PERMUTATIONS AND COMBINATIONS

In this unit, you will first examine some probability topics such as the "Fundamental Counting Principal", permutations, and combinations. Permutations and combinations are different ways to make arrangements out of objects. A permutation is an arrangement of objects in which the order of the arrangement is important to the outcome. A combination is an arrangement of objects where order is not important.

Fundamental Counting Principal

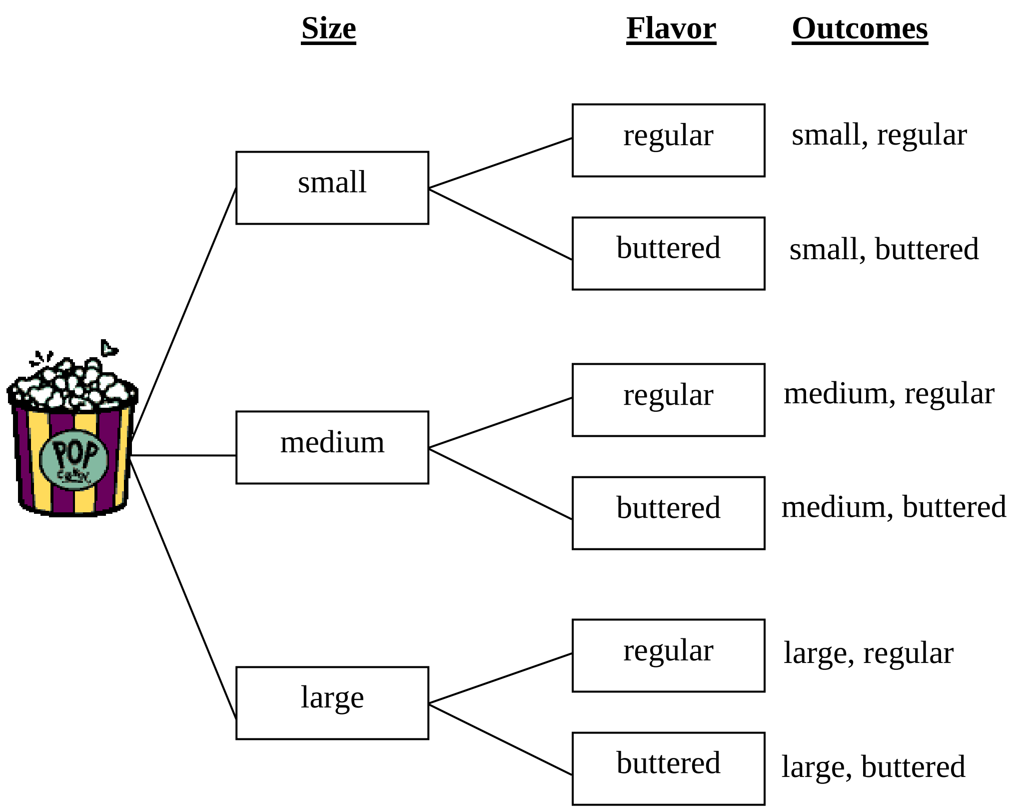
Permutations

Combinations

Fundamental Counting Principle

If there are *m* ways that one event can occur and *n* ways that another event can occur, then there are $m \times n$ ways that both events can occur.

Example 1: A movie theatre sells popcorn in small, medium, and large containers. Each size is also available in regular or buttered popcorn. How many options for buying popcorn does the movie theatre provide?



There are 6 possible options for buying popcorn at the movie theatre.

Example 2: How many options are there if the theatre adds three new flavors - caramel apple, jelly bean, and bacon cheddar?

For each size, you would add the three new flavors to the tree diagram. Applying the fundamental counting theorem, you would multiply 3 $\times$ 5 to get 15.

There would be a total of 15 options.

Example 3: Emily is choosing a password to gain access to the Internet. She decides not to use the digit 0 or the letters A, E, I, O, or U. Each letter or number may be used more than once. How many passwords of 3 letters followed by 2 digits are possible?

Use the fundamental counting principle. There are 21 possible letters and 9 possible digits.

first letter		second letter		third letter		first digit		second digit
?		?		?		?		?
21 choices	$\times$	21 choices	$\times$	21 choices	$\times$	9 choices	$\times$	9 choices

The number of possible passwords for Emily is $21^3 \times 9^2$ or 750,141.

Permutations

Another way to arrange objects is called permutations. A **permutation** is an arrangement of objects in a specific order. Such arrangements could include the batting order of a softball team, seat assignments in a classroom, or items displayed on a store shelf.

The following is the formula for finding the number of permutations of *n* objects taken *r* at a time.

Permutation of "n objects taken r at a time"

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Example: Find the number of ways to listen to 5 CDs from a selection of 12 CDs.

$$_{12}P_{5} = \frac{12!}{(12-5)!}$$

$$_{12}P_{5} = \frac{12!}{7!}$$

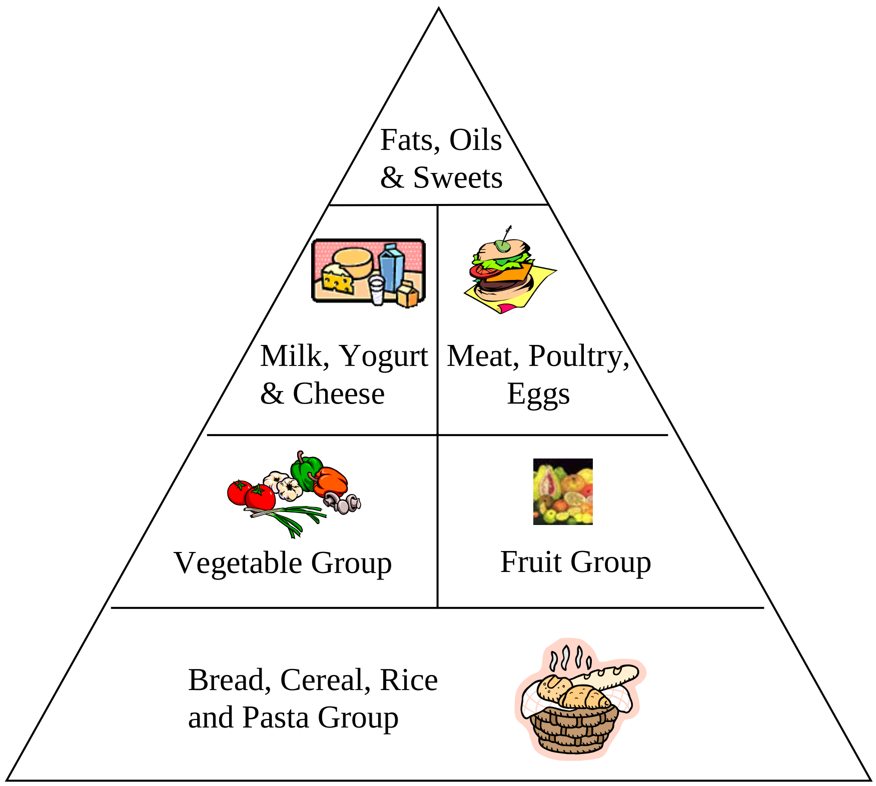
$$_{12}P_{5} = \frac{12\times 11\times 10\times 9\times 8\times \cancel{7!}}{\cancel{7!}}$$

$$_{12}P_{5} = 95,040$$

There are 95,040 different ways to listen to 5 CDs from a selection of 12 CDs.

Combinations

A well-planned meal or balanced diet gives you all the nutrients you need each day. To plan a balanced diet, you need to select foods from each of the main food groups. The food pyramid below is a practical tool to help you make food choices that are consistent with the dietary guidelines for Americans.



We are going to take a look at the different types of foods Hanna has for her friends and separate them into the food groups:

Meats: chicken and fish

Dairy: milk and cheese

Breads: spaghetti, brown rice, crackers, mixed nuts, dinner rolls

Fruits: mixed fruit, peaches

Vegetables: spaghetti sauce, lettuce

To determine the number of possible meals, Hanna will multiply the number of each type as shown below:

# of Meats	×	# of Dairy	×	# of Breads	×	# of Fruits	×	# of Vegetables	=	Total Possible Meals
2	×	2	×	5	×	2	×	2	=	80

Thus, Hanna could plan 80 different meals.

This example is an introduction to combinations**, an arrangement of objects in which order is not important. Such situations may occur when choosing members for a committee, drawing numbers for bingo , or determining your chances of winning the lottery. The following formula for combinations is given below.**

Combination of "*n* objects taken *r* at a time"

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

To understand this formula, we need to understand the symbolism.

The symbol "!" is a mathematical expression called "factorial".

This means that for an integer *n*, *n!* means to multiply all integers 1, 2, 3, 4, …, *n* together to produce a result called "*n factorial*". In combination problems, "*n* factorial" is written as "$n \ldots \times 4 \times 3 \times 2 \times 1$".

Example 1: $5! = 5\times4\times3\times2\times1 = 120$

Example 2: $\frac{9!}{5!} = \frac{9\times8\times7\times6\times\cancel{5\times4\times3\times2\times1}}{\cancel{5\times4\times3\times2\times1}} = 3024$

Notice that 5!, which is, $5 \times 4 \times 3 \times 2 \times 1$, can be cancelled from 9!,

so $\frac{9!}{5!}$ could actually be written as

$$\frac{9 \times 8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}}$$

This now brings us to the formula, ${}_nC_r = \frac{n!}{r!(n-r)!}$. The n represents the number of things that are available, and the r represents the number of things you are choosing.

Example 3: A pizza parlor offers a selection of 8 different toppings. In how many ways can a pizza be made with 3 toppings?

8 represents *n*, the number of total toppings

3 represents *r*, the number of toppings you are choosing.

Replace these numbers in the formula and solve.

$${}_8C_3 = \frac{8!}{3!(8-3)!}$$

$${}_8C_3 = \frac{8!}{3!(5!)}$$

$${}_8C_3 = \frac{8 \times 7 \times 6 \times \cancel{5!}}{3!(\cancel{5!})}$$

$${}_8C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$$

$${}_8C_3 = 56$$

There are 56 combinations of choosing 3 toppings from 8 selections.

Sometimes you will need to find combinations of more than one thing at a time. In this case, multiply the combinations together to find the total amount of combinations.

Let's go back to the pizza example and add different sizes to the list of choices.

Follow the example below.

Example 4: A pizza parlor offers a selection of 8 different toppings and 3 different sizes. In how many ways can a pizza be ordered with the following selections: 2 sizes and 4 toppings?

3 sizes	$\times$	8 toppings
choosing 2		choosing 4

$$
\begin{aligned}
& {}_3C_2 \times {}_8C_4 \\
&= \frac{3!}{2!(3-2)!} \times \frac{8!}{4!(8-4)!} \\
&= \frac{3\times\cancel{2\times1}}{\cancel{2\times1}(1!)} \times \frac{\overset{2}{\cancel{8}}\times7\times\cancel{6}\times5\times\cancel{4!}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1(\cancel{4!})} \\
&= \frac{3}{1} \times \frac{2\times7\times5}{1} \\
&= 3 \times 70 \\
&= 210
\end{aligned}
$$

Thus, there are 210 combinations of 2 sizes and 4 toppings.