WHOLE NUMBERS AND DECIMALS MULTIPLICATION AND DIVISION

This unit is about computing with whole numbers and decimals using multiplication and division. In addition, many types of problems that use these skills including calculating averages, determining geometric sequences, and solving multi-step problems will be investigated.

Multiply Whole Numbers

Multiply Decimals

Divide Whole Numbers

Divide Decimals

Average (Mean)

Geometric Sequences

Multi-Step Word Problems

Multiply Whole Numbers

One Digit Numbers

Finding patterns can simplify the work and help with computing problems mentally.

Look at the pattern below.

$$\begin{aligned} 8 \quad &\times 3 = 24 \\ 80 \quad &\times 3 = 240 \\ 800 \quad &\times 3 = 2400 \\ 8000 \quad &\times 3 = 24,000 \end{aligned}$$

What is the shortcut for multiplying the following problem?

Example 1: Calculate: $6000 \times 3 = ?$

$6000 \times 3 = 18,000$

Shortcut: Multiply the nonzero numbers $(6 \times 3 = 18)$, and then count the zeros (3 zeros) and place that number of zeros after the $18\,(18,000)$.

$6000 \times 3 = (6 \times 3 = 18 \text{ with 3 zeros}) = 18,000$

Now let's take a look at basic multiplication.

Product is the answer to a multiplication problem.

Estimating is important because estimates can be used to check that the actual answer is reasonable.

Example 2: Estimate the product: $7342 \times 6 = ?$

Round 7342 to the nearest thousand, and then multiply.

$7342 \times 6 = ?$

$7000 \times 6 = 42,000$

The estimated product is 42,000.

Example 3: Calculate the actual product: $7342 \times 6 = ?$

Line up the numbers from right to left.

Multiply by the ones.

7342×6

2

$6 \times 2 = 12$ ones

12 ones = 1 ten + 2 ones

Put down the 2 and *carry* the 1 to tens place.

Multiply by the tens.

7342×6

52

$6 \times 4 = 24 + 1 = 25$ tens

25 tens = 2 hundreds + 5 tens

Put down the 5 and *carry* the 2 to hundreds place.

Multiply by the hundreds.

7342×6

052

$6 \times 3 = 18 + 2 = 20$ hundreds

20 hundreds = 2 thousands + 0 hundreds

Put down the 0 and *carry* the 2 to thousands place.

Now, finish the problem by multiplying by the thousands.

7342×6

$44,052$

$6 \times 7 = 42 + 2 = 44$ thousands

There are no more digits to multiply, so just put down the 44.

The product is 44,052.

Is the answer reasonable? The estimated product is 42,000 (from *Example 2*). The actual product is 44,052. The answer is indeed reasonable.

Two Digit Numbers

Example 4: Estimate the product: $637 \times 48 = ?$

Round 637 to the nearest hundred and 48 to the nearest ten, and then multiply.

$637 \times 48 = ?$

$600 \times 50 = 30,000$

The estimated product is 30,000.

Example 5: Calculate the actual product: $637 \times 48 = ?$

Write the problem vertically.

$$\begin{array}{r} 637 \\ \times\ 48 \\ \hline \end{array}$$

Line up the two numbers from right to left.

Multiply by the ones in 48.

$$\begin{array}{r} {}^{2}6^{5}37 \\ \times\ \ 48 \\ \hline 5096 \end{array}$$

Multiply 637 times 8 ones.

Multiply by the tens in 48.

$$\begin{array}{r} {}^{1}6^{2}37 \\ \times\ \ 4\cancel{8}\ \checkmark \\ \hline 5096 \\ 25480 \end{array}$$

Multiply by 4 tens. Use a 0 as a place holder because 637×40 (4 tens) is actually 25,480.

Ignore the 8 in 48 as it was used in the previous step.

Now add.

$$\begin{array}{r} 637 \\ \times\ 48 \\ \hline 5096 \\ 25480 \\ \hline 30576 \end{array}$$

Add 5096 + 25480 to get 30,576.

The product is 30,576.

Is the answer reasonable? The estimated product is 30,000 (from *Example 4*). The actual product is 30,576. The answer is indeed reasonable.

Problem Solving

Example 6: The computer club is planning to attend a baseball game. There are 26 members in the club and each ticket to the game costs $12. What is the cost of the tickets for all of the members in the computer club?

In this problem you know the cost of one and want to find the cost of many.

$12 ↓ 26 ↓

This is a multiplication problem.

$$\begin{array}{r} 12 \\ \times 26 \\ \hline 72 \\ 240 \\ \hline 312 \end{array}$$

The cost of 26 tickets is $312.

Multiply Decimals

Multiplying Decimals Less Than One

To place the decimal point when multiplying decimals, count the decimal places in each factor and total them. The total is the number of decimal places that will be in the product (answer to a multiplication problem).

Example 1: Estimate the product: $0.7 \times 0.9 = ?$

Round each decimal number to the nearest whole number.

$0.9 \times 0.7 = ?$

$1 \times 1 = 1$

The estimated product is 1.

Example 2: Calculate the actual product: $0.7 \times 0.9 = ?$

Line up the numbers from right to left.

Multiply.

$$\begin{array}{r} 0.9 \\ \times 0.7 \\ \hline 63 \end{array} \qquad 9 \times 7 = 63$$

Place the decimal point by adding up the decimal places in the two decimals being multiplied.

$$\begin{array}{rl} 0.9 & (1 \text{ decimal place}) \\ \times 0.7 & (1 \text{ decimal place}) \\ \hline 0.63 & (2 \text{ decimal places needed in answer}) \end{array}$$

The product is 0.63.

Is the answer reasonable? The estimated product is 1 (from *Example 1*). The actual product is 0.63. The answer is indeed reasonable.

Example 3: Calculate the actual product using fractions instead of decimals and write the final answer in decimal form: $0.7 \times 0.9 = ?$

Write both decimals as fractions.

$$0.7 = \frac{7}{10} \qquad 0.9 = \frac{9}{10}$$

Multiply the fractions.

$$\frac{7}{10} \times \frac{9}{10} = \frac{7 \times 9}{10 \times 10} = \frac{63}{100}$$

Write the fraction as a decimal

$$\frac{63}{100} = 0.63$$

The product is 0.63 and is read as 63 hundredths.

*Note: The fraction form of the problem connects the decimal answer to the procedure explained in *Example 2* and shows why two decimal places are necessary in the answer.

Example 4: Calculate the product: $0.12 \times 0.36 = ?$

Line up the numbers from right to left.

Multiply.

$$\begin{array}{r} 0.12 \\ \times 0.36 \\ \hline 72 \\ 360 \\ \hline 432 \end{array}$$

Place the decimal point by adding up the decimal places in the two decimals being multiplied.

$$\begin{array}{r} 0.12 \text{ (2 decimal places)} \\ \times 0.36 \text{ (2 decimal places)} \\ \hline 72 \\ 360 \\ \hline 0.0432 \text{ (4 decimal places needed in answer)} \end{array}$$

*Note: A zero is placed in front of the 4 as a place holder so that there will be four places since the multiplication only gave three digits.

The product is 0.0432.

Example 5: Verify the number of decimal places in the answer to the previous problem by calculating the problem using fractions, and then writing a decimal for the answer.

$$0.12 \times 0.36 = ?$$

Write both decimals as fractions.

$$0.12 = \frac{12}{100} \qquad 0.36 = \frac{36}{100}$$

Multiply the fractions.

$$\frac{12}{100} \times \frac{36}{100} = \frac{12 \times 36}{100 \times 100} = \frac{432}{10,000}$$

Write the fraction as a decimal

$$\frac{432}{10,000} = 0.0432$$

The product is 0.0432 and is read as 432 ten thousandths.

Now, let's look at a few more examples of using decimals when multiplying.

Example 6: Calculate the product: $5.23 \times 7.9 = ?$

$$\begin{array}{r} 5.23 \text{ (2 decimal places)} \\ \times \quad 7.9 \text{ (1 decimal place)} \\ \hline 4707 \\ 36610 \\ \hline 41.317 \text{ (3 decimal places needed in answer)} \end{array}$$

The product is 41.317 and is read as 41 and 317 thousandths.

Is the answer reasonable? The estimated product would be $5 \times 8 = 40$. The actual product is 41.317 and could be written in fraction form as $41\frac{317}{1000}$. The answer is indeed reasonable.

Example 7: Calculate the product: $46 \times 2.8 = ?$

$$\begin{array}{r} 46 \text{ (0 decimal places)} \\ \times \quad 2.8 \text{ (1 decimal place)} \\ \hline 368 \\ 920 \\ \hline 128.8 \text{ (1 decimal place needed in answer)} \end{array}$$

The product is 128.8 and is read as 128 and 8 tenths.

Is the answer reasonable? The estimated product would be $50 \times 3 = 150$. The actual product is 128.8 and could be written in fraction form as $128\frac{8}{10}$. The answer is indeed reasonable.

Example 8: Calculate the product: $5.23 \times 3.79 = ?$

$$
\begin{array}{rl}
 & 5.23 \text{ (2 decimal places)} \\
\times & 3.79 \text{ (2 decimal places)} \\
\hline
 & 4707 \\
 & 36610 \\
 & 156900 \leftarrow \textit{Two zeros needed in the third multiplication.} \\
 & 19.8217 \text{ (4 decimal places needed in answer)}
\end{array}
$$

The product is19.8217 and is read as 19 and 8,217 ten thousandths.

Is the answer reasonable? The estimated product would be $5 \times 4 = 20$. The actual product is 19.8217 and could be written in fraction form as $19\frac{8217}{10,000}$. The answer is reasonable.

Divide Whole Numbers

Division by One Digit

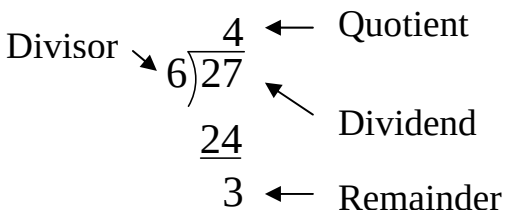
In a division problem, each part has a name.

Quotient is the answer to a division problem.

Dividend is the number that is being divided.

Divisor is the number by which the dividend is being divided.

Remainder is the amount left over when the division problem does not come out even.



Example 1: Estimate the quotient: $25,798 \div 7$

Since 28 is divisible by 7 evenly, we'll round 25,798 to 28,000.

$$28,000 \div 7 =$$

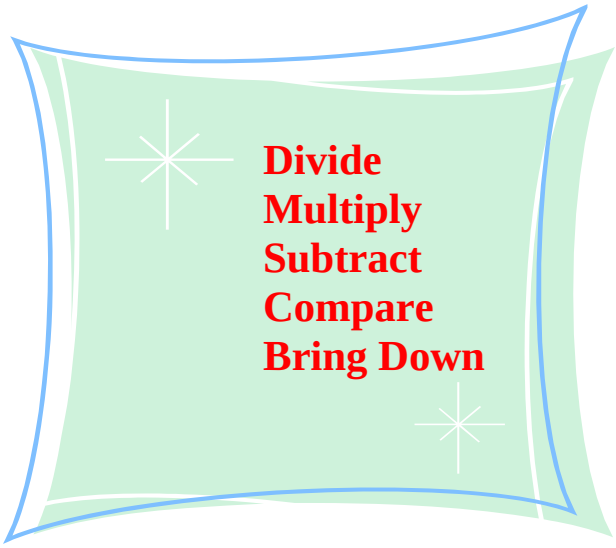
$$\begin{array}{r} 4000 \\ 7\overline{)28000} \\ \underline{28} \\ 0000 \end{array}$$

The estimated quotient is 4,000.

**Shortcut*: Divide the nonzero numbers $(28 \div 7 = 4)$, and then count the zeros in 28,000 (3 zeros) and place that number of zeros after the $4 (4,000)$.

$$28000 \div 7 = (28 \div 7 = 4 \text{ with 3 zeros}) = 4,000$$

To use the division algorithm correctly, follow these five steps:



Example 2: Find the actual quotient: $25,798 \div 7$

$$25,798 \div 7 =$$

Step 1:

$$\begin{array}{r} 3 \\ 7\overline{)\,25798} \\ \underline{21} \\ 4 \end{array}$$

Divide 7 into 25 to get 3. Four would be too high and give 28.

Multiply $7 \times 3 = 21$.

Subtract $25 - 21 = 4$

$$\begin{array}{r} 3 \\ 7\overline{)\,25\ 798} \\ \underline{21}\downarrow \\ 4\ 7 \end{array}$$

Compare 4 with 7. Is 4 less than 7? Yes, so it is OK. If the number were higher than 7, we would need to check the division.

Bring down the next digit, 7.

Step 2:

$$\begin{array}{r} 36 \\ 7\overline{)25798} \\ \underline{21} \\ 47 \\ \underline{42} \\ 5 \end{array}$$

Divide 7 into 47 to get 6. Seven would be too high and give 49.

Multiply $7 \times 6 = 42$.

Subtract $47 - 42 = 5$

$$\begin{array}{r} 36 \\ 7\overline{)257\,98} \\ \underline{21}\ \downarrow \\ 47 \\ \underline{42}\downarrow \\ 5\ 9 \end{array}$$

Compare 5 with 7. Is 5 less than 7? Yes, we continue with the algorithm.

Bring down the next digit, 9.

Step 3:

$$\begin{array}{r} 368 \\ 7\overline{)25798} \\ \underline{21} \\ 47 \\ \underline{42} \\ 59 \\ \underline{56} \\ 3 \end{array}$$

Divide 7 into 59 to get 8. Nine would be too high and give 63.

Multiply $7 \times 8 = 56$.

Subtract $59 - 56 = 3$

$$\begin{array}{r} 368 \\ 7\overline{)25798} \\ \underline{21}\ \downarrow \\ 47 \\ \underline{42}\ \downarrow \\ 59 \\ \underline{56}\downarrow \\ 3\ 8 \end{array}$$

Compare 3 with 7. Is 3 less than 7? Yes, we continue with the algorithm.

Bring down the next digit, 8.

Step 4:

$$
\begin{array}{r}
3685 \\
7\overline{)25798} \\
\underline{21} \\
47 \\
\underline{42} \\
59 \\
\underline{56} \\
38 \\
\underline{35} \\
3
\end{array}
$$

Divide 7 into 38 to get 5. Six would be too high and give 42.

Multiply $7 \times 5 = 35$.

Subtract $38 - 35 = 3$

There are no more digits to bring down, so the process is complete.

Following the division algorithm, we found the answer of 3685 with a remainder of 3.

The quotient is 3685 R3 where R3 means "remainder of 3".

Is the answer reasonable? The estimated quotient is 4000 (from *Example 1*). The actual quotient is 3685 R3. The answer is indeed reasonable.

To check the answer to a division, you can multiply the divisor times the quotient and add on the remainder.

$$\text{Quotient } (3685) \times \text{ Divisor } (7) + \text{ Remainder } (3) = \text{ Dividend } (25,798)$$

$$3685 \times 7 + 3 =$$

$$25795 + 3 =$$

$$25,798 = 25,798 \checkmark$$

Division by One Digit with Zeros

Example 3: Find the actual quotient: $1227 \div 4$

$$\begin{array}{r} 3 \\ 4\overline{)12\,27} \\ \underline{12\downarrow} \\ 0\ 2 \end{array} \qquad \begin{array}{r} 30 \\ 4\overline{)1227} \\ \underline{12\ \downarrow} \\ 02 \\ \underline{00} \\ 27 \end{array} \qquad \begin{array}{r} 306 \\ 4\overline{)1227} \\ \underline{12} \\ 02 \\ \underline{00} \\ 27 \\ \underline{24} \\ 3 \end{array}$$

*Notice that on the second step, 4 will not divide into 2, so a zero must be placed in the quotient. After placing the zero, proceed on with the multiplication $(4 \times 0 = 0)$.

The quotient is 306 R3.

Is the answer reasonable? The estimated quotient would be $1200 \div 4 = 300$. The actual quotient is 306 Remainder 3. The answer is reasonable.

Check the answer by multiplying the divisor times the quotient and adding on the remainder.

Quotient (306) × Divisor (4) + Remainder (3) = Dividend (1227)

$306 \times 4 + 3 =$

$1224 + 3 =$

$1227 = 1227$ ✓

Division by Two Digits

Example 4: Find the actual quotient: $3691 \div 39$

$$3691 \div 39 =$$

Step 1:

Hint: Round 39 to 40, then think "How many 40's are in 369?"

$$\begin{array}{r} 9 \\ 39\overline{)3691} \\ \underline{351} \\ 18 \end{array}$$

Divide 39 into 369 to get 9.

Multiply $39 \times 9 = 351$.

Subtract $369 - 351 = 18$

Compare 18 with 39. Is 18 less than 39? Yes, so it is OK. If the number were higher than 39, we would need to check the division.

$$\begin{array}{r} 9 \\ 39\overline{)3691} \\ \underline{351}\downarrow \\ 181 \end{array}$$

Bring down the next digit, 1.

Step 2:

Hint: Round 39 to 40, then think "How many 40's are in 181?"

$$\begin{array}{r} 94 \\ 39\overline{)3691} \\ \underline{351}\downarrow \\ 181 \\ \underline{156} \\ 25 \end{array}$$

Divide 39 into 181 to get 4. Five would be too high and give 195.

Multiply $39 \times 4 = 156$.

Subtract $181 - 156 = 25$

There are no more digits to bring down, so the process is complete.

The quotient is 94 R25.

Is the answer reasonable? The estimated quotient would be $4000 \div 40 = 100$. The actual quotient is 94 Remainder 25. The answer is reasonable.

Check the answer by multiplying the divisor times the quotient and adding on the remainder.

Quotient (94) × Divisor (39) + Remainder (25) = Dividend (3691)

$$94 \times 39 + 25 =$$

$$3666 + 25 =$$

$$3691 = 3691 \checkmark$$

Division by Two Digits with Zeros

Example 5: Find the actual quotient: $1859 \div 23$

$$1859 \div 23 =$$

Step 1:

Hint: Round 23 to 20, then think "How many 20's are in 185?"

Try 9, but that gives 207, so drop down to 8.

$$\begin{array}{r} 8 \\ 23\overline{)1859} \\ \underline{184} \\ 1 \end{array}$$

Divide 23 into 185 to get 8.

Multiply $23 \times 8 = 184$.

Subtract $185 - 184 = 1$

Compare 1 with 23. Is 1 less than 23? Yes, so it is OK.

$$\begin{array}{r} 8 \\ 23\overline{)1859} \\ \underline{184}\downarrow \\ 19 \end{array}$$

Bring down the next digit, 9.

$$\begin{array}{r} 80 \\ 23\overline{)1859} \\ \underline{184}\downarrow \\ 19 \end{array}$$

Since 19 is smaller than 23 after bringing down the 9, a zero must be placed in the quotient in the divide step.

Nothing more needs to be done, the problem is complete.

The quotient is 80 R19.

Is the answer reasonable? The estimated quotient would be $2000 \div 20 = 100$. The actual quotient is 80 Remainder 19. The answer is reasonable.

Check the answer by multiplying the divisor times the quotient and adding on the remainder.

Quotient (80) × Divisor (23) + Remainder (19) = Dividend (1859)

$80 \times 23 + 19 =$

$1840 + 19 =$

$1859 = 1859$✓

Problem Solving

Example 6: There are 26 members in the PALS Club. The club members raised $500 to attend the local hockey game. The hockey game tickets totaled $318. After purchasing the tickets, the club members decided to share the money that was left equally. How much did each member receive?

This is a two step problem.

Step #1: Figure the amount of money that was left after purchasing the tickets (subtract).

$$500 - 318 = 182$$

The amount of money left after purchasing the tickets was $182.

Step #2: Calculate the equal shares for each of the 26 members. To figure the equal shares, divide by the number of members.

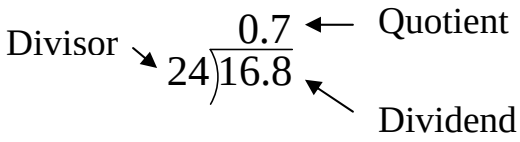
$$\begin{array}{r} 7 \\ 26\overline{)182} \\ \underline{182} \end{array}$$

Each member received $7.

Divide Decimals

Divide by Whole Numbers

In decimal division, the division algorithm works the same as when dividing whole numbers; BUT, placing the decimal point correctly must be considered carefully!



When using the division algorithm and **dividing a decimal by a whole number**, the decimal point in the quotient is placed directly about the location of the decimal point in the dividend.

Example 1: Calculate the quotient: $16.8 \div 24$

$$\begin{array}{r} 0.7 \\ 24\overline{)16.8} \\ \underline{16\,8} \end{array}$$

In the quotient, place the decimal point right about its location in the dividend.

The quotient is 0.7.

Quotient × Divisor = Dividend
0.7 × 24 = 16.8

Divide by Tenths

To divide a decimal number by a decimal number, make the divisor a whole number by multiplying it by a power of ten needed to **move the decimal to the right of all of the digits in the divisor**, and then multiply the dividend by the same power of ten.

Example 2: Calculate the quotient: $29.24 \div 3.4$

*Recall that the shortcut for multiplying by 10 is to move the decimal point right once place.

$$\frac{29.24}{3.4} \times 1 = \frac{29.24}{3.4}$$

$$\frac{29.24}{3.4} \times \frac{10}{10}^{(1)} = \frac{292.4}{34.}$$

Multiplying both the dividend and the divisor by 10 is the same as multiplying the whole division problem by 10/10 which equals 1. This, in turn, does not alter the true value of the answer to the problem.

Therefore,

$$\frac{29.24}{3.4} = \frac{292.4}{34.}$$

*Carats (^) are used to show the movement of the decimal point in the division algorithm.

In this problem, to make the divisor (3.4) a whole number (34), move the decimal point one place to the right.

Complete the process by moving the decimal point one place to the right in the dividend, and then placing the decimal point above the carat in the quotient.

$$3.4\overset{\wedge}{)}\overline{29.2\overset{\wedge}{4}}$$

Now, divide, keeping all numbers lined up as is done in division of whole numbers.

$$\begin{array}{r} 8.6 \\ 3.4\overset{\wedge}{)}\overline{29.2\overset{\wedge}{4}} \\ 27\,2\downarrow \\ \hline 2\,0\,4 \\ 2\,0\,4 \\ \hline \end{array}$$

The quotient is 8.6.

Quotient	×	Divisor	=	Dividend
8.6	×	3.4	=	29.24

Divide by Hundredths

Example 3: Calculate the quotient: $8.0124 \div 1.32$

*Recall that the shortcut for multiplying by 100 is to move the decimal point right two places.

$$\frac{8.0124}{1.32} \times 1 = \frac{8.0124}{1.32}$$

$$\frac{8.0124}{1.32} \times \overset{(1)}{\frac{100}{100}} = \frac{801.24}{132.}$$

Multiplying both the dividend and the divisor by 100 is the same as multiplying the whole division problem by 100/100 which equals 1. This, in turn, does not alter the true value of the answer to the problem.

Therefore,

$$\frac{8.0124}{1.32} = \frac{801.24}{132.}$$

*Carats (^) are used to show the movement of the decimal point in the division algorithm.

In this problem, to make the divisor (1.32) a whole number (132), move the decimal point two places to the right.

Complete the process by moving the decimal point two places to the right in the dividend, and then placing the decimal point above the carat in the quotient.

$$1.32_{\wedge}\overline{)8.01_{\wedge}24}$$

Now, divide, keeping all numbers lined up as is done in division of whole numbers.

$$
\begin{array}{r}
6.07 \\
1.32\overset{\wedge}{)}\overline{8.01\overset{\wedge}{2}4} \\
\underline{7\,92}\downarrow \\
9\,2 \\
\underline{\quad 0}\downarrow \\
9\,24 \\
\underline{9\,24} \\
\end{array}
$$

The quotient is 6.07.

Quotient × Divisor = Dividend
6.07 × 1.32 = 8.0124

Divide and Round Quotients

In division of decimals, sometimes the answer does not come out even. In this case, we divide until the quotient has one extra decimal place, and then round to the given place.

Example 4: Calculate the quotient of $95.8 \div 0.24$ and round the answer to the nearest hundredth.

*Recall that the shortcut for multiplying by 100 is to move the decimal point right two places.

$$\frac{95.8}{0.24} \times 1 = \frac{95.8}{0.24}$$

$$\frac{95.8}{0.24} \times \overset{(1)}{\frac{100}{100}} = \frac{9580.}{24.}$$

Multiplying both the dividend and the divisor by 100 is the same as multiplying the whole division problem by 100/100 which equals 1. This, in turn, does not alter the true value of the answer to the problem.

Therefore,

$$\frac{95.8}{0.24} = \frac{9580.}{24.}$$

*Carats (∧) are used to show the movement of the decimal point.

In this problem, to make the divisor (0.24) a whole number (24), move the decimal point two places to the right.

Complete the process by moving the decimal point two places to the right in the dividend, and then place the decimal point above the carat in the quotient.

*In this problem, **a zero is added as a place holder** to show two places for moving the decimal point two places to the right in the dividend.

$$0.24_{\wedge}\overline{)95.80_{\wedge}}$$

Now, divide, keeping all numbers lined up as done in division of whole numbers.

Additional zeros may be added, as needed, to divide and have enough digits in the quotient to express the answer rounded to the nearest hundredth. *Three* more zeros must be added so that the quotient will be in thousandths giving an extra digit for rounding to the nearest hundredth.

$$\begin{array}{r} 399.166 \\ 0.24_{\wedge}\overline{)95.80_{\wedge}000} \\ \underline{72}\downarrow \\ 238 \\ \underline{216}\downarrow \\ 220 \\ \underline{216}\downarrow \\ 40 \\ \underline{24}\downarrow \\ 160 \\ \underline{144}\downarrow \\ 160 \\ \underline{144} \\ 16 \end{array}$$

Do not continue the division as the quotient has enough decimal places (3) for rounding to nearest hundredth (2 decimal places).

In the quotient, identify the number in hundredths place (6) and look at the number to the right of hundredths place (6). If that number is 5 or higher, round up; otherwise, write hundredths place as is.

399.166 rounds to 399.17

The symbol for approximately equal to is "≈".

399.166... ≈ 399.17

*Note: If the answer would have been 72.333… and given to be rounded to the nearest hundredth, the rounded answer would be 7.33 since the number to the right of hundredths place (3) is less than 5.

Average (Mean)

When analyzing data, the study of the "measures of central tendency" is a necessity. In this unit we will one of these measures, the *mean.*

The **mean** is the **average** of the numbers in the data list. To calculate the mean, **add** the numbers, and then **divide** by the number of numbers.

Example: The ninth grade class collects cans of dog food for the animal shelter. The following number of cans of dog food was collected per day. Find the mean to show an overall average of the number of cans collected per day.

What is the mean (average) number of cans collected per day?

25 32 15 47 25 38 40 32 25 45 50 27

First find the sum, and then divide by the number of numbers (12) to calculate the mean.

Step 1: Find the sum of the numbers.

25
32
15
47
25
38
40 } 12 numbers
32
25
45
50
27
Sum → 401

Step 2: Divide by the number of numbers.

$$12\overline{)401.0}$$

33.4
36
41
36↓
5 0
4 8
2

33.4 ≈ 33

The **mean** is about **33** cans per day.

Quotient × Divisor = Dividend

33.4 × 12 = 400.8 ≈ 401

Geometric Sequences

sequence – A sequence is a list of numbers in a certain order connected through a pattern.

term – A term is any of the numbers in a sequence.

geometric sequence – A geometric sequence is a sequence in which the ratio between any two successive terms is the same.

Example 1: Find the next three numbers in the sequence.

{6, 12, 24, 48 ...}

Notice that each number in the above sequence is twice as much as the number before it. Each successive term has a ratio of 2.

x2 x2 x2
{6, 12, 24, 48 ...}

To find the next three numbers in the sequence, **follow the pattern**, multiplying by two each time.

x2 x2 x2
{6, 12, 24, 48, 96, 192, 384 ...}

The next three numbers in the sequence are 96, 192, and 384.

Example 2: Find the next three numbers in the sequence.

{1.5, 1.8, 2.16, 2.592 ...}

Notice that each number in the above sequence is 1.2 as much as the number before it. Each successive term has a ratio of 1.2.

x1.2 x1.2 x1.2
{1.5, 1.8, 2.16, 2.592 ...}

To find the next three numbers in the sequence, **follow the pattern**, multiplying by 1.2 each time.

x 1.2 x 1.2 x 1.2

{1.5, 1.8, 2.16, 2.592, 3.1104, 3.73248, 4.478976 …}

The next three numbers in the sequence are 3.1104, 3.73248, and 4.478976.