SCIENTIFIC NOTATION AND FORMULAS

There are lots of helpful patterns in mathematics. This unit begins with an investigation of the patterns that exist in powers of tens. These patterns are then used to understand how to represent and compute numbers that are expressed in scientific notation. The unit concludes with examining various formulas in math and how they can be used to solve practical problems.

Powers of Ten

Standard Form for Numbers in Scientific Notation

Express Numbers in Scientific Notation

Compute Using Scientific Notation

Formulas

Powers of Ten

Numbers that are powers of ten have patterns that provide interesting ways to express them in shorter notation.

Study the chart.

Let's start with 10,000	10,000	=	$10 \times 10 \times 10 \times 10$	*or*	10^4
Divide 10,000 by 10 to get 1000	1,000	=	$10 \times 10 \times 10$	*or*	10^3
Divide 1,000 by 10 to get 100	100	=	10×10	*or*	10^2
Divide 100 by 10 to get 10	10	=	10	*or*	10^1
Divide 10 by 10 to get 1	1	=	1	*or*	10^0
Divide 1 by 10 to get 0.1	0.1	=	$\frac{1}{10}$ or $\frac{1}{10^1}$	*or*	10^{-1}
Divide $\frac{1}{10}$ by 10 to get 0.01	0.01	=	$\frac{1}{100}$ or $\frac{1}{10^2}$	*or*	10^{-2}
Divide $\frac{1}{100}$ by 10 to get 0.001	0.001	=	$\frac{1}{1000}$ or $\frac{1}{10^3}$	*or*	10^{-3}
Divide $\frac{1}{1000}$ by 10 to get 0.0001	0.0001	=	$\frac{1}{10,000}$ or $\frac{1}{10^4}$	*or*	10^{-4}

*Notice that for 10^0 to fit into the pattern, 10^0 must equal 1.

Exponent Rule: Any number raised to a power of zero equals one.

In general terms:

$$a^0 = 1$$

where *a* can equal any real number except zero ($a \neq 0$).

*Notice that *negative* powers are just a more concise way to represent the fraction form of powers of ten.

$$10^{-1} = \frac{1}{10} \qquad 10^{-2} = \frac{1}{100} \qquad 10^{-3} = \frac{1}{1000} \qquad 10^{-4} = \frac{1}{10,000}$$

Exponent Rule: For a negative exponent, write the expression in a fraction form so that the numerator is 1 and the denominator is the number to the positive power.

In general terms:

$$a^{-b} = \frac{1}{a^{b}}$$

where *a* can equal any real number except zero ($a \neq 0$).

* *a cannot equal zero because division by zero is undefined* ($\frac{1}{0}$ *is undefined*).

Standard Form for Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:

- a number that is less than 10, but greater than or equal to 1

- a power of ten

$$5.6 \times 10^4 = 56,000$$

In this section, we will take numbers given in scientific notation and write them in standard form.

Example 1: What is the standard form for 7×10^4 ?

$7 \times 10^4 =$

$= 7 \times (10 \times 10 \times 10 \times 10)$ $10^4 = 10 \times 10 \times 10 \times 10$

$= 7 \times 10,000$ $10 \times 10 \times 10 \times 10 = 10,000$

$= 70,000$

The standard form for 7×10^4 is 70,000.

Example 2: What is the standard form for 8.2×10^5 ?

$8.2 \times 10^5 =$

$= 8.2 \times (10 \times 10 \times 10 \times 10 \times 10)$ 8.2

$= 8.2 \times 100,000$ $\times 100000$

$= 820,000$ $820000.\cancel{0} = 820,000$

The standard form for 8.2×10^5 is $820,000$.

A shortcut to multiply by a number that is a power of ten is to start at the decimal point's location and move it to the **right** as many places as the given power.

$$8.2 \times 10^5 = 8\underline{20000}. = 820,000$$

*Use zeros as place holders. In this case, the 2 takes up one place, so four zeros are needed to complete the move of 5 places to the right for the power of 5.

Express Numbers in Scientific Notation

To express a number in scientific notation, determine the first part that is written as a number between 1 and 10, and then determine the power of 10 that will make the expression equal to the given number.

Large Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:

- a number that is less than 10, but greater than or equal to 1

- a power of ten

$$56,000 = 5.6 \times 10^4$$

Example 1: What is the scientific notation for 9,000,000?

Look at the first nonzero digit of the number and write as a **number between 1 and 10**.

9×10^6

Multiply the number by a **power of ten** that will give the standard number.

$$9,000,000 =$$
$$= 9 \times 1,000,000$$
$$= 9 \times 10^6$$

A shortcut to calculate the power is to place a decimal point after the first digit of the number, and then count the number of places **right** to the end of the whole number.

$$9,000,000 = 9.000000 = 9 \times 10^6$$

Count six places to the end of the number, so the power is 6. Counting right gives a positive power.

The scientific notation for 9,000,000 is 9×10^6.

Example 2: What is the scientific notation for 673,000?

| Look at the nonzero digits at the beginning of the number and write as a **number between 1 and 10** by placing a decimal point after the first digit. | 6.73×10^5 | Multiply the number by a **power of ten** that will give the standard number. |

$$673,000 \quad =$$
$$= 6.73 \times 100,000$$
$$= 6.73 \times 10^5$$

A shortcut to calculate the power is to place a decimal point after the first digit of the number, and then count the number of places to the **right** to the end of the number.

$$673,000 = 6.73000 = 6.73 \times 10^5$$

Count five places to the end of the number, so the power is 5. Counting right gives a positive power.

The scientific notation for 673,000 is 6.73×10^5.

Small Numbers in Scientific Notation

> When a number is expressed in scientific notation, it is written as a product of two parts:
>
> - a number that is less than 10, but greater than or equal to 1
>
> - a power of ten
>
> $$0.00347 = 3.47 \times 10^{-3}$$

To express a number in scientific notation, determine the first part that is a number between 1 and 10, and then determine the power of 10 that will make the expression equal to the given number.

Example 3: What is the scientific notation for 0.00000004?

Look at the nonzero digit at the end of the number and write as a **number between 1 and 10**.

$$4 \times 10^{-8}$$

Multiply the number by a **power of ten** that will give the standard number.

$$0.00000004 =$$
$$= 4 \times 0.00000001$$
$$= 4 \times 10^{-8}$$

A shortcut to calculate the power is to place a decimal point after the first non-zero digit of the number, and then count the number of places to the **left** to the original position of the decimal point.

$$\cancel{0}.00000004 = \underset{\text{\textcolor{red}{00000004}}}{.} = 4 \times 10^{-8}$$

Count eight places to the left to the original position of the decimal point, so the power is –8. Counting left gives a negative power.

*The zero located left of the decimal point has no value. It is just a place-holder used to indicate there is no whole number in the decimal number; thus, it is disregarded when calculating the scientific notation.

The scientific notation for 0.00000004 is 4×10^{-8}.

Example 4: What is the scientific notation for 0.0000295?

Look at the nonzero digits at the end of the number and write as a **number between 1 and 10** by placing a decimal point after the first digit.

$$2.95 \times 10^{-5}$$

Multiply the number by a **power of ten** that will give the standard number.

$$
\begin{aligned}
0.0000295 \quad &= \\
&= 2.95 \times 0.00001 \\
&= 2.95 \times 10^{-5}
\end{aligned}
$$

A shortcut to calculate the power is to place a decimal point after the first non-zero digit of the number, and then count the number of places to the **left** to the original position of the decimal point.

$$\cancel{0}.0000295 = \underset{\sim}{00002}.95 = 2.95 \times 10^{-5}$$

Count five places to the left to the original position of the decimal point, so the power is –5. Counting left gives a negative power.

*The zero located left of the decimal point has no value as it is just a place-holder used to indicate there is no whole number in the decimal number; thus, it is disregarded when calculating the scientific notation.

The scientific notation for 0.0000295 is 2.95×10^{-5}.

Compute in Scientific Notation

In this section, we will first review scientific notation and then, examine how to add, subtract, multiply, and divide numbers when they are written in scientific notation. Please recall that numbers in scientific notation have two parts; one part has a value between one and ten and the other part is a power of ten.

Example 1: Explain the meaning of 2.334×10^5.

The first value, 2.334, is a number between 1 and 10 while the second part is 10^5, a power of ten. The number written in standard form is 233,400.

Example 2: Which is larger, 4.34×10^5 or 4.34×10^4?

A number like 4.34×10^5 will be larger than 4.34×10^4 because the power of 5 places the first number in a larger place value.

In standard form, 434,000 > 43,400.

Using scientific notation will make some calculations easier, especially multiplication and division of large numbers.

Multiplying in Scientific Notation

Example 3: Consider 4,250,000,000 times 3,000. Use scientific notation to simplify the problem.

To enter the large number into a normal handheld calculator may be difficult in some instances. In other instances, the results will be shown in scientific notation.

Let's examine the problem this way.

   Step 1: Write each number in scientific notation.

   $$4,250,000,000 \ \times \ 3,000$$
   $$4.25 \times 10^9 \quad \times \quad 3 \times 10^3$$

   Step 2: Multiply the first values of each scientific notation.

   $$4.25 \quad \times \quad 3 = 12.75$$

   Step 3: Multiply the powers of ten.

   $$10^9 \ \times \ 10^3 \ = \ 10^{12}$$

   *Note: Since this is exponent multiplication and the base is the same (10), we only have to add the exponents. Nine factors of ten and 3 factors of ten make 12 factors of ten or 10^{12}.

   Step 4: Put the parts together.

   $$12.75 \ \times \ 10^{12}$$

The answer is correct, but not in scientific notation. (The first value must fall between one and ten.)

   Step 5: We must move the decimal…

   $12.75 \rightarrow 1.275$ smaller by one place value (ie. divide by 10)

   Step 6: We will adjust the 10's…

   $10^{12} \rightarrow 10^{13}$ which is larger by one place value (ie. multiply by 10)

*Multiplying by 10 in *Step 6* balances dividing by 10 in *Step 5*, and thus, the value of the answer remains the same.

By adjusting one part of the number up and one part down, we have balanced and kept the answer the same while also being correct with our scientific notation rules.

The product of 4,250,000,000 times 3,000 is 1.275×10^{13}.

Summarizing the work:

$$
\begin{aligned}
4{,}250{,}000{,}000 \text{ times } 3{,}000 \quad &= 4.25 \times 10^9 \quad \times \quad 3 \times 10^3 \\
&= 4.25 \times 10^9 \quad \times \quad 3 \times 10^3 \\
&= 4.25 \times \quad 3 \quad \times \quad 10^{9+3} \\
&= \quad 12.75 \quad \times \quad 10^{12} \\
&= \quad 1.275 \quad \times \quad 10^{13} \\
&= 12{,}750{,}000{,}000{,}000
\end{aligned}
$$

The product of 4,250,000,000 times 3,000 is 12,750,000,000,000.

Adding and Subtracting in Scientific Notation

When we try addition (or subtraction) in scientific notation, it is important to note the powers of ten should match. This is needed in order for the shortcut to work.

Example 4: Compute $(4.1 \times 10^4) + (5.3 \times 10^4)$ using scientific notation.

*Note: the powers are the same and do not need adjusted.

Step 1: Values first… 4.1 + 5.3 = 9.4

Step 2: Powers next, but since we are adding, the place value will remain the same; thus the power gets copied… 10^4.

Step 3: Put the parts together…

$$(4.1 \times 10^4) + (5.3 \times 10^4) =$$
$$= (4.1 \times 10^4) + (5.3 \times 10^4)$$
$$= 9.4 \times 10^4$$

*Note: This answer does not need adjusted as it is in scientific notation because 9.4 is a number between 1 and 10.

For subtraction follow the same format as addition, only subtract.

Example 5: Compute $(8.5 \times 10^6) - (2.9 \times 10^6)$ using scientific notation.

*Note: the powers are the same and do not need adjusted.

   Step 1: Values first…$8.5 - 2.9 = 5.6$

   Step 2: Powers next, but since we are subtracting, the place value will remain the same; thus the power gets copied… 10^6.

   Step 3: Put the parts together…

$$(8.5 \times 10^6) - (2.9 \times 10^6) =$$
$$= (8.5 \times 10^6) - (2.9 \times 10^6)$$
$$= 5.6 \times 10^6$$

*Note: This answer does not need adjusted as it is in scientific notation because 5.6 is a number between 1 and 10.

Dividing in Scientific Notation

For division follow the same format as multiplication, only divide the values and subtract the powers.

Example 6: Consider $\dfrac{9.42 \times 10^{16}}{3 \times 10^{12}}$ or $(9.42 \times 10^{16}) \div (3 \times 10^{12})$.

Step 1: Numbers… $9.42 \div 3 = 3.14$

Step 2: Powers… $10^{16} \div 10^{12} = 10^{16-12} = 10^4$

Step 3: Answer… 3.14×10^4

Step 4: Putting it all together…

$$\frac{9.42 \times 10^{16}}{3 \times 10^{12}} =$$

$$= \frac{9.42 \times 10^{16}}{3 \times 10^{12}}$$

$$= \frac{9.42}{3} \times 10^{16-12}$$

$$= 3.14 \times 10^4$$

*Note: This answer does not need adjusted as it is in scientific notation because 3.14 is a number between 1 and 10.

Formulas

A **formula** shows how certain quantities are related. A formula may be used to find an unknown quantity if the other quantities are known.

Using the Distance Formula

Step 1: Write the appropriate formula.

Step 2: Replace each variable with the appropriate number.

Step 3: Solve.

Example 1: William traveled 825 miles to St. Helen's Mountain. If he traveled at an average rate of 55 miles per hour, how long did it take William to travel to Mount St. Helen's?

The formula that relates distance, rate or speed, and time is $d = rt$.

- the variable d represents distance
- r represents rate or speed
- t represents time

Step 1: Write the appropriate formula: $\qquad\qquad\quad d = rt$

Step 2: Replace each variable with the appropriate number.

William traveled a distance of 825 miles. $\qquad d = 825$

William traveled at a rate of 55 MPH. $\qquad\quad r = 55$

Replace d with 825 and r with 55 in $d = rt$. $\qquad 825 = 55t$

Step 3: Solve.

$$825 = 55t$$

$$\frac{825}{55} = \frac{\cancel{55}t}{\cancel{55}}$$　　　　Divide both sides of the equation by 55.

$$15 = t$$　　　　　　Simplify.

It took William 15 hours to drive to Mount St Helen's.

Using the Interest Formula

Interest is the amount of money that can be earned or charged on a banking account. If the account is a savings account, then the interest will be added on to the savings periodically. If the account is a loan, the interest is the extra charges that are added on to the loan amount. The formula for simple interest is $I = prt$.
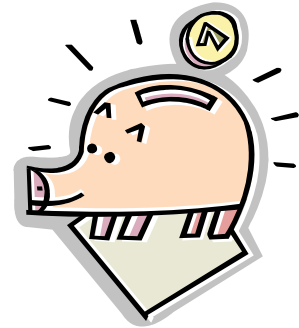
$I = p\,r\,t$		
I	**Interest**	▪ Amount of money earned if the account is savings ▪ Amount of money charged if the account is a loan
p	**Principal**	▪ Original amount of money placed in the account ▪ Amount of the loan
r	**Rate**	Interest rate (a percent expressed as a decimal)
t	**Time**	Time (in years)

Example 2: Jack has $2000 in a savings account that pays simple interest yearly at 3%. How much interest will be earned at the end of the first year?

To solve this problem use the simple interest formula. Replace each variable with the appropriate number, and then solve.

Amount of savings (p) = $2000

Interest rate (r) = 3%

Time of savings (t) = 1 years

$I = p\,r\,t$
$I = 2000 \times 3\% \times 1$
$I = 2000 \times 0.03 \times 1$ $3\% = 0.03$*
$I = \$60$

*To calculate the interest, the rate must be changed to its decimal equivalent. $(3\% = 3/100 = 0.03)$

The amount of interest earned in one year is $60.

Example 3: In the previous problem, calculate the amount of money Jack will have in his savings account once the interest has been added.

Amount of Principal + Interest accrued = Total Amount

$2000   + $60   = $2060

Jack will have $2060 in his savings account after one year.

Example 4: Jenna's bank calculates interest on a semi-annual basis; that is, every 6 months. How much interest will $3000 earn at a 3.5% annual interest rate for 6 months?

Amount of savings $(p) = \$3000$

Interest rate $(r) = 3.5\%$

Time of savings $(t) = 6$ months $= \dfrac{1}{2}$ year*

*Since the interest rate is an annual interest rate, the time must also be based on a year. (6 months = 6/12 of a year = 1/2)

$$I \ = \ p\,r\,t$$
$$I \ = \ 3000 \ \times \ 3.5\% \ \times \ \dfrac{1}{2}$$
$$I \ = \ 3000 \ \times \ 0.035 \ \times \ \dfrac{1}{2} \qquad 3\% = 0.035**$$
$$I \ = \ 105 \ \times \ 0.5 \qquad\qquad \dfrac{1}{2} = 2\overline{)1.0}^{\,0.5} = 0.5$$
$$I \ = \ \$52.50$$

**To calculate the interest, the rate must be changed to its decimal equivalent. $(3\% = 3/100 = 0.03)$

The interest earned for six months is $52.50.