INTRODUCTION TO GEOMETRY

This unit is about basic geometry including points, lines, angles, angle relationships, and circles. The topic about circles includes finding the circumference of a circle when given the radius or the diameter.

Points, Lines, and Planes

Points, Lines, and Line Segments

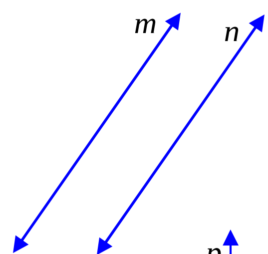
Point - A point is a location on a line. It has no dimensions but is represented by a dot.

Line - A line is a straight length that extends indefinitely into space. Lines have no width or thickness but are represented by straight edge marks.

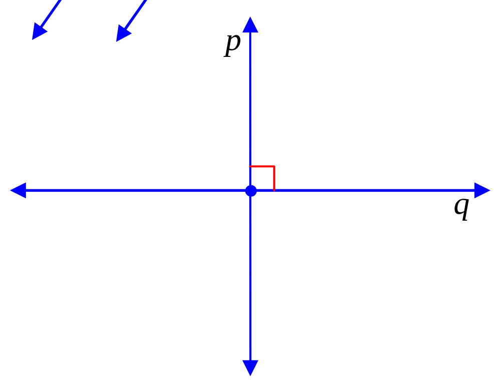
Line segment – A line segment is a part of a line usually named with its endpoints.

Intersection - Intersection is the point or line where two geometric figures meet. When two lines cross each other, there is one point at the place where they cross called the point of intersection. Two planes meet at and share a line of intersection.

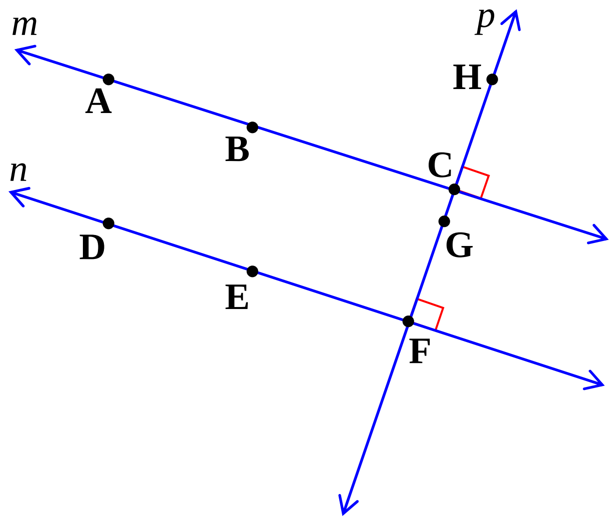
Parallel lines - Parallel lines are lines that lie in the same plane, are equidistant apart, and never meet.



Perpendicular lines - Perpendicular lines are lines that intersect and make right angles at the point of intersection. Right angles are denoted by a square shape as in the figure.



Example 1: Refer to the given figure to answer the questions.



(a) *Name a point.*

One point in the figure is Point A. Other points are B, C, D, E, F, G, H.

(b) *Name a line.*

Line *m* is one line in the figure.

This line can be named another way by using any two points named on the line.

Have you ever heard the expression "the shortest distance between two points is a straight line"?

Well, a line can be named by using any two points on it. So line *m* can also be named as $\overleftrightarrow{AB}$, read Line AB. Other names for this line are $\overleftrightarrow{AC}, \overleftrightarrow{BC}, \overleftrightarrow{BA}, \overleftrightarrow{CA},$ and $\overleftrightarrow{CB}$.

* When writing the name of a line on paper, draw a mini-line above the two letters, $\overleftrightarrow{AB}$. When referring to a line in text, just type in "Line AB" via the keyboard.

(c) *Name a line segment.*

One line segment starts at Point A and ends at Point B. Its name is $\overline{AB}$, read segment AB. Recall that a line segment is part of a line. Some other segments are $\overline{BC}, \overline{GH}$, and $\overline{EF}$. There are many more segments in the figure.

* When writing the name of a line segment on paper, draw a mini-segment above the two letters as shown in the previous paragraph. When referring to a line segment in text, type in "Segment AB" via the keyboard.

(d) *Name a point of intersection.*

Point C is a point of intersection. It is the point where line *m* intersects with line *p*.

(e) *Name a pair of parallel lines.*

Lines *m* and *n* are parallel because they are equidistant apart. The lines may also be named $\overleftrightarrow{AB}$ and $\overleftrightarrow{DF}$.

Another way to state this answer is $m \,||\, n$ or $\overleftrightarrow{AB} \,||\, \overleftrightarrow{DF}$.

* When referring to parallel lines on paper, draw mini-parallel lines between the names for the lines as shown in the previous paragraph. When referring to parallel lines in text, type in "line m is parallel to line n" or "Line AB is parallel to Line DF".

(f) *Name a pair of perpendicular lines.*

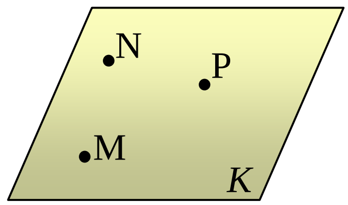
Line *m* is perpendicular to line *p* since the two lines intersect to make right angles. Another way to state this answer is $m \perp p$.

* When referring to perpendicular lines on paper, draw mini-perpendicular lines between the names for the lines as shown in the previous paragraph. When referring to perpendicular lines in text, type in "line m is perpendicular to line n".

Planes

Plane - A plane is a flat surface. A plane extends forever in all directions. Flat tables, floors, ceilings, and walls are examples of parts of planes.

Example 2: Name the plane.



A plane can be named several ways.

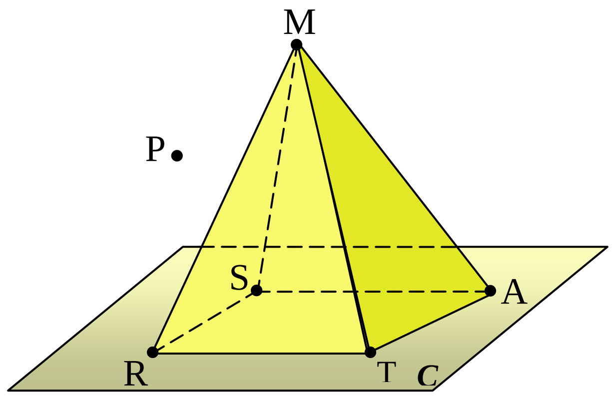
(a) A plane can be denoted by one italicized letter.

This plane could be named Plane *K*.

(b) When points are denoted by letters in a plane, the plane can be named by using three of the letters.

This plane could be named Plane MPN or Plane PMN or any combination of the three letters that represent three points on the plane.

Example 3: Refer to the 3-dimensional pyramid to answer the questions.



a) Name a plane that contains point S.

Point S lies in Plane *C*. Point S also lies in Plane MSR.

b) Name a plane that does not contain point S.

Point S does NOT lie in Plane MAT. It also does not lie in Plane MTR.

c) True or False. The bottom of the pyramid is part of Plane SAR.

True. The bottom of the pyramid and the portion of plane SAR that is displayed in the figure are both parts of the same plane the extends on forever.

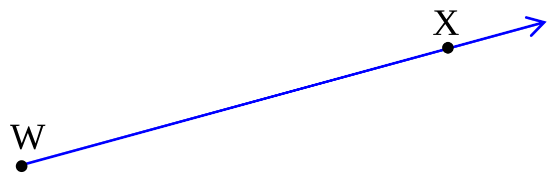
d) True or False. Point P lies in Plane MSR.

False. Point P is not part of any of the planes represented in the figure.

Angles

Features of Angles

Ray – A ray is a set of points that has a starting point and extends infinitely in a straight path in one direction.

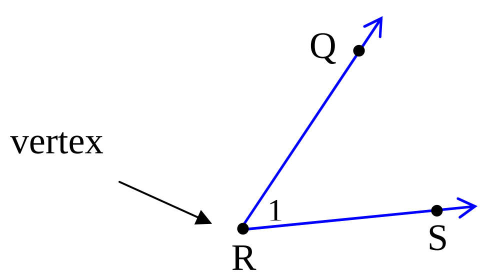


The name of this ray is $\overrightarrow{WX}$ or Ray WX. Since the ray starts at point W, W *must* be the first letter of its name.

Angle – An angle is formed when two rays meet at a common point.

Degree - The measure of an angle is the amount of circular rotation about a point starting with a ray and ending with a second ray. An angle is measured in degrees.

Vertex – A vertex is the point where two rays meet.



This angle can be named several ways.

(1) First, a number has been placed inside the angle and is used to identify the angle quickly; thus, it can be named Angle 1.

(2) A angle may also be named by its vertex letter. In this angle, since R is the vertex, it could be named Angle R.

(3) A third way to name this angle is to use three letters. Two of the letters represent a point on each of the rays and the third letter represents the vertex.

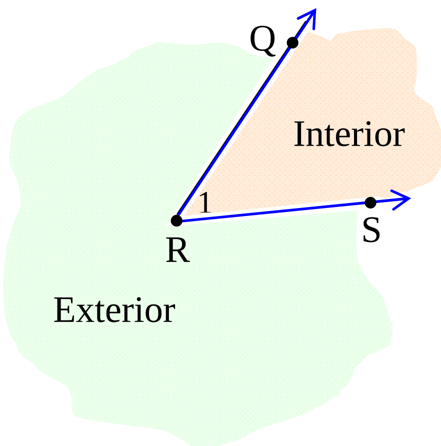
*When using three letters to name an angle, make sure to put the letter that represents the vertex as the middle letter. This angle could be named Angle QRS or Angle SRQ.

When writing the name of the angle on paper, it can be denoted with an angle mark preceding the number or letter(s) that represent the angle.

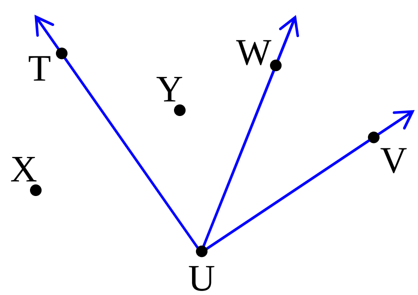
For example: $\angle 1$ or $\angle R$ or $\angle QRS$

Interior – The interior of an angle is the area within the two rays.

Exterior – The exterior of an angle is the area outside the two rays.



Example 1: Refer to the figure to answer the questions.



a) Name a vertex.

The vertex is Point U.

b) Name a ray.

There are three rays. They are $\overrightarrow{UT}, \overrightarrow{UW}, \overrightarrow{UV}$.

*Notice that Point U is the starting point of each ray and is therefore the first letter of the name of each ray.

c) Name three angles.

The three angles are $\angle TUV, \angle TUW,$ and $\angle WUV$.

d) What points lie in the interior of $\angle TUV$?

The two points named in the interior of $\angle TUV$ are Points Y and W.

*Note: There are an infinite number of points that lie in the interior of the angle; but, only two points are named in this figure.

e) Name a point that lies in the exterior of $\angle TUV$.

Point X is the only named point that lies in the exterior of $\angle TUV$.

*Note: Again, there are an infinite number of points that lie in the exterior of the angle; but, only one point is named in this figure.

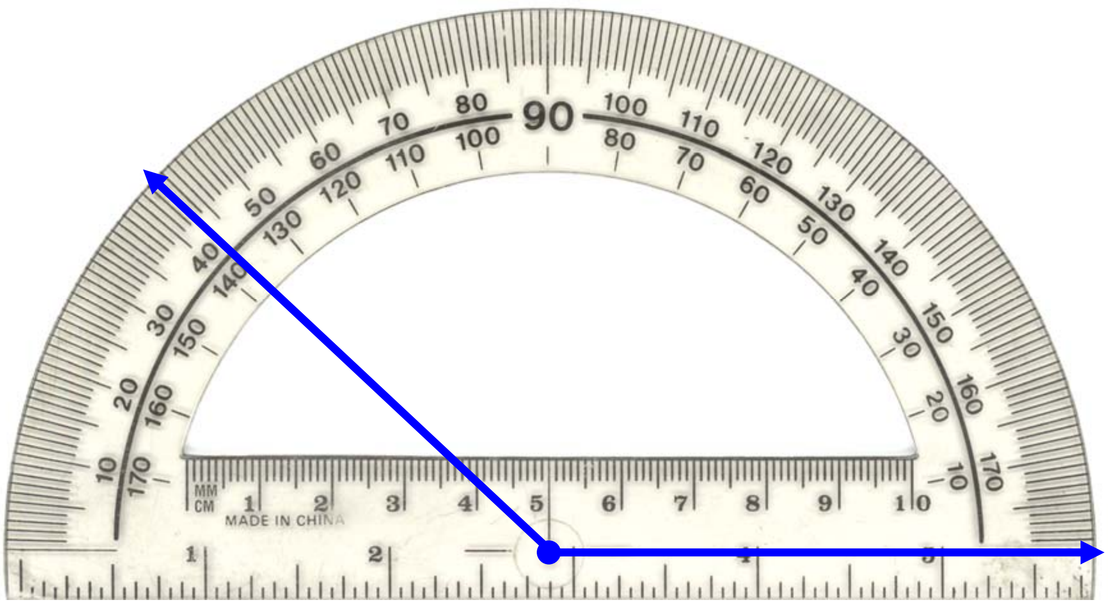
Measuring Angles

Measurement of an angle – Angles are measured in degrees.

degree – A degree is a unit of rotation around a point that may be used to measure angles. One degree is $\frac{1}{360}$ of a rotation around a point.

Protractor – A protractor is a measurement tool used to measure angles in degrees.

Example 1: What kind of angle is shown below? What is the angle's measure?

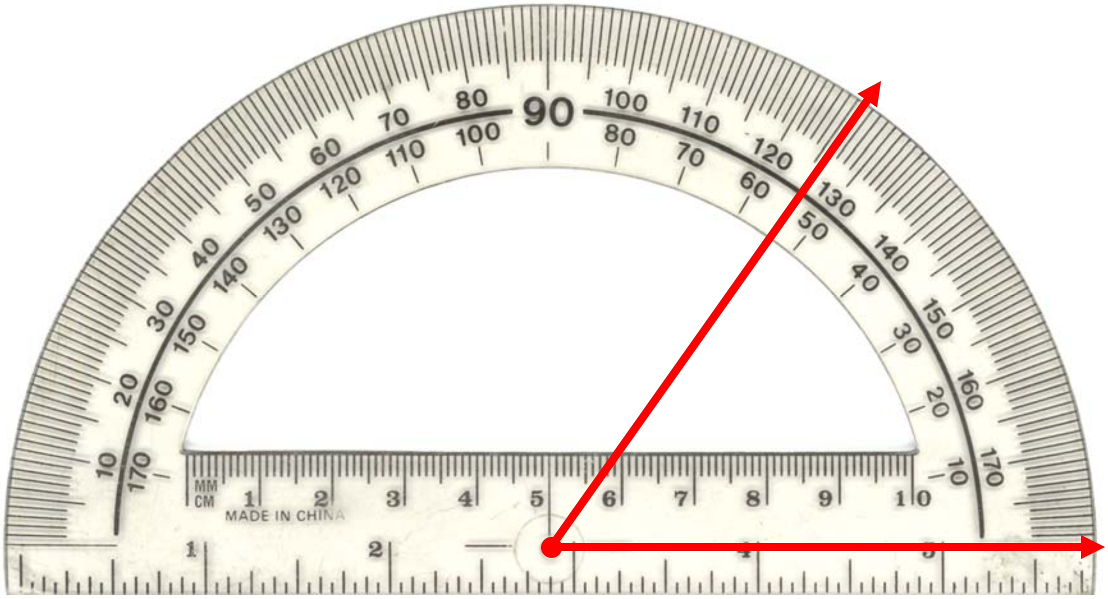


This angle starts on the **right** at $0°, 10°, 20°...$ passes by $90°$ $\ldots$ $130°, 135°,$ and goes up to $137°$. (Read the bottom numbers since the bottom ray is pointing at zero, the starting point of the set of lower numbers.)

This angle can be labeled as an **obtuse angle**. Obtuse angles are angles that measure ***more than*** *90 degrees* ***and less than*** *180 degrees***.**

The angle is an obtuse angle that measures 137 degrees.

Example 2: What kind of angle is shown below? What is the angle's measure?

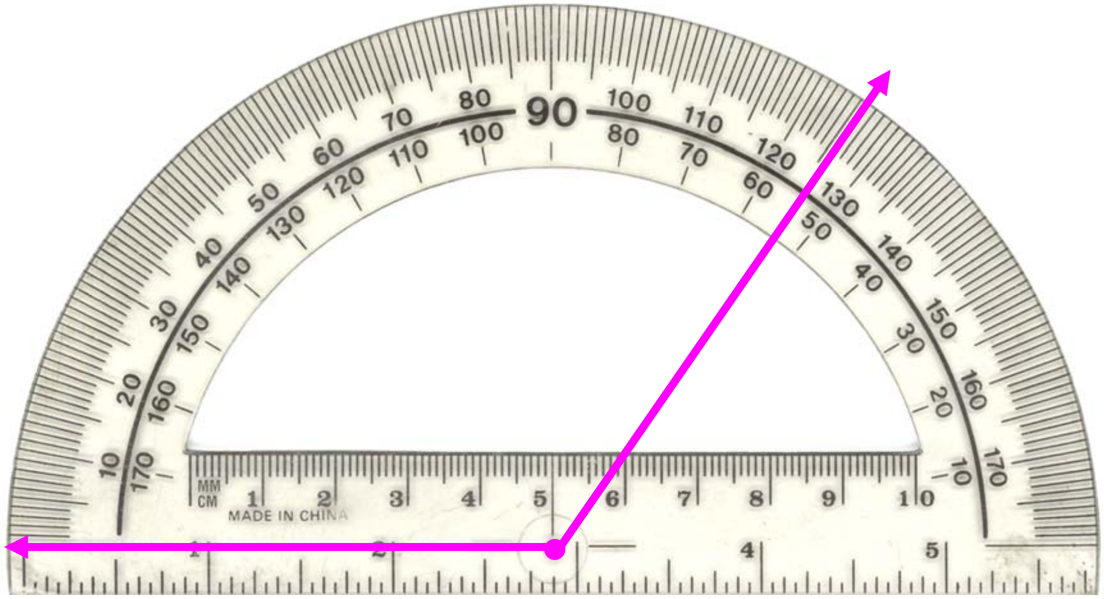


This angle starts on the **right** at $0°, 10°, 20° ... 50°$, up to $54°$. (Read the bottom numbers since the bottom ray is pointing at zero, the starting point of the set of lower numbers.)

This angle can be labeled as an **acute angle**. Acute angles are angles that measure ***less than** 90 degrees*.

The angle is an acute angle that measures 54 degrees.

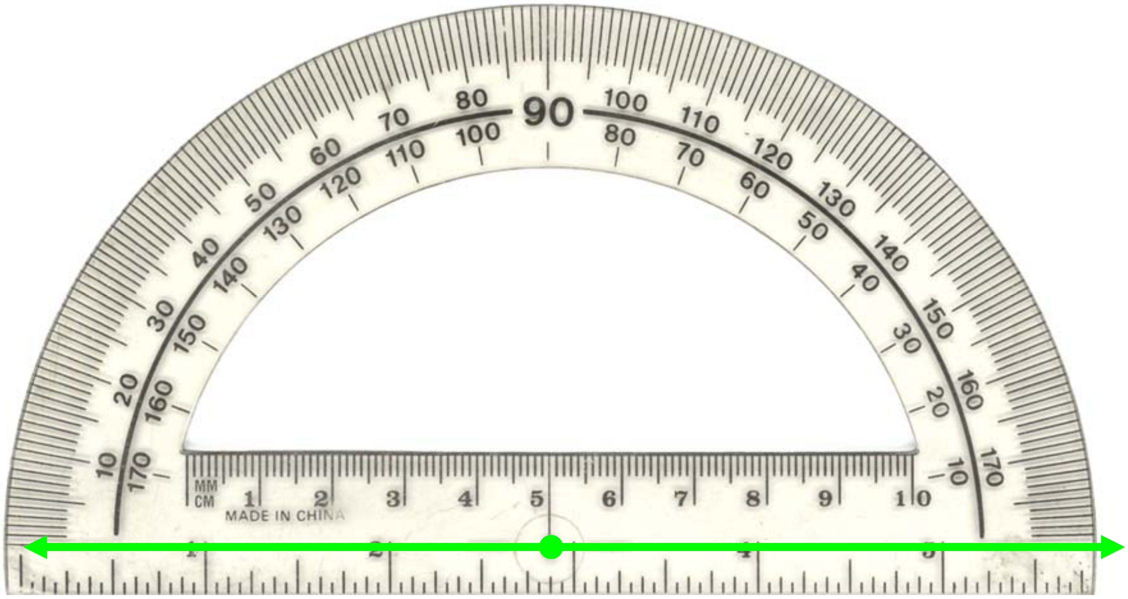
Example 3: What kind of angle is shown below? What is the angle's measure?



This angle starts on the **left** at $0°, 10°, 20°...$ passes by $90°$ … $120°, 125°,$ and goes up to $126°$. (Read the top numbers since the bottom ray is pointing at zero, the starting point of the set of upper numbers.)

The angle is an obtuse angle that measures 126 degrees.

Example 4: What kind of angle is shown below? What is the angle's measure?

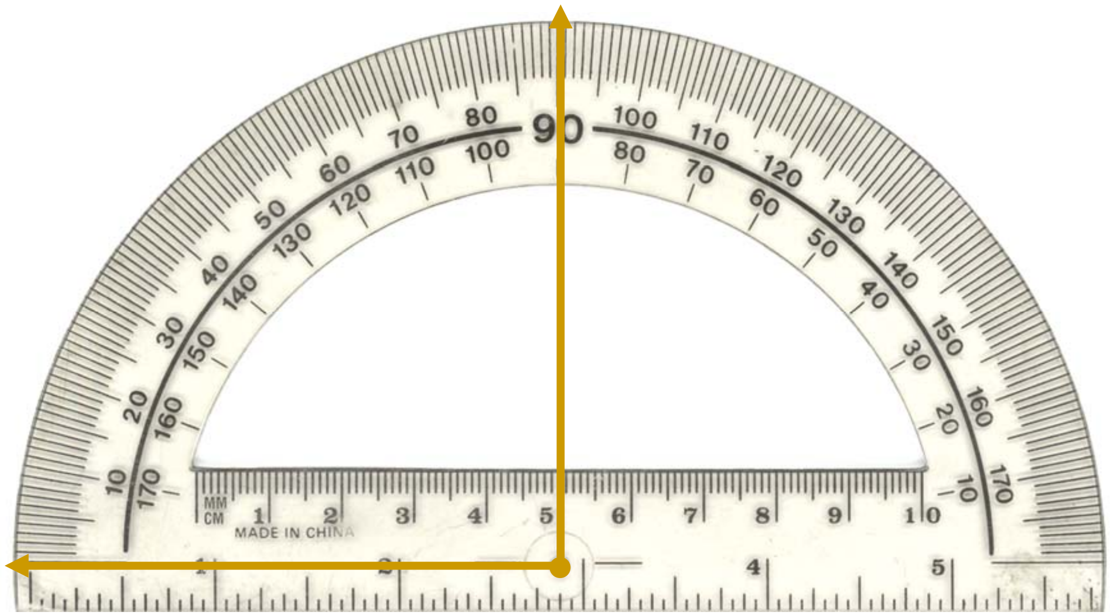


The angle below appears to be a straight line?   Start at $0°, 10°, 20°...$ on either end and go past $90°$ up to $180°$.

This angle can be labeled as a **straight angle**. Straight angles are angles that measure *exactly 180 degrees*

The angle is a straight angle that measures 180 degrees.

Example 5: What kind of angle is shown below? What is the angle's measure?



This angle starts on the **left** at $0°, 10°, 20°...$ and goes up to $90°$. (Read the top numbers since the bottom ray is pointing at zero, the starting point of the set of upper numbers.)

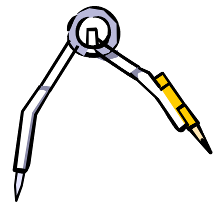
This angle can be labeled as a **right angle**. Right angles are angles that measure *exactly 90 degrees*.

The angle is a right angle that measures 90 degrees.

Constructions

Line Segment

To make geometric constructions, use a compass and a straight edge such as a ruler.



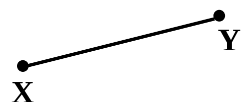
Compass – A compass is a measurement tool used to draw arcs and circles.

Arc – An arc is a portion of a circle.

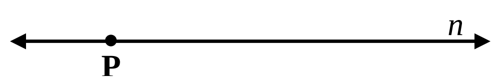
Congruent figures – Congruent figures are geometric figures that have the same size and shape.

Example 1: On line *n* draw segment PQ so that it is congruent to segment XY.

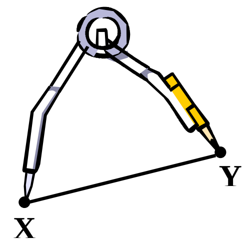
Step 1: Draw segment *XY*,



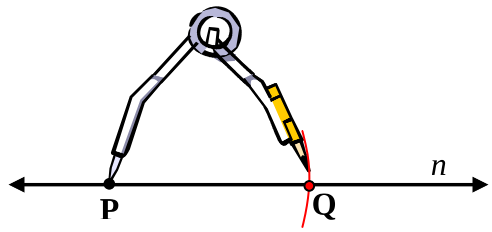
Step 2: Draw line *n* and label point *P* on the line.



Step 3: Place the metal point of the compass on segment *XY* and adjust the pencil point to touch point *Y*.



Step 4: Move the compass to line *n* and without changing the setting of the compass, place the metal point at *P* and draw an arc on line *n*. Label the point *Q* as the point where the arc crosses the line.



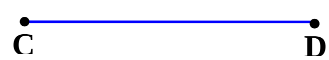
Segment *PQ* is congruent to segment *XY*.

Perpendicular Bisector

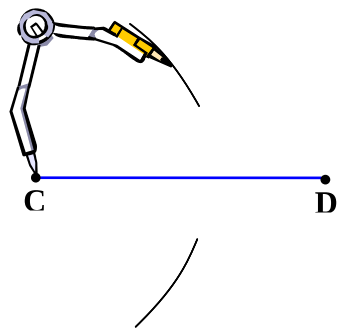
Perpendicular bisector – A perpendicular bisector is a line or segment that intersects another line segment forming 90-degree angles at the point of intersection and divides the segment into two congruent segments.

Example 2: Draw line PQ through segment CD so that it line PQ is a perpendicular bisector of segment CD.

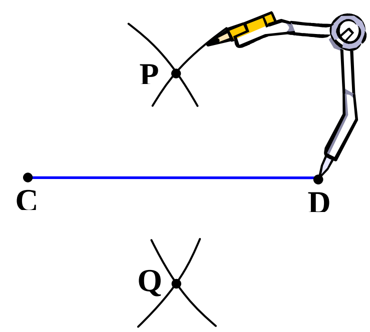
Step 1: Start with $\overline{CD}$.



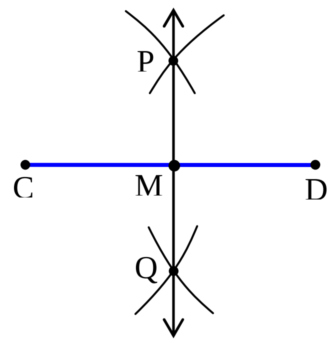
Step 2: Place the metal point of a compass on one of the endpoints of the segment. Adjust the setting of the compass so that it is greater than halfway across the segment. Draw an arc above and below the segment.



Step 3: Place the metal point of the compass on the other end point. Without changing the setting of the compass, draw an arc above and below the segment so that the arcs intersect. Mark the points of intersection as points *P* and *Q*.



Step 4: Draw a line that passes through the two points of intersection, points *P* and *Q*.



$$\overleftrightarrow{PQ} \perp \overline{CD}$$

Line PQ is perpendicular to segment CD. In this construction, line PQ bisects segment CD.

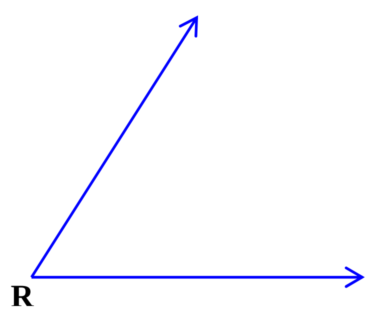
Therefore, $\overleftrightarrow{PQ}$ is a perpendicular bisector of $\overline{CD}$.

Angle Bisector

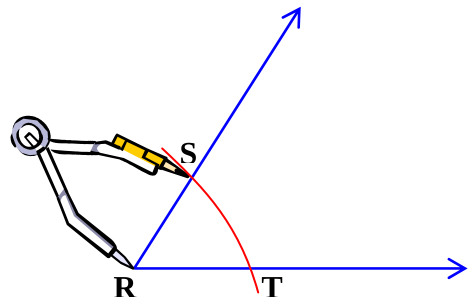
Angle bisector – An angle bisector is a ray that divides an angle into two congruent angles.

Example: Draw Ray RU so that it bisects Angle R.

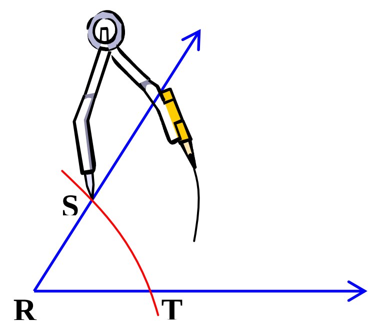
Step 1: Draw $\angle R$.



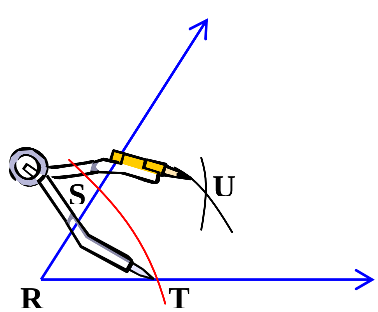
Step 2: Place the metal point of the compass at point R and draw an arc through the angle rays, naming the points of intersection, S and T.



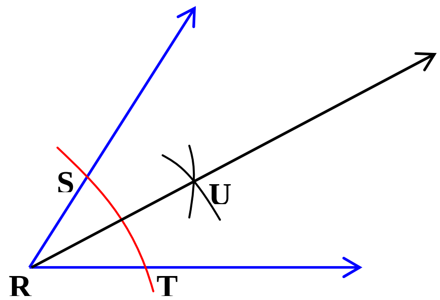
Step 3: Adjust the compass settings a little wider and place the metal point of the compass at point S. Draw an arc in the interior of the angle.



Step 4: Keep the compass setting the same and place the metal point of the compass at point T. Draw a second arc in the interior of the angle letting it cross the other arc.



Step 5: Draw $\overrightarrow{RU}$ so that it starts at the vertex R and extends though the intersection of the two arcs, point U.

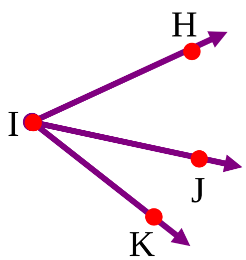


Ray RU is the angle bisector of $\angle SRT$.

Therefore, $m\angle SRU = m\angle URT$ and $\angle SRU \cong \angle URT$.

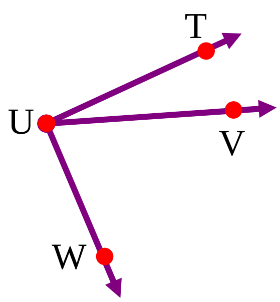
Angle Relationships

Adjacent Angles are angles that share a common vertex and a common ray.



Angle HIJ is adjacent to Angle JIK. The adjacent angles share vertex I and ray IJ.

Complementary Angles are adjacent angles that form a right angle (90°).



Angle TUV and Angle VUW are complementary angles. Together these angles form right angle TUW.

Example 1: In the complementary angles shown above, if the measure of angle TUV is 23°, what is the measure of angle VUW?

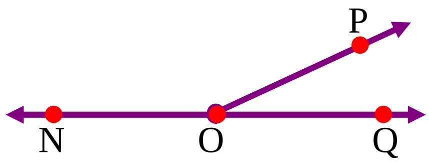
By definition, complementary angles total 90°.

Let x represent the measure of $\angle VUW$.

$$\begin{aligned} 23 + x &= 90 \\ -23 \quad &\quad -23 \\ x &= 67 \end{aligned}$$

The measure of $\angle VUW$ is 67°.

Supplementary Angles are adjacent angles that form a straight angle (180°).



Angle NOP and Angle POQ are supplementary angles.

Together the outer rays of these angles form straight Angle NOQ.

Example 2: In the supplementary angles shown above, if the measure of Angle QOP is 23°, what is the measure of Angle PON?

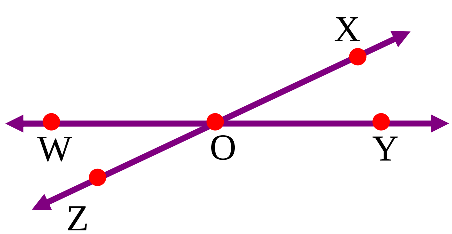
By definition, supplementary angles total 180°.

Let x represent the measure of $\angle PON$.

$$\begin{aligned} 23 + x &= 180 \\ {\color{red}-23} \quad & {\color{red}-23} \\ x &= 157 \end{aligned}$$

The measure of $\angle PON$ is 157°.

Vertical Angles are opposite angles formed when two lines intersect. Vertical angles are equal in measure (congruent).

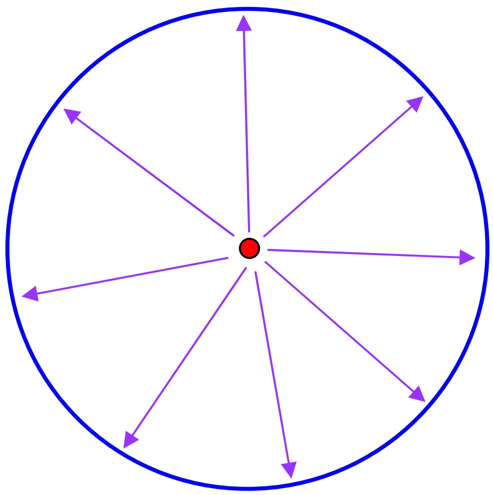


Angle WOZ and Angle XOY are vertical angles. These angles are congruent acute angles

Angle WOX and Angle ZOY are vertical angles. These angles are congruent obtuse angles.

Circles and Circumference

circle – A circle is a set of points that are equidistant from a given center point all of which lie in a plane.

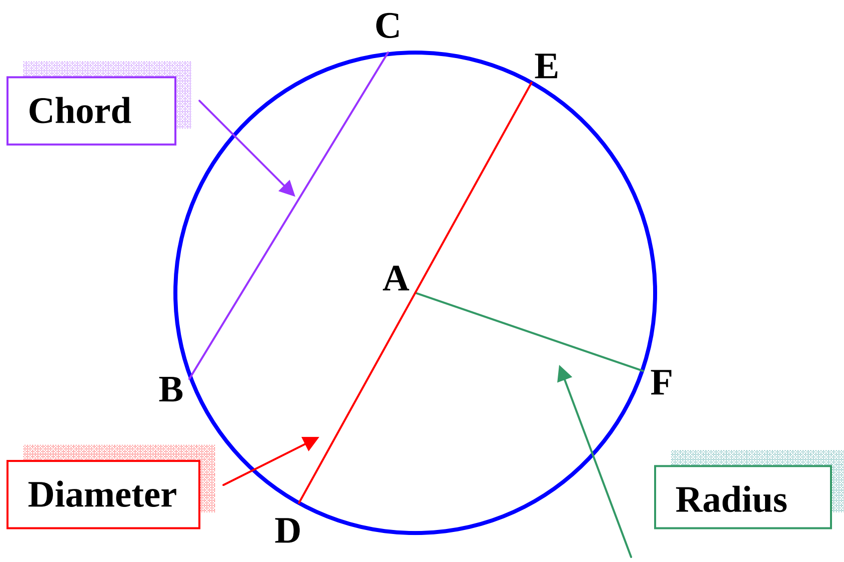


radius – A radius of a circle is a line segment that has its endpoints on the center of the circle and a point on the circle. (See figure below.)

chord – A chord of a circle is a line segment that has its endpoints on the circle. (See figure below.)

diameter –A diameter of a circle is a chord which passes through the center of a circle. (See figure below.)

circumference – The circumference of a circle is the distance around a circle.

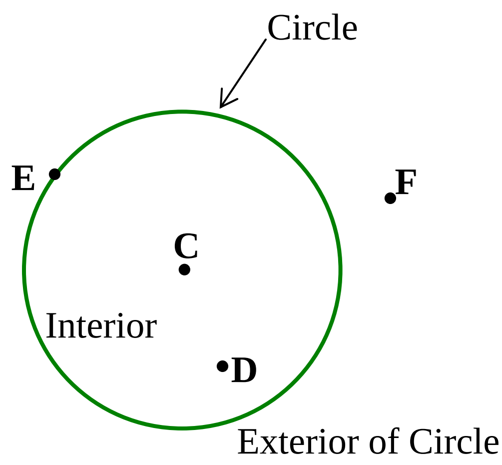


- The name of this circle is Circle A which is represented by ⊙A. Circles are generally named by their center point.
- Segments BC and DE are chords.
- Segment DE is also a diameter.
- Segments AD, AE, and AF are radii. (Radii is the plural of radius.)

circle (another definition) – A circle is a set of points whose distance from the center is equal to the radius of the circle.

interior of a circle – The interior of a circle is the set of points where the distance of these points from the center is less than the length of the radius of the circle.

exterior of a circle – The exterior of a circle is the set of points where the distance of these points from the center is greater than the length of the radius of the circle.



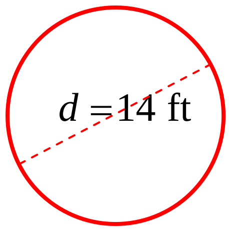
Point	*Location*
E	on $\odot$**C**
C, D	Interior of $\odot$**C**
F	Exterior of $\odot$**C**

Circumference

To determine the circumference of a circle, multiply the diameter of the circle times “pi”. To simplify the calculation, round “pi” to 3.14.

$$C = \pi d$$

Example 1: Find the circumference of a circle that has a diameter equal to 14 feet.



$$C = \pi d$$
$$C = (3.14)(14)$$
$$C = 43.96$$

The circumference of the circle is 43.96 feet.

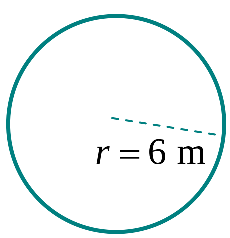
Since a radius is half the length of a diameter, we can say that a diameter equals two radii (plural of radius). Thus, a second formula can be written for calculating circumference.

$$C = \pi \times d \quad \rightarrow \quad C = \pi \times (2 \times r) \quad or \quad C = 2 \times \pi \times r$$

To determine the circumference of a circle when given the radius, multiply two times "pi" times the radius of the circle.

$$C = 2\pi r$$

Example 2: Find the circumference of a circle that has a radius equal to 6 meters.

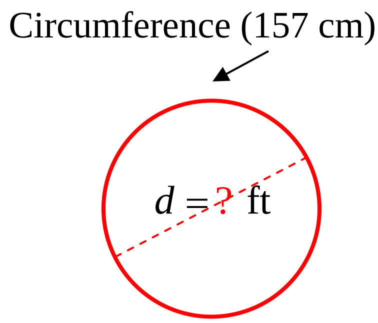


$$C = 2\pi r$$
$$C = 2(3.14)(6)$$
$$C = 37.68$$

The circumference of the circle is 37.68 meters.

If the circumference of a circle is known, then the diameter and/or radius can be calculated.

Example 3: What is the length of the diameter of a circle that has a circumference of 157 centimeters?

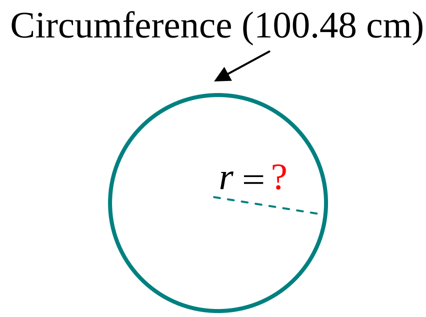


Substitute into the formula.

$C = \pi d$	Formula for circumference when givin radius
$157 = 3.14r$	Substitute ($C = 157$ $\pi = 3.14$)
$\frac{157}{3.14} = \frac{\cancel{3.14}r}{\cancel{3.14}}$	Divide both sides by 3.14
$r = 50$	Simplify

The radius of the circle is 16 centimeters.

Example 4: What is the length of the radius of a circle that has a circumference of 100.48 centimeters?



Substitute into the formula.

$C = 2\pi r$	Formula for circumference when given radius
$100.48 = 2(3.14)r$	Substitute ($C = 100.48$ $\pi = 3.14$)
$100.48 = 6.28r$	Simplify
$\dfrac{100.48}{6.28} = \dfrac{\cancel{6.28}r}{\cancel{6.28}}$	Divide both sides by 6.28
$r = 16$	Simplify

The radius of the circle is 16 centimeters.