

# VOLUME

In this unit, solids and their volumes will be investigated. The volume of a solid is the amount the solid contains. Volume is a measure of capacity and is measured in cubic units. Formulas for each solid will be derived and used to calculate the volume. In addition, a close look will be given to the relationship that exists between the volume of a cylinder and a cone with the same measurements, along with the volume of a prism and a cylinder with the same measurements.

You may use a calculator to help with the computations, but work with the calculator carefully and double check all calculations.

Volume of a Rectangular Prism

Volume of a Cube

Volume of a Prism

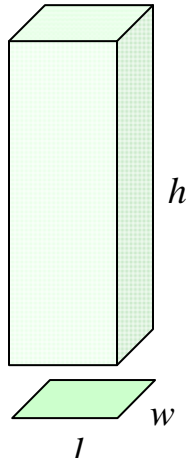
Volume of a Cylinder

Volume of a Cone and a Pyramid

Volume of a Sphere

## Volume of a Rectangular Prism

To calculate the volume of a **rectangular prism**, multiply the area of the base (length  $\times$  width) times height.

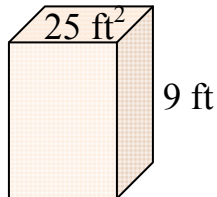


$$\begin{aligned}V &= \text{Area of Base} \times \text{Height} \\V &= (\text{length} \times \text{width}) \times \text{height} \\V &= l \times w \times h\end{aligned}$$

*Example 1:* Compute the volume of a square prism with a base area of 25 square feet and a height of 9 feet.

\*A square prism is a prism with a square base.

Since the base area is given, simply multiply the area of the base times the height to calculate the volume.



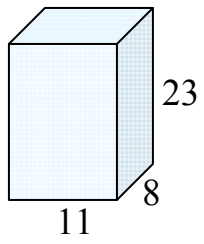
$$\begin{aligned}V &= \text{Area of base} \times \text{Height} \\V &= 25 \times 9 \\V &= 225\end{aligned}$$

\**Reminder:* Volume is measured in cubic units.

The volume of the square prism is 225 cubic feet.

*Example 2:* Calculate the volume of a rectangular prism with a length of 11 feet, width of 8 feet, and a height of 23 feet.

Since all dimensions of the prism are given, use the volume formula for a rectangular prism to calculate the volume.

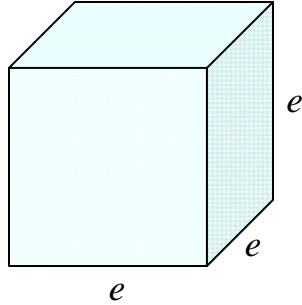


$$V = l \times w \times h$$
$$V = 11 \times 8 \times 23$$
$$V = 2024$$

The volume of the rectangular prism is 2024 cubic feet.

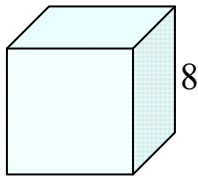
## Volume of a Cube

To calculate the volume of a **cube**, multiply the edge times itself three times.



$$\begin{aligned}V &= \text{Area of Base} \times \text{Height} \\V &= (\text{length} \times \text{width}) \times \text{height} \\V &= (e \times e) \times e \\V &= e^3\end{aligned}$$

*Example:* Calculate the volume of a cube with an edge length of eight feet.



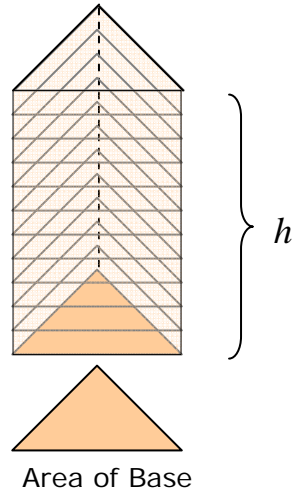
$$\begin{aligned}V &= e^3 \\V &= 8^3 \\V &= 8 \times 8 \times 8 \\V &= 512\end{aligned}$$

The volume of the cube is 512 cubic feet.

## Volume of a Prism

The **volume of a prism** is the amount the prism can hold and is measured in cubic units. To calculate the volume of a prism, multiply the area of its base times its height.

Triangular Prism



$$V = \text{Area of Base} \times \text{height}$$
$$V = B \times h$$

*Example:* Find volume of a triangular prism with a height of 16 inches, and the dimensions of the triangular base are a base of six inches and a height of four inches.

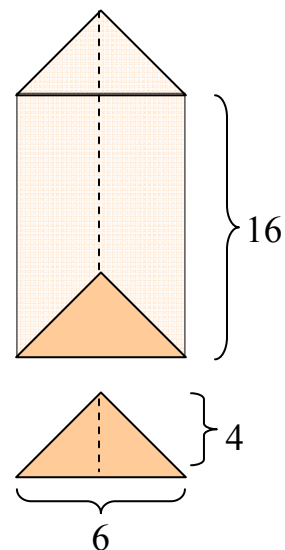
*Step 1:* First, find the area of the triangular base.

$$A = \frac{1}{2} b \times h$$

$$A = \frac{1}{2} \times 6 \times 4$$

$$A = \frac{1}{2} \times 24$$

$$A = 12 \text{ square inches (in}^2\text{)}$$



*Step 2:* Let ***B*** represent the area of the base ( $12 \text{ in}^2$ ) and ***h*** represent the height (16 in) of the prism.

$$V = B \times h$$

$$V = 12 \times 16$$

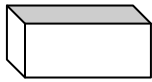
$$V = 192$$

*\*Reminder: Volume is measured in cubic units.*

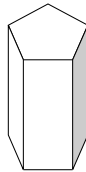
The volume of the triangular prism is 192 cubic inches ( $\text{in}^3$ ).

There are many kinds of prisms. Some prisms are named for the shape of their bases. The formula,  $V = B \times h$ , may be used to determine the volume of prisms.

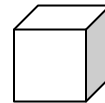
Here are some examples of prisms:



Rectangular Prism



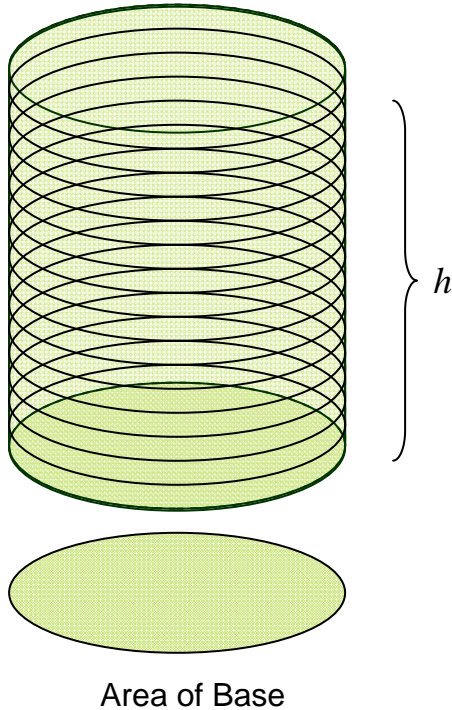
Pentagonal Prism



Cube

## Volume of a Cylinder

The **volume of a cylinder** is the amount a cylinder can hold and is measured in cubic units. To calculate the volume of a cylinder, multiply the area of the base times the height.



$$V = \text{Area of Base} \times \text{Height}$$

*The base of a cylinder is a circle.  
To calculate the area of a circle,  
use  $A = \pi \times r^2$ .*

$$V = (\pi \times r^2) \times h$$

*Example 1:* Find the volume of a cylinder with a radius of 12 centimeters and the height is 23 centimeters. (Use 3.14 for “pi”.)

$$V = \pi \times r^2 \times h$$

$$V = 3.14 \times 12^2 \times 23$$

$$V = 3.14 \times 144 \times 23$$

$$V = 10,399.68$$

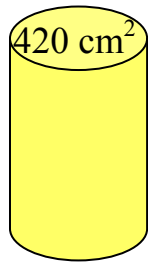


*\*Reminder: Volume is measured in cubic units.*

The volume of the cylinder is 10,399.68 cubic centimeters (cm<sup>3</sup>).

*Example 2:* Compute the volume of a cylinder with a base area of 420 square centimeters and a height of 14 centimeters.

Since the base area is given, simply multiply the area of the base times the height to calculate the volume.



$$V = \text{Area of base} \times \text{Height}$$

$$V = 420 \times 14$$

$$V = 5880$$

*\*Reminder: Volume is measured in cubic units.*

The volume of the cylinder is 5880 cubic centimeters.



*Example 3:* If the volume of a cylinder is 628 cubic feet and the height of the cylinder is eight feet, what is the radius of the cylinder? (Use 3.14 for “pi”.)

$$V = \pi \times r^2 \times h$$

$$628 = 3.14 \times r^2 \times 8 \quad \text{Substitute (} V = 628 \text{ and } h = 8 \text{)}$$

$$628 = 3.14 \times 8 \times r^2 \quad \text{Apply the commutative property to switch the positions of 8 and } r^2.$$

$$628 = 25.12 \times r^2 \quad \text{Simplify (} 3.14 \times 8 = 25.12 \text{)}$$

$$\frac{628}{25.12} = \frac{\cancel{25.12} \times r^2}{\cancel{25.12}} \quad \text{Divide both sides by 25.12.}$$

$$25 = r^2 \quad \text{Simplify.}$$

$$r = 5 \quad \text{Since } r^2 = 25, \text{ take the square root of 25 to determine } r.$$

The radius of the cylinder is 5 feet.

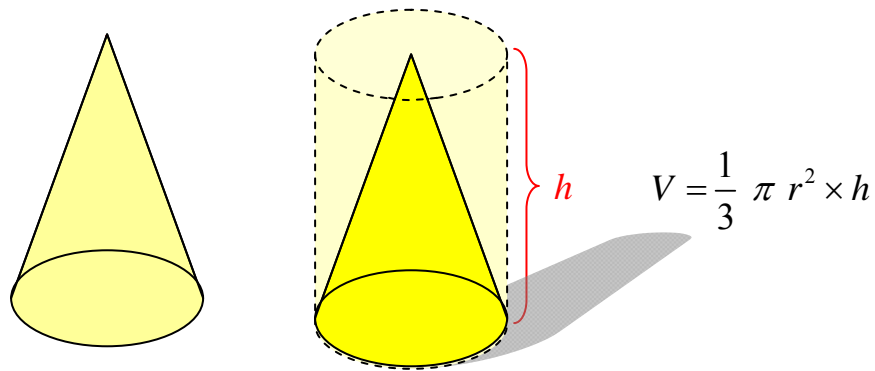
\*Since the answer is the length the radius, a linear measure, use feet (not cubic feet) to label the answer.

## Volume of a Cone and a Pyramid

### Cone

The volume of a cone can be found in a similar way to finding the volume of a cylinder. The difference is that the volume of a cone is *considerably less* than the volume of a cylinder with the same dimensions (radius and height).

\*The volume of a cone is equal to 1/3 the volume of a cylinder with the same dimensions.



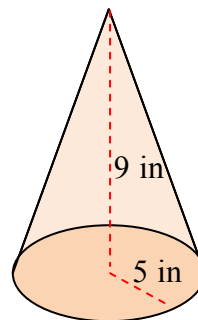
*Example 1:* Find volume of a cone with a radius of five inches and the height is nine inches.

$$V = \frac{1}{3} \pi r^2 \times h$$

$$V = \frac{1}{3} \times 3.14 \times 5^2 \times 9$$

$$V = \frac{1}{3} \times 3.14 \times 25 \times 9$$

$$V = 235.5$$



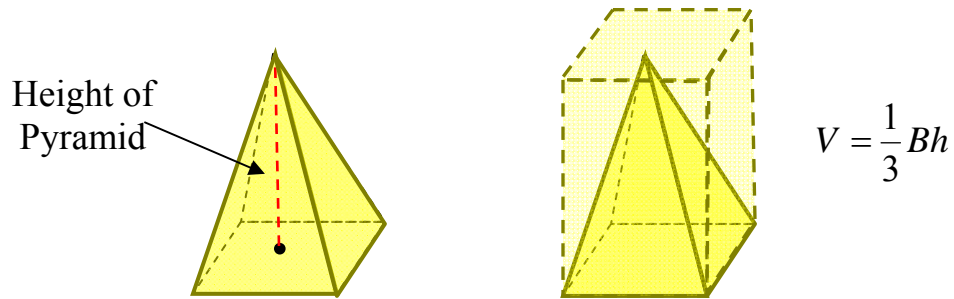
\**Reminder:* Volume is measured in cubic units.

The volume of the cone is 235.5 cubic inches ( $\text{in}^3$ ).

## Pyramid

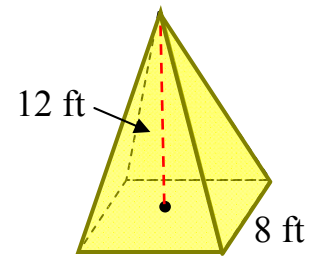
The volume of a pyramid is related to a rectangular prism that has the same dimensions (length, width, and height). As with the comparison of the volume of a cylinder and cone that have the same dimensions, the volume of a pyramid is *considerably less* than the volume of the rectangular prism with the same dimensions.

\*The volume of a pyramid is equal to 1/3 the volume of a rectangular prism with the same dimensions.



When calculating the volume of a pyramid, multiply 1/3 times the area of the base ( $B$ ) times the height.

*Example 2:* Find the volume of a square pyramid (base is a square) with the edge of the square base measuring eight feet and the height of the pyramid measuring 12 feet.



$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{3} \times (8 \times 8) \times 12$$

Area of square base equals  $8^2$

or  $8 \times 8$ .

$$V = \frac{1}{3} \times 64 \times 12$$

Simplify

$$V = 256$$

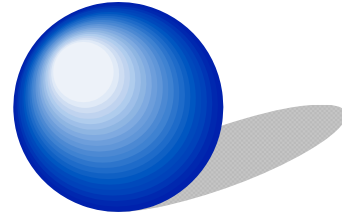
*\*Reminder: Volume is measured in cubic units.*

The volume of the pyramid is 256 cubic feet ( $\text{ft}^3$ ).

## Volume of a Sphere

The volume of a sphere is calculated using the

$$\text{formula } V = \frac{4}{3} \pi r^3.$$



*Example:* Calculate the volume of a sphere with a radius of seven centimeters, and round the answer to the nearest hundredth.

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} (3.14) (7^3)$$

Substitute the given radius.

$$V = \frac{4}{3} \times 3.14 \times (343)$$

$$7 \times 7 \times 7 = 343$$

$$V = 1436.0266\dots$$

Simplify

$$V = 1436.03$$

Round to the nearest hundredth.

*\*Reminder: Volume is measured in cubic units.*

The volume of the sphere is 1436.03 cubic centimeters ( $\text{cm}^3$ ).