **19.22167969**	1537.734375 $\underline{19.22167969}$ 1556.95605469 ≈ 1556.956055
4	**1,556.956055**	$1{,}556.956055\times 0.05\times\frac{1}{4}=$ $1{,}556.956055\times 0.05\times 0.25=$ **19.46195068**	1556.956055 $\underline{19.46195068}$ 1576.41800568 ≈ 1576.418006 ≈ $1576.42

A calculator was used to make the calculations in the table.
* Instead of multiplying by 1/4, the decimal equivalent of 0.25 was used.
** In steps 3 and 4, as the decimal number grew larger, some rounding was used to keep the decimal a reasonable size. This did not affect the final outcome of rounding to the nearest cent.

The savings total at the end of one year is **$1,576.42**.

The amount of interest that Teresa earned over the year was **$76.42**.

$$\begin{array}{r}1576.42\\ \underline{-1500.00}\\ 76.42\end{array}$$

Example 2: Find the savings total for the given account.

Principal: $175
Annual rate: 6%
Compounded semiannually (Every 6 months – 1/2 year)
Time: 1 year

Period	**Principal (P)**	**Interest (I)** **P • r • t = I**	**Total at End of Period(T)** **P + I**
1	175	$175\times0.06\times\frac{1}{2}=$ $175\times0.06\times0.5=$**5.25**	175.00 5.25 180.25
2	**180.25**	$180.25\times0.06\times\frac{1}{2}=$ $180.25\times0.06\times0.5=$ **5.4075**	180.25 5.4075 $185.6575\approx\$185.66$

A calculator was used to make the calculations in the table.
* Instead of multiplying by 1/2, the decimal equivalent of 0.5 was used.

The savings total at the end of one year is **$185.66**.

Example 3: Find the savings total after 3 years for a deposit of $650 at an annual rate of 8% compounded quarterly.

Principal: $650
Annual rate: 8%
Compounded quarterly (Every 3 months – 1/4 year)
Time: 3 years

This problem would have 12 steps and would get very cumbersome to calculate using a table.

There is a formula for computing compound interest and we'll take advantage of that formula in this problem.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$\boldsymbol{P}$ = principal amount (the initial amount you borrow or deposit)

$\boldsymbol{r}$ = annual rate of interest (as a decimal)

$\boldsymbol{t}$ = number of years over which the amount is deposited or borrowed

$\boldsymbol{A}$ = amount of money accumulated after n years, including interest.

$\boldsymbol{n}$ = number of times the interest is compounded per year

Principal: $650	$\boldsymbol{P} = 650$
Annual rate: 8%	$\boldsymbol{r} = 0.08$
Compounded quarterly (4 times per year)	$\boldsymbol{n} = 4$
Time: 3 years	$\boldsymbol{t} = 3$

Now, substitute into the formula.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 650\left(1+\frac{.08}{4}\right)^{(4)(3)}$$

$$\begin{array}{r} .02 \\ 4\overline{\smash{)}.08} \end{array} \qquad (3)(4)=12$$

$$A = 650(1+.02)^{12}$$

$$1+.02=1.02$$

$$A = 650(1.02)^{12}$$

$$1.02^{12}=1.268241795$$

$$A = 650(1.268241795)$$

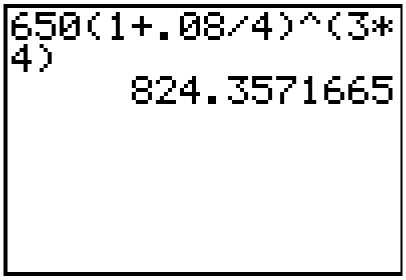
$$A = 824.3571665$$

$$A = \$824.36$$

Round the answer to money.

The savings total at the end of three years is **$824.36**.

*Note: When entering the data into a graphing calculator, the formula may look similar to the figure below.



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650(1+.08/4)^(3*
4)
      824.3571665
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