CIRCLES

In this unit, you will investigate various aspects of the circle and related geometric figures. You will study central angles, arc measure, and circumference. You explore the meaning of "Pi" and it relates to circles. You will construct circles graphs and examine theorems about the properties of circles and related geometric figures.

Parts of a Circle

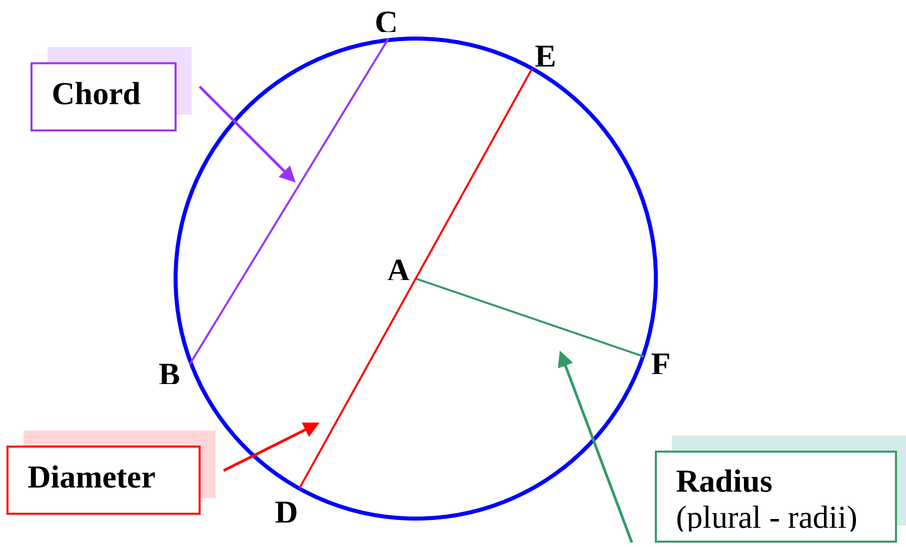
circle – A circle is a set of points that are a given distance from a given center point all of which lie in a plane

radius – A radius of a circle is a line segment that has its endpoints on the center of the circle and a point on the circle.

chord – A chord of a circle is a line segment that has its endpoints on the circle.

diameter –A diameter of a circle is a chord which passes through the center of a circle.

circumference – The circumference of a circle is the distance around a circle.



The circle above is called Circle A which is represented by A. Circles are generally named by their center point.

$\overline{BC}$ and $\overline{DE}$ are chords.

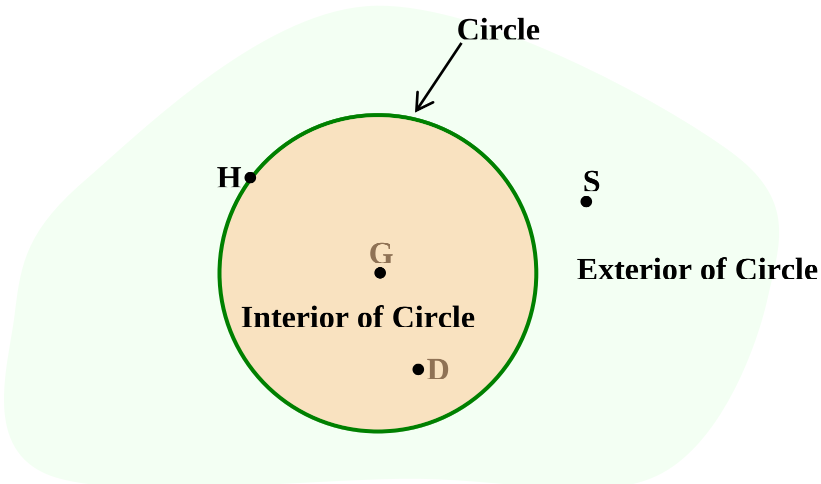
Since $\overline{DE}$ passes through the center and is a chord, $\overline{DE}$ is a diameter.

$\overline{AD}$, $\overline{AE}$, and $\overline{AF}$ are radii.

circle (another definition) – A circle is a set of points whose distance from the center is equal to the radius of the circle.

interior of a circle – The interior of a circle is the set of points whose distance from the center is less than the length of the radius of the circle.

exterior of a circle – The exterior of a circle is the set of points whose distance from the center is greater than the length of the radius of the circle.



Point	**Location**
H	on ⊙ G
G	in interior of ⊙ G
D	in interior of ⊙ G
S	in exterior of ⊙ G

In a circle, a diameter is twice as long as the radius, and conversely, a radius is half the length of a diameter.

$$d = 2r \qquad\qquad r = \frac{1}{2}d$$

Let's take a look at why the statement above can be made.

$\overline{DE}$ is a diameter.	Given
DA, AE, and AF are radii.	Given
$\overline{DE} = \overline{DA} + \overline{AE}$	Segment Addition
$d = r + r$	Substitution
$d = 2r$	Simplify
$\frac{1}{2}d = r$	Division property

Therefore, $d = 2r$ or $r = \frac{1}{2}d$.

Pi (π) is an irrational number that is used with circles. For our purposes in this course, we will round "pi" to 3.14.

Here are the first 500 digits of "Pi". How many digits can you memorize?

3.141592653589793238462643383279502884197169399375105820974944592
307816406286208998628034825342117067982148086513282306647093844609
550582231725359408128481117450284102701938521105559644622948954930
381964428810975665933446128475648233786783165271201909145648566923
460348610454326648213393607260249141273724587006606315588174881520
920962829254091715364367892590360011330530548820466521384146951941
511609433057270365759591953092186117381932611793105118548074462379
9627495673518857527248912279381830119 49

*Note: A repeating pattern of numbers never develops in "Pi". Pi (π) is an irrational number.

Many mathematicians worldwide are fascinated with "Pi". Scroll through the unit link to "The Life of Pi" to review past and present explorations of "Pi".

The circumference of a circle is equal to the diameter of the circle times “Pi” or two times the radius of the circle times “Pi”.

$C = \pi d$ or $C = 2\pi r$

$C = \pi d$	
$C = \pi(2r)$	$d = 2r$
$C = 2\pi r$	Commutative Property

Therefore, $C = \pi d$ or $C = 2\pi r$.

*Note: Circumference is a measurement of length and measured in plain units. Examples: inches, feet, centimeters, millimeters, kilometers.

Example 1: Find the circumference of a circle with a radius that measures 15.3 centimeters. Express the answer in terms of “Pi” and rounded to the nearest tenth.

$C = 2\pi r$	Formula for Circumference
$C = 2\pi(15.3)$	Substitution
$C = 30.6\pi$	Simplify all numbers except "Pi".

The circumference is 30.6π centimeters.

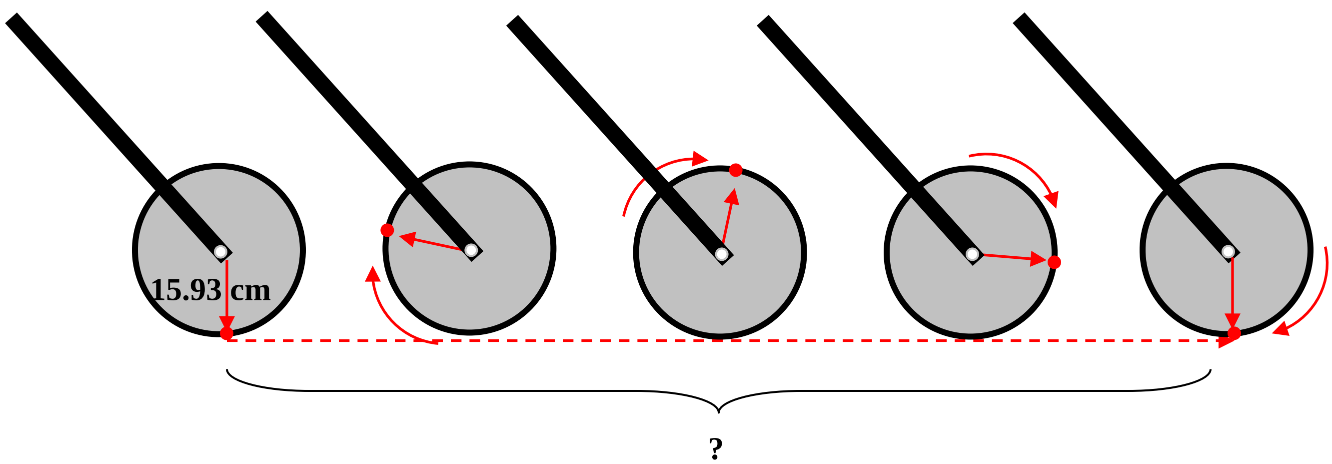
Example 2: Find the approximate diameter and radius of a circle with a circumference of 3256 inches. Round the answer to the nearest whole number.

$C = 2\pi r$	Formula for Circumference
$\dfrac{C}{2\ \pi} = r$	Division
$\dfrac{3256}{2(3.14)} = r$	Substitution ($C = 3256,\ \pi \approx 3.14$)
$r \approx 518$	Simplify
$d = 2\ r$	Definition of Diameter
$d = 2(518)$	Substitution
$d \approx 1036$	Simplify

The diameter of the circle is approximately 1036 centimeters.

The radius of the circle is approximately 518 centimeters.

Example 3: Read the following scenario, and then answer the questions below: Eleanor's teacher gave her a trundle wheel to use for measuring the distance across the field used for physical education class. The radius of the trundle wheel was 15.93 centimeters. Eleanor began by setting the trundle wheel right after it clicked. Eleanor then rolled the wheel across the field along a line that was perpendicular to the length of the field. The trundle wheel clicked 15 times.



(a) What distance is covered in one complete revolution of the wheel (1 click)? Round the answer to nearest whole number.

If we relate the distance covered to one complete revolution of the wheel, we realize that this distance is just simply the circumference of the wheel.

$C = 2\pi r$	Formula for circumference
$C = 2(3.14)(15.93)$	Substitution
$C \approx 100$	Simplify

The distance covered by one complete revolution (circumference) of the trundle wheel is about 100 centimeters.

(b) How many meters wide is the field?

Since the trundle wheel clicked 15 times for Eleanor, this means that the wheel made 15 complete revolutions.

1 revolution = 100 cm -determined in the question a.

15 revolutions = 15(100) =1500 cm

HOWEVER, the question asks "How many METERS wide is the field?"

So, we must consider that 1 meter = 100 centimeters.

We can write a proportion to solve the problem.

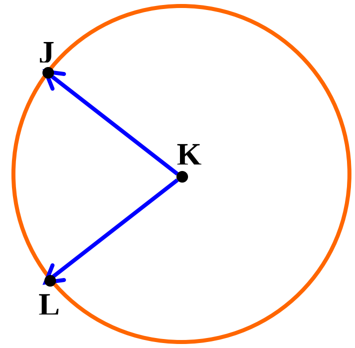
$$\frac{1 \text{ m}}{100 \text{ cm}} = \frac{x}{1500 \text{ cm}}$$

$$100x = 1500$$
$$x = 15$$

The field is approximately 15 meters wide.

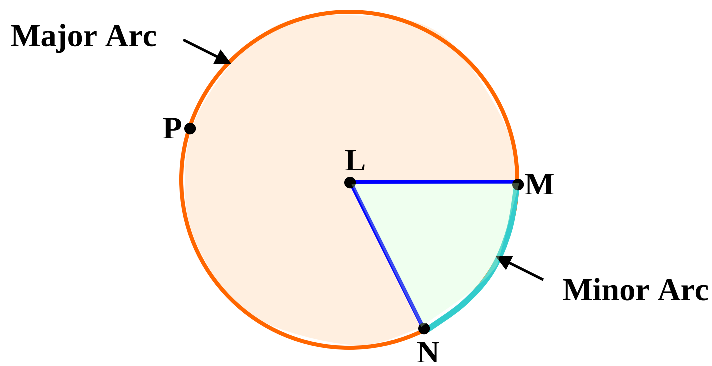
Arcs and Angles

central angle – A central angle is an angle whose vertex is at the center of a circle and its rays are radii.



$\angle JKL$ is a central angle in $\odot$ K.

minor arc – Given central angle MLN in circle L with points M and N on the circle, then points M and N and all points that are on the circle and within the interior of angle MLN form minor arc MN, written as $\overset{\frown}{MN}$. Minor arcs are denoted with two letters.

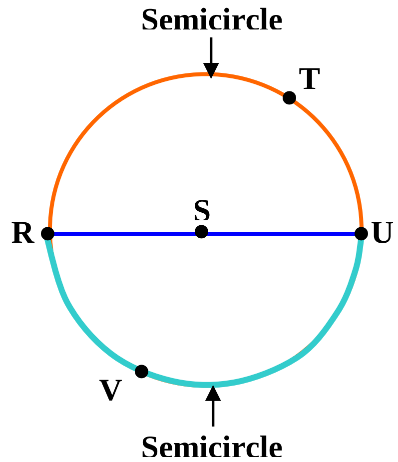


major arc – Given central angle MLN, in circle L with points M and N on the circle, then points M and N and all points that are on the circle and in the exterior of angle MLN form major arc MPN, written as $\overset{\frown}{MPN}$. Major arcs are denoted with three letters.

To Summarize:

Circle L	$\rightarrow$	$\odot L$
Central Angle	$\rightarrow$	$\angle MLN$
Minor Arc	$\rightarrow$	$\overset{\frown}{MN}$
Major Arc	$\rightarrow$	$\overset{\frown}{MPN}$

semicircle – A semicircle is either half-circle formed by the endpoints of a diameter intersecting a circle.

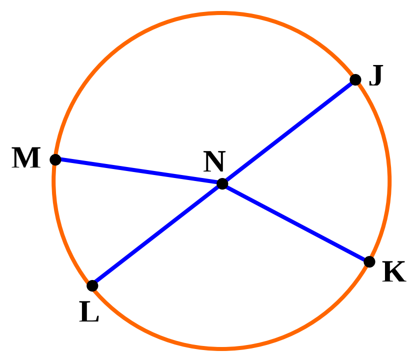


To summarize:

Circle S	$\rightarrow$	$\odot S$
Diameter	$\rightarrow$	$\overline{RU}$
Semicircle	$\rightarrow$	$\overset{\frown}{RTU}$
Semicircle	$\rightarrow$	$\overset{\frown}{RVU}$

The sum of the central angles in a circle is 360°.

Let's examine $\odot N$ and make some statements based on the given information.



Circle N	$\rightarrow$	$\odot N$
Diameter	$\rightarrow$	$\overline{LJ}$
Supplementary Angles	$\rightarrow$	$\angle LNM$, $\angle MNJ$
Supplementary Angles	$\rightarrow$	$\angle LNK$, $\angle KNJ$
$\angle LNM + \angle MNJ = 180°$	$\rightarrow$	Definition of Supplementary Angles
$\angle JNK + \angle KNJ = 180°$	$\rightarrow$	Definition of Supplementary Angles

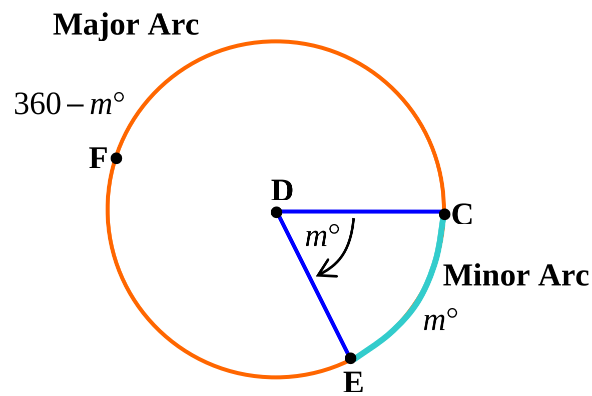
$\angle LNM + \angle MNJ + \angle JNK + \angle KNJ = 360° \rightarrow$ Addition Property

Definition of Arc Measure

The measure of a minor arc is the same as the measure of its central angle.

The measure of a major arc is the 360˚ minus the measure of its central angle.

The measure of a semicircle is 180˚.

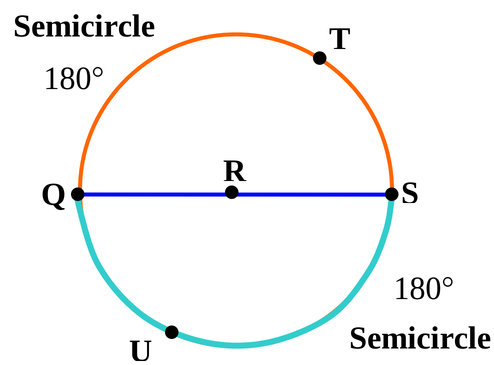


To summarize:

measure of minor arc = measure of its central angle

measure of major arc = 360° – measure of its central angle

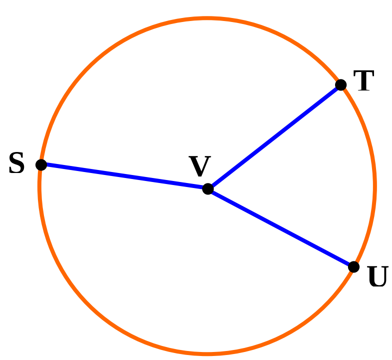
measure of a semicircle = 180°



adjacent arcs – Adjacent arcs are arcs that share a common endpoint on a circle.

Postulate 24–A	**When an arc is formed by two adjacent arcs, the measure of the arc is the sum of the measures of the two adjacent arcs.**

The figure below illustrates Postulate 24-A



Circle *N*	$\rightarrow$	$\odot N$
Adjacent Angles	$\rightarrow$	$\angle SVT$ and $\angle TVU$
$m\overset{\frown}{ST} + m\overset{\frown}{TU} = m\overset{\frown}{STU}$	$\rightarrow$	Postulate 24-A

arc length – Arc length is the length of an arc.

concentric circles – Concentric circles are circles that lie in the same plane and share the same center.

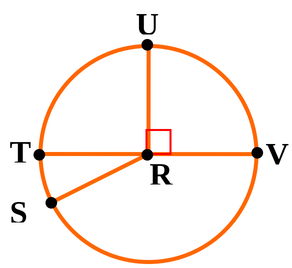
Refer to the diagram and information given below to solve the next five problems.

Given:

$\odot R$

$\overline{UR} \perp \overline{TV}$

$m\angle TRS = 25°$



Example 1: Find $m\overset{\frown}{TS}$.

$m\overset{\frown}{TS} = m\angle TRS$	Definition of Arc Measure
$m\angle TRS = 25°$	Given
$m\overset{\frown}{TS} = 25°$	Substitution

Example 2: Find $m\overset{\frown}{TU}$.

$\overline{UR} \perp \overline{TV}$	Given
$m\angle TRU$ is a right angle.	Definition of Perpendicular Lines
$m\angle TRU = 90°$	Definition of a Right Angle
$m\overset{\frown}{TU} = 90°$	Definition of Arc Measure (Minor Arc)

Example 3: Find $m\overset{\frown}{SU}$.

$m\overset{\frown}{SU} = m\overset{\frown}{ST} + m\overset{\frown}{TU}$	Arc Addition Postulate
$m\overset{\frown}{SU} = 25° + 90°$	Substitution
$m\overset{\frown}{SU} = 115°$	Simplify

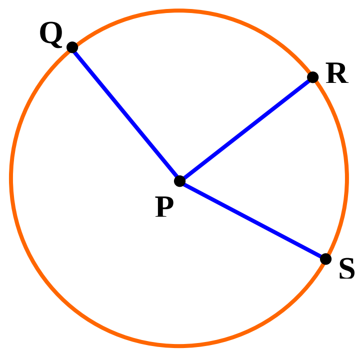
Example 4: Find $m\overset{\frown}{SVU}$.

$m\overset{\frown}{SVU} = 360 - m\overset{\frown}{SU}$	Definition of Arc Measure (Major Arc)
$m\overset{\frown}{SVU} = 360° - 115°$	Substitution
$m\overset{\frown}{SVU} = 245°$	Simplify

Example 5: Find $m\overset{\frown}{TSV}$

Diameter TV	Given
$\overset{\frown}{TSV}$ is a semicircle.	Definition of Semicircle
$m\overset{\frown}{TSV} = 180°$	Definition of Arc Measure (Semicircle)

Example 6: In $\odot P$ shown below, the radius equals 12 inches and $m\angle RPS = 60°$. What is the length of $\overset{\frown}{RS}$?



a) Determine what fractional part of the circle $(360°)$ central angle $RPS(60°)$ represents.

$$\frac{60}{360} = \frac{1}{6}$$

b) The length of $\overset{\frown}{RS} = \frac{1}{6}$ of the circumference of ⊙ P.

Length of $\overset{\frown}{RS} = \frac{1}{6}C$	
Length of $\overset{\frown}{RS} = \frac{1}{6}(2\pi r)$	Substitution ($C = 2\ \pi\ r$)
$C = \frac{1}{6}(2)\pi(12)$	Substitution ($r = 12$)
$C = 4\pi$	Simplify
$C = 12.56$	Simplify

The arc length of $\overset{\frown}{RS}$ in ⊙ P is 12.56 centimeters.

Constructing a Circle Graph

In a survey Madison determined her classmates' favorite colors based on the following choices: blue, green, yellow, red. She decided to display the results in a **circle graph**.

First she made a **tally** chart to record the responses.

She then made another chart to **organize** her calculations for drawing the graph.

Madison made fractions based on 20 (total responses) and changed them to percent. She then multiplied the percent by 360 since there are $360°$ in a circle to determine the central angle to represent each color.

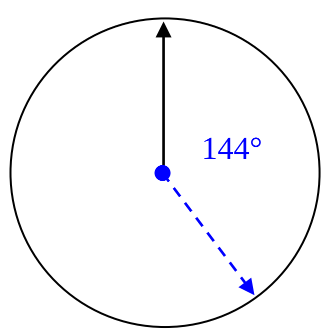
Tally of Favorite Color		
Blue	𝍸 𝍷𝍷𝍷	**8**
Green	𝍸	**5**
Yellow	𝍷	**1**
Red	𝍸 𝍷	**6**

Favorite Color		
Blue	$8/20$ = **40%**	$40\% \times 360 = 144°$
Green	$5/20$ = **25%**	$25\% \times 360 = 90°$
Yellow	$1/20$ = **5%**	$5\% \times 360 = 18°$
Red	$6/20$ = **30%**	$30\% \times 360 = 108°$
Check	$20/20$ = **100%**	**360°**

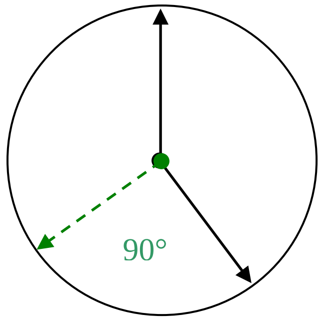
To check the calculations, Madison added the results. The fractions total **20/20**, the percents total **100%**, and the degrees total **360°** which makes **1 whole circle**.

Now Madison is ready to draw the circle graph.

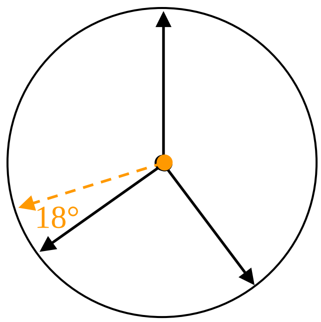
Step 1: Draw a circle with one radius. Place a protractor on the radius, using the center of the circle as the vertex, and draw the first angle (144° - blue).



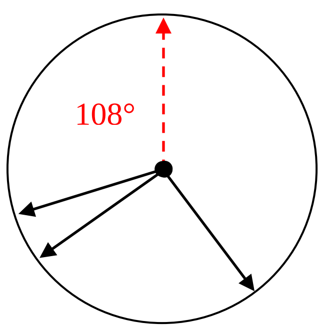
Step 2: Move the protractor to rest on the ray just drawn and proceed to make the second angle (90° -green).



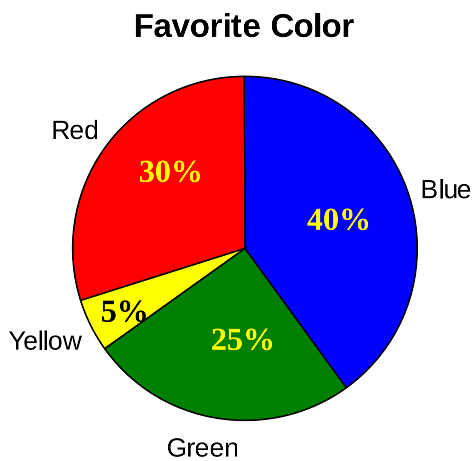
Step 3: Then move the protractor to rest on the ray she drew and proceed to make the third angle (18° - yellow).



Step 4: Madison will not have to draw the last angle, but will measure it. It should measure 108° (red).



Now the circle has been divided proportionally to the percent for each category. Complete the graph by adding color, category labels, and a title.

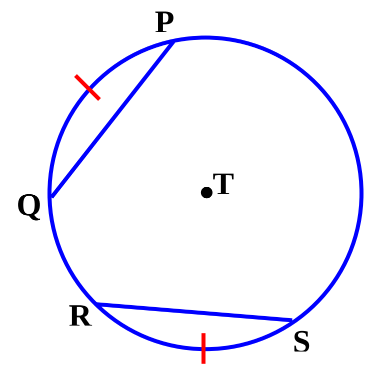


Arcs and Chords

In this section of the unit, we will examine three theorems about circles.

Theorem 24-A

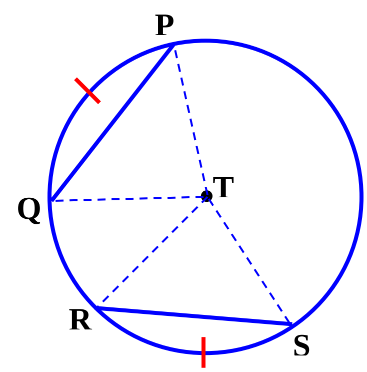
In a circle or in congruent circles, two minor arcs are congruent, if and only if, their corresponding chords are congruent.



Given: $\odot T$

$\overset{\frown}{QP} \cong \overset{\frown}{RS}$

Prove: $\overline{QP} \cong \overline{RS}$



Draw radii TP and TQ so that they intersect with the endpoints of chord QP.

Draw radii TR and TS so that they intersect with the endpoints of chord RS.

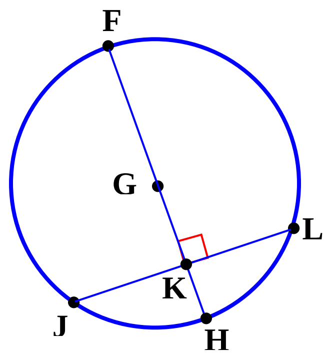
Statement	**Reason**
$\overset{\frown}{QP} \cong \overset{\frown}{RS}$	Given
$\overline{TP} \cong \overline{TQ} \cong \overline{TR} \cong \overline{TS}$	Radii of the same circle are congruent.
$m\overset{\frown}{QP} = m\angle QTP$	Definition of Arc Measure
$m\overset{\frown}{RS} = m\angle RTS$	Definition of Arc Measure
$m\angle QTP = m\angle RTS$	Substitution
$\triangle QTP \cong \triangle RTS$	SAS
$\therefore \overline{QP} \cong \overline{RS}$	CPCTC

Theorem 24-B

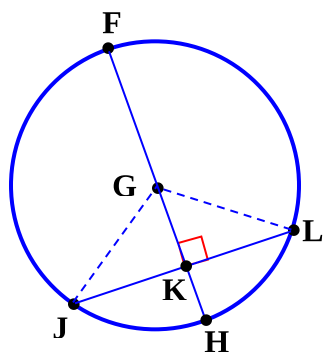
In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.

Given: ⊙ G
Diameter FH
Chord JL
$\overline{FH} \perp \overline{JL}$

Prove: $\overline{JK} \cong \overline{KL}$



Draw radii GJ and GL.

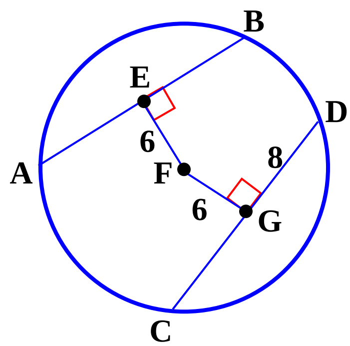


Statement	**Reason**
$\overline{FH} \perp \overline{JL}$	Given
$\angle FKL$ is right angle.	Definition of Perpendicular Lines
$\angle FKJ$ is right angle.	Definition of Perpendicular Lines
$\overline{GJ} \cong \overline{GL}$	Radii of the same circle are congruent.
△ JKG is a right triangle.	Definition of a Right Triangle
△ LKG is a right triangle.	Definition of a Right Triangle
$\overline{GK} \cong \overline{GK}$	Reflexive Property
△ $JKG \cong$ △ LKG	Hy-Leg Postulate
$\therefore \overline{JK} \cong \overline{KL}$	CPCTC

Theorem 24-C

In a circle or in congruent circles, two chords are congruent, if and only if, they are equidistant from the center.

Example: Find the length of AB if GD measures 8 centimeters and EF and FG each measure 6 centimeters.



Step 1: Chords AB and CD are equidistant from the center since segments EF and FG are perpendicular to chords AB and CD, respectively, and each measure 6 cm.

$\therefore$ Chords AB and CD are congruent. (Theorem 24-C)

Step 2: Segments EF and GF pass through the center; thus, they lie on diameters. They also are given as perpendicular to chords AB and CD, respectively.

$\therefore$ Segments EF and GF bisect chords AB and CD. (Theorem 24-B)

Step 3: Since segment DG measures 8 centimeters, segment CG measures 8 centimeters.

$\therefore$ Segment CD equals $8 \times 2 = 16$ centimeters.

Step 4: Since segment AB is congruent to segment CD (from Step 1), AB and CD are equal.

$\therefore$ AB =16 centimeters