DIVIDE FRACTIONS

This unit is about division of fractions and mixed fractions. Also, you will examine changing fractions to decimals and vice versa. The unit concludes with unit pricing.

Reciprocals

Division of Fractions

Fractions to Decimals and Vice Versa

Unit Prices and Comparative Shopping

Reciprocals

The **reciprocal** (inverse) of a fraction is the fraction that is multiplied by the original fraction to equal one (1).

For example, the **reciprocal** of $\frac{1}{2}$ is $\frac{2}{1}$ because $\frac{1}{2}\times\frac{2}{1}=\frac{1}{\cancel{2}^{1}}\times\frac{\cancel{2}^{1}}{1}=\frac{1}{1}=1$.

*To find the reciprocal of a proper fraction, just "flip" the fraction.

Example 1: What is the reciprocal of 3/5?

Flip the fraction: $\frac{3}{5}\rightarrow\frac{5}{3}$.

The reciprocal of 3/5 is 5/3.

*To find the reciprocal of a mixed number, first write the mixed number as an improper fraction, and then "flip" the fraction.

Example 2: What is the reciprocal of 2 3/4?

First, write the fraction as an improper fraction.

$$2\frac{3}{4}=\frac{4\times2+3}{4}=\frac{11}{4}$$

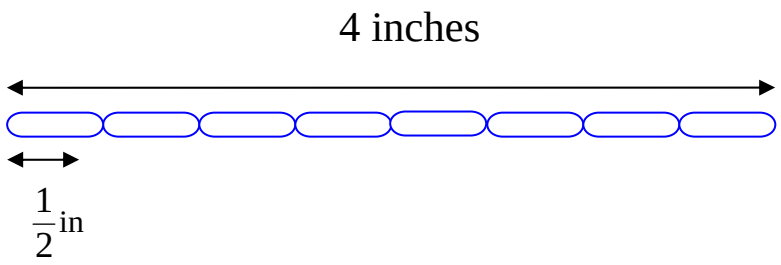
Then, flip the fraction: $\frac{11}{4}\rightarrow\frac{4}{11}$.

The reciprocal of 2 3/4 is 4/11.

Division of Fractions

Introduction to Division of Fractions

A child's bracelet is four (4) inches long. Each of the links is a half-inch long. How many links are in the chain?



Count the 1/2-inch links to determine the number of links in the bracelet. There are eight. Four divided by one-half equals eight.

$$4 \div \frac{1}{2} = 8$$

It takes eight half-inch links to make up a bracelet four inches long.

*Recall that the answer to a division problem is called the **quotient**, the number being divided is the **dividend**, and the number that the dividend is being divided by is the **divisor**.

$$4 \div \frac{1}{2} = 8$$

*Notice, that in this division problem of fractions, the dividend (4) and the divisor (1/2) are both smaller than the quotient (8). Interesting!!!

To divide fractions, multiply by the **reciprocal or inverse** of the second fraction.

Let's revisit the problem above, and solve it using the algorithm (mathematical process) for dividing fractions.

Example 1: Find the quotient of 4 divided by 1/2.

*Note: When entering a division problem via the keyboard, it would look like 4 / (1/2) . The first slash (/) represents division and the second slash is part of the fraction.

$$4 \div \frac{1}{2} =$$

$$= \frac{4}{1} \div \frac{1}{2}$$ (Write 4 as a fraction $(\frac{4}{1})$.)

$$= \frac{4}{1} \times \frac{2}{1}$$ (Multiply by the reciprocal.)

$$= \frac{4 \times 2}{1} = \frac{8}{1} = 8$$ (Multiply the numerators and the denominators.)

Dividing Fractions and Mixed Numbers

Now, we'll take a look at several examples of division of fractions and mixed numbers.

Example 2: Find the quotient of 3/4 divided by 4/5.

$$\frac{3}{4} \div \frac{4}{5} =$$

$$= \frac{3}{4} \times \frac{5}{4}$$ (Multiply by the reciprocal.)

$$= \frac{3 \times 5}{4 \times 4}$$ (Multiply the numerators and the denominators.)

$$= \frac{15}{16}$$

Example 3: Find the quotient of 2/3 divided by 8/9.

$$\frac{2}{3} \div \frac{8}{9} =$$

$$= \frac{2}{3} \times \frac{9}{8}$$ (Multiply by the reciprocal.)

$$= \frac{\cancel{2}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{9}^{3}}{\cancel{8}^{4}}$$ (Use cancelling.)

$$= \frac{1 \times 3}{1 \times 4} = \frac{3}{4}$$

Example 4: Find the quotient of 12 divided by 3/5.

$$12 \div \frac{3}{5} =$$

$$= \frac{12}{1} \times \frac{5}{3}$$ (Multiply by the reciprocal.)

$$= \frac{\cancel{12}^{4}}{1} \times \frac{5}{\cancel{3}^{1}}$$ (Use cancelling.)

$$= \frac{4 \times 5}{1 \times 1} = \frac{20}{1} = 20$$

Example 5: Find the quotient of 2 2/7 divided by 6 2/3.

$$2\frac{2}{7} \div 6\frac{2}{3} =$$

$$= \frac{16}{7} \div \frac{20}{3}$$ Change both mixed numbers to improper fractions.

$$= \frac{16}{7} \times \frac{3}{20}$$ Multiply by the reciprocal.

$$= \frac{\cancel{16}^{4}}{7} \times \frac{3}{\cancel{20}^{5}}$$ Use cancelling.

$$= \frac{4 \times 3}{7 \times 5} = \frac{12}{35}$$

Fractions to Decimals and Vice Versa

Fractions to Decimals

To write a fraction as a decimal, recall that the fraction bar means divide.

Example 1: Find the decimal for 3/4.

To find the decimal for $\frac{3}{4}$, divide the denominator into the numerator, and then add a decimal point and zeros. In this example, the decimal comes out even after adding two zeros.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ \end{array} \qquad \frac{3}{4} = 0.75$$

The decimal for 3/4 is 0.75.

Some fractions do not have decimals that come out even and, when dividing, continue on forever. For this type of fraction, divide and round to the given place.

Example 2: Find the decimal for 2/3 and round to nearest hundredth.

To find the decimal for $\frac{2}{3}$, divide to get one extra decimal place for rounding, stop, and then round. In this problem, divide through thousandths place (3 decimal places).

$$
\begin{array}{r}
.666 \\
3\overline{)2.000} \\
\underline{18} \\
20 \\
\underline{18} \\
20 \\
\underline{18} \\
2
\end{array}
$$

$\frac{2}{3} \approx 0.67$

Sometimes repeating decimals are expressed with a bar over the repeating pattern in the decimal.

Two-thirds may be written as $0.\overline{6}$.

Decimals to Fractions

Decimals may be written as fractions and simplified, if needed.

Example 3: How is 4.53 read?

4	.	53
four	**and**	$\frac{53}{100}$

Answer: 4 and 53 hundredths

*Notice that when there are **two decimal places**, there are **two zeros** in the denominator of the fraction.

Example 4: How is 7.5 read?

7	.	5
seven	**and**	$\frac{5}{10}$

Answer: 7 and 5 tenths

*Notice that when there is **one decimal place**, there is **one zero** in the denominator of the fraction.

Example 5: Write the mixed number for 7.5 in simplest form.

To solve this problem, first write the fraction for the mixed number, and then reduce.

$$7.5 = 7\frac{5}{10} \qquad \left(\frac{5}{10} \div \frac{5}{5} = \frac{1}{2}\right)$$

$$= 7\frac{1}{2}$$

The mixed number for 7.5 is 7 1/2 in simplest form.

Example 6: Write the mixed number for 6.225 in simplest form.

To solve this problem, first write the fraction for the mixed number, and then reduce.

$$6.225 = 6\frac{225}{1000} \qquad \left(\frac{225}{1000} \div \frac{25}{25} = \frac{9}{40}\right)$$

$$= 6\frac{9}{40}$$

The mixed number for 6.225 is 6 9/40 in simplest form.

*Notice that when there are **three decimal places**, there are **three zeros** in the denominator of the fraction.

Unit Prices and Comparative Shopping

The **unit price** for an item is the cost for one unit of measure. The unit price may be the cost per ounce, per pound, per item, or any other unit. When shopping, unit prices can be used to determine the better buy. To find the unit price of any item, divide the price by the number of units it contains.

Example: Alex notices two jars of pickles on the grocery story shelf with the following labels:

Jar A	**Jar B**
Pickles - $1.29 1lb 2oz	Pickles - $0.89 15 oz.

Compare the unit prices for each jar to see which jar of pickles is the better buy.

In this problem, the best comparison for unit price would be to compare both jars in ounces, the smaller of the two units used to express the weight of the jars.

Jar A: *First, change 1lb 2oz to ounces.

There are 16 ounces in 1 pound so
1lb 2oz =16 + 2 = 18 ounces.

Divide the cost of the Jar A by 18.

$$18\overline{)1.290} = .071$$

1 26

30

18

12

Jar A ≈$0.07 per ounce

Jar B: Divide the cost of the Jar B by 15.

The weight of Jar B is given in ounces.

Jar B $\approx$ \$0.06 per ounce

$$
\begin{array}{r}
.059 \\
15\overline{)0.890} \\
\underline{75} \\
140 \\
\underline{135} \\
5
\end{array}
$$

Jar B, 15oz for \$.89, is the better buy at \$.06 per ounce.

*Note: Sometimes the unit prices are close and rounding to the nearest cent (hundredths) will give the same unit price. In these cases, add one more step and divide to ten-thousandths; and then, round to the nearest thousandth to compare instead of the nearest hundredth.