## Theorems and Postulates

**Postulate 2-A**
**Protractor Postulate**

**Given $\overrightarrow{AB}$ and a number $r$ between 0 and 180, there is exactly one ray with endpoint $A$, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is $r$.**

**Definition of Right, Acute and Obtuse Angles**

**$\angle A$ is a right angle if $m\angle A$ is 90.**
**$\angle A$ is an acute angle if $m\angle A$ is less than 90.**
**$\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180.**

**Postulate 2-B**
**Angle Addition**

**If $R$ is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.**
**If $m\angle PQR + m\angle RQS = m\angle PQS$, then $R$ is in the interior of $\angle PQS$.**

**Vertical angles are congruent.**

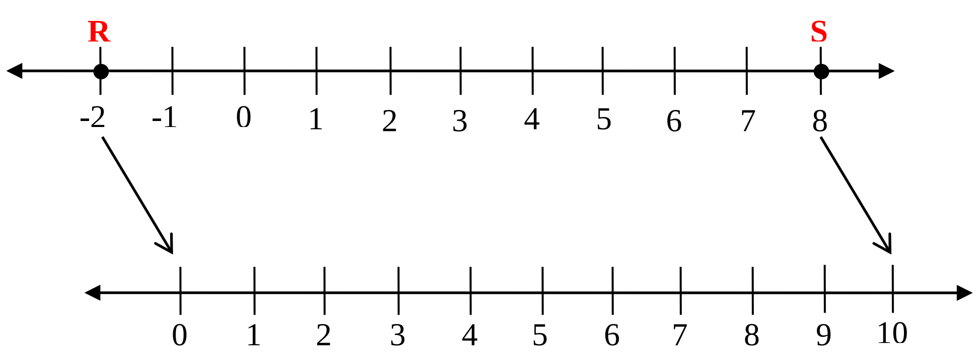
**The sum of the measures of the angles in a linear pair is 180$^\circ$.**

**The sum of the measures of complementary angles is 90$^\circ$.**

**Postulate 3-A**
**Ruler**

Two points on a line can be paired with real numbers so that, given any two points R and S on the line, R corresponds to zero, and S corresponds to a positive number.

Point R could be paired with 0, and S could be paired with 10.




**Postulate 3-B**
**Segment Addition**

If N is between M and P, then MN + NP = MP.
Conversely, if MN + NP = MP, then N is between M and P.

**Theorem 4-A Pythagorean Theorem**

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

**Distance Formula**

The distance $d$ between any two points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**Midpoint Definition**

The midpoint, M, of $\overline{AB}$ is the point between A and B such that AM = MB.

**Midpoint Formula Number Line**

With endpoints of A and B on a number line, the midpoint of $\overline{AB}$ is $\frac{A+B}{2}$.

**Midpoint Formula Coordinate Plane**

In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates $(x_1, y_1)$ and $(x_2, y_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

**Theorem 4-B Midpoint Theorem**

If M is the midpoint of $\overline{PQ}$, then $\overline{PM} \cong \overline{MQ}$.

**Postulate 5-A Law of Detachment**

If $p \Rightarrow q$ is true, and $p$ is true, then $q$ is true.

**Postulate 5-B Law of Syllogism**

If $p \Rightarrow q$ is true and $q \Rightarrow r$ is true, then $p \Rightarrow r$ is true.

**Postulate 6-A Reflexive Property**

Any segment or angle is congruent to itself.

$$\overline{QS} \cong \overline{QS}$$

**Postulate 6-B**
**Symmetric**
**Property**

**If** $\overline{AB} \cong \overline{CD}$, **then** $\overline{CD} \cong \overline{AB}$.
**If** $\angle CAB \cong \angle DOE$, **then** $\angle DOE \cong \angle CAB$.

**Theorem 6-A**
**Transitive**
**Property**

**If any segments or angles are congruent to the same angle, then they are congruent to each other.**

**If** $\overline{AB} \cong \overline{CD}$ **and** $\overline{CD} \cong \overline{EF}$, **then** $\overline{AB} \cong \overline{EF}$.
**If** $\angle 1 \cong \angle 2$ **and** $\angle 2 \cong \angle 3$, **then** $\angle 1 \cong \angle 3$.

**Theorem 6-B**
**Transitive**
**Property**

**If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 6-A.)**

**Theorem 7-A**
**Addition**
**Property**

**If a segment is added to two congruent segments, then the sums are congruent.**

**Theorem 7-B**
**Addition**
**Property**

**If an angle is added to two congruent angles, then the sums are congruent.**

**Theorem 7-C**
**Addition**
**Property**

**If congruent segments are added to congruent segments, then the sums are congruent.**

**Theorem 7-D**
**Addition**
**Property**

**If congruent angles are added to congruent angles, then the sums are congruent.**

**Theorem 7-E**
**Subtraction**
**Property**

**If a segment is subtracted from congruent segments, then the differences are congruent.**

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| **Theorem 7-F Subtraction Property** | If an angle is subtracted from congruent angles, then the differences are congruent. |
| **Theorem 7-G Subtraction Property** | If congruent segments are subtracted from congruent segments, then the differences are congruent. |
| **Theorem 7-H Subtraction Property** | If congruent angles are subtracted from congruent angles, then the differences are congruent. |
| **Theorem 7-I Multiplication Property** | If segments are congruent, then their like multiples are congruent. |
| **Theorem 7-J Multiplication Property** | If angles are congruent, then their like multiples are congruent. |
| **Theorem 7-K Division Property** | If segments are congruent, then their like divisions are congruent. |
| **Theorem 7-L Division Property** | If angles are congruent, then their like divisions are congruent. |
| **Theorem 10-A** | Congruence of angles is reflexive, symmetric, and transitive. |
| **Theorem 10-B** | If two angles form a linear pair, then they are supplementary angles. |

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| **Theorem 10-C** | **Angles supplementary to the same angle are congruent.** |
| **Theorem 10-D** | **Angles supplementary to congruent angles are congruent.** |
| **Theorem 10-E** | **Angles complementary to the same angle are congruent.** |
| **Theorem 10-F** | **Angles complementary to congruent angles are congruent.** |
| **Theorem 10-G** | **Right angles are congruent.** |
| **Theorem 10-H** | **Vertical angles are congruent.** |
| **Theorem 10-I** | **Perpendicular lines intersect to form right angles.** |
| **Postulate 10-A** | **If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.** |
| **Theorem 10-J** | **If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.** |

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| **Theorem 10-K** | **If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.** |
| **Theorem 10-L** | **If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.** |
| **Theorem 10-M** | **If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.** |

The definition of slope states that, given two points $(x_1, y_1)$ and $(x_2, y_2)$, the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

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| **Postulate 11-A** | **Two non-vertical lines have the same slope if and only if they are parallel.** |
| **Postulate 11-B** | **Two non-vertical lines are perpendicular if and only if the product of their slopes is –1.** |
| **Postulate 11-C** | **If two lines in a plane are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.** |
| **Postulate 11-D** | **If there is a line and a point that is not on the line, then there exists exactly one line that passes through the point that is parallel to the given line.** |
| **Theorem 11-A** | **If two lines in a plane are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.** |
| **Theorem 11-B** | **If two lines in a plane are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.** |

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| **Theorem 11-C** | **If two lines in a plane are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.** |
| **Theorem 11-D** | **If two lines in a plane are perpendicular to the same line, then the lines are parallel.** |

**The distance from a point, which is not on a line, and a line is the length of a line segment that is perpendicular from the point to the line.**

**The distance between two parallel lines is the distance between one line and any point on the other line.**

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| **Theorem 12-A Angle Sum Theorem** | **The sum of the measures of the angles of a triangle is 180.** |
| **Theorem 12-B Third Angle Theorem** | **If two of the angles in one triangle are congruent to two of the angles in a second triangle, then the third angles of each triangle are congruent.** |
| **Theorem 12-C Exterior Angle Theorem** | **In a triangle, the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.** |
| **Corollary 12-A-1** | **The acute angles of a right triangle are complementary.** |
| **Corollary 12-A-2** | **There can be at most one right angle in triangle.** |
| **Corollary 12-A-3** | **There can be at most one obtuse angle in triangle.** |

**Corollary 12-A-4**

**The measure of each angle in an equiangular triangle is 60.**

**Definition of Congruent Triangles (CPCTC)**

**Two triangles are congruent if and only if their corresponding parts are congruent.**

**Postulate 12-A**

**Any segment or angle is congruent to itself. (Reflexive Property)**

**Postulate 13-A**

**SSS Postulate**

**If the sides of a triangle are congruent to the sides of a second triangle, then the triangles are congruent.**

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| **SSS** | The three sides of one triangle must be congruent to the three sides of the other triangle. |

**Postulate 13-B**

**SAS Postulate**

**If two sides and the included angle of a triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.**

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| **SAS** | Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle. |

**Postulate 13-C**

**ASA Postulate**

**If two angles and the included side of a triangle are congruent to the two angles and included side of a second triangle, then the triangles are congruent.**

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| **ASA** | Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle. |

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| **Theorem 13-A**<br>**AAS Theorem** | **If two angles and a non-included side of a triangle are congruent to two angles and a non-included side of a second triangle, then the two triangles are congruent.** |

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| **AAS** | Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and side of the other triangle. |

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| **Theorem 13-B**<br>**Isosceles Triangle Theorem** | **If two sides of a triangle are congruent, then the angles that are opposite those sides are congruent.** |
| **Theorem 13-C** | **If two angles of a triangle are congruent, then the sides that are opposite those angles are congruent.** |
| **Corollary 13-B-1** | **A triangle is equilateral if and only if it is equiangular.** |
| **Corollary 13-B-2** | **Each angle of an equilateral triangle measures 60°.** |
| **Postulate 14-A**<br>**HL Postulate** | **If there exists a correspondence between the vertices of two right triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the second triangle, then the two right triangles are congruent.** |

**The shortest distance between two points is a straight line.**

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| **Postulate 14-B** | **A line segment is the shortest path between two points.** |
| **Theorem 14-A** | **A point on a perpendicular bisector of a segment is equidistant from the endpoints of the segment.** |

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| **Theorem 14-B** | **A point that is equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.** |
| **Theorem 14-C** | **A point on the bisector of an angle is equidistant from the sides of the angle.** |
| **Theorem 14-D** | **A point that is in the interior of an angle and is equidistant from the sides of the angle lies on the bisector of the angle.** |

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| **Comparison Property** | $a < b$, $a = b$, or $a > b$. |
| **Transitive Property** | 1. If $a < b$ and $b < c$, then $a < c$.<br>2. If $a > b$ and $b > c$, then $a > c$. |
| **Addition Property** | 1. If $a > b$, then $a + c > b + c$.<br>2. If $a < b$, then $a + c < b + c$. |
| **Subtraction Property** | 1. If $a > b$, then $a - c > b - c$.<br>2. If $a < b$, then $a - c < b - c$. |
| **Multiplication Properties** | 1. If $c > 0$ and $a < b$, then $ac < bc$.<br>2. If $c > 0$ and $a > b$, then $ac > bc$.<br>3. If $c < 0$ and $a < b$, then $ac > bc$.<br>4. If $c < 0$ and $a > b$, then $ac < bc$. |
| **Division Properties** | 1. If $c > 0$ and $a < b$, then $\frac{a}{c} < \frac{b}{c}$.<br>2. If $c > 0$ and $a > b$, then $\frac{a}{c} > \frac{b}{c}$.<br>3. If $c < 0$ and $a < b$, then $\frac{a}{c} > \frac{b}{c}$.<br>4. If $c < 0$ and $a > b$, then $\frac{a}{c} < \frac{b}{c}$. |

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| **Theorem 15-A Exterior Angle Inequality Theorem** | **If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its remote interior angles.** |

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| **Theorem 15-B** | If a side of a triangle is longer than another side, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side. |
| **Theorem 15-C** | In a triangle, if the measure of an angle is greater than the measure of a second angle, then the side that is opposite the larger angle is longer than the side opposite the smaller angle. |
| **Theorem 15-D** | The shortest segment from a point to a line is a perpendicular line segment between the point and the line. |
| **Theorem 15-E Triangle Inequality Theorem** | The sum of the lengths of any two sides of a triangle is greater than the length of the third side. |
| **Theorem 15-F SAS Inequality (Hinge Theorem)** | If two sides of a triangle are congruent to two sides of a second triangle, and if the included angle of the first triangle is greater than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle. |
| **Theorem 15-G SSS Inequality** | If two sides of a triangle are congruent to two sides of a second triangle, and if the third side in the first triangle is longer than the third side in the second triangle, then the included angle between the congruent sides in the first triangle is greater than the included angle between the congruent sides in the second triangle. |
| **Theorem 16-A** | The opposite sides of a parallelogram are congruent. |
| **Theorem 16-B** | The opposite angles of a parallelogram are congruent. |
| **Theorem 16-C** | The consecutive pairs of angles of a parallelogram are supplementary. |

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| **Theorem 16-D** | **The diagonals of a parallelogram bisect each other.** |
| **Theorem 16-E** | **Either diagonal of a parallelogram separates the parallelogram into two congruent triangles.** |
| **Theorem 16-F** | **In a quadrilateral if both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.** |
| **Theorem 16-G** | **In a quadrilateral if both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.** |
| **Theorem 16-H** | **In a quadrilateral if its diagonals bisect each other, then the quadrilateral is a parallelogram.** |
| **Theorem 16-I** | **In a quadrilateral if one pair of opposite sides is both congruent and parallel, then the quadrilateral is a parallelogram.** |
| **Theorem 16-J** | **If a parallelogram is a rectangle, then its diagonals are congruent.** |
| **Theorem 16-K** | **If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.** |
| **Theorem 19-A** | **The diagonals of a rhombus bisect its four angles.** |

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| **Theorem 19-B** | **The diagonals of a rhombus are perpendicular.** |
| **Theorem 19-C** | **If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.** |
| **Theorem 19-D** | **In an isosceles trapezoid, both pairs of base angles are congruent.** |
| **Theorem 19-E** | **In an isosceles trapezoid, the diagonals are congruent.** |
| **Theorem 19-F Mid-Segment Theorem** | **The median of a trapezoid is parallel to the bases and its length is one-half the sum of the lengths of the bases.** |
| **Theorem 19-G** | **The diagonals of a kite are perpendicular.** |

**Equality of Cross Products**

For any real numbers, *a*, *b*, *c*, and *d*, where *b* and *d* are not equal to zero,

$\frac{a}{b} = \frac{c}{d}$ if and only if, $ad = bc$.

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| **Postulate 20-A AA Similarity** | **If two angles of one triangle are congruent to two angles of a second triangle, then the triangles are similar.** |
| **Theorem 20-A SSS Similarity** | **If the measure of the corresponding sides of two triangles is proportional, then the triangles are similar.** |

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| **Theorem 20-B SAS Similarity** | If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of a second triangle, and the included angles are congruent, then the triangles are similar. |
| **Theorem 20-C** | The similarity of triangles is reflexive, symmetric, and transitive. |
| **Theorem 21-A** | If a line is parallel to one side of a triangle and intersects the other two sides, then those sides are separated into segments of proportional lengths. |
| **Theorem 21-B** | A line that divides two sides of a triangle proportionally is parallel to the third side of the triangle. |
| **Theorem 21-C Triangle Mid-segment Theorem** | If a segment's endpoints are the midpoints of two sides of a triangle, then it is parallel to the third side of the triangle and one-half its length. |
| **Corollary 21-A-1** | If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally. |
| **Corollary 21-A-2** | If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. |
| **Theorem 21-D** | If two triangles are similar, then their perimeters are proportional to the measures of the corresponding sides. |
| **Theorem 21-E** | If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides. |

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| **Theorem 21-F** | **If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.** |
| **Theorem 21-G** | **If two triangles are similar, then the measures of the corresponding angle bisectors of the two triangles are proportional to the measures of the corresponding sides.** |
| **Theorem 21-H Angle Bisector Theorem** | **In a triangle an angle bisector separates the opposite side into segments that have the same ratio as the other two sides.** |
| **Theorem 22-A** | **In a right triangle, if an altitude is drawn from the vertex of the right angle to the hypotenuse, then the two triangles formed are similar to each other and to the given triangle.** |
| **Theorem 22-B** | **In a right triangle, the measures of the altitude drawn from the vertex of the right angle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse created by the intersection of the hypotenuse and the altitude.** |
| **Theorem 22-C** | **In a right triangle with the altitude drawn to the hypotenuse, the measure of a leg is the geometric mean between the measure of the hypotenuse and the measure of the segment of the hypotenuse that is adjacent to the leg.** |
| **Theorem 22-D Converse of the Pythagorean Theorem** | **If the sum of the squares of the measures of the two legs of a right triangle equals the square of the hypotenuse, then the triangle is a right triangle.** |

Suppose that $m$ and n are two positive integers with $m < n$, then $n^2 - m^2$, $2mn$, and $n^2 + m^2$ is a **Pythagorean triple**.

**Theorem 22-E**

**In a 45-45-90 degree right triangle, the length of the hypotenuse can be determined by multiplying $\sqrt{2}$ times the leg.**

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| leg $a$ = leg $b$ | $\rightarrow$ | $x$ |
| hypotenuse | $\rightarrow$ | $x\sqrt{2}$ |

**Theorem 22-F**

**In a 30-60-90 degree right triangle, the length of the hypotenuse is twice as long as the shorter leg, and the longer leg equals the shorter leg multiplied by $\sqrt{3}$.**

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| shorter leg | $\rightarrow$ | $x$ |
| longer leg | $\rightarrow$ | $x\sqrt{3}$ |
| hypotenuse | $\rightarrow$ | $2x$ |

## Law of Sines

**When given any triangle, ABC, with sides named *a, b,* and *c* representing the measures of the sides opposite the angles with measures A, B, and C, respectively; the following ratios exist:**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Sines can be used to solve a triangle when the following conditions are met:

*Case I*: Two angles and a side are given. (The third angle can be found using the Angle Sum Theorem.)

*Case II*: Two sides and an angle opposite one of these sides is given.

## Law of Cosines

**When given any triangle, ABC, with sides named *a, b,* and *c* representing the measures of the sides opposite the angles with measures A, B, and C, respectively; the following is true:**

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The Law of Cosines may be used in the following cases:

*Case I*: Two sides and the included angle are given.

*Case II*: All three sides are given

**nd**

**The circumference of a circle is equal to the diameter of the circle times "Pi" or two times the radius of the circle times "Pi".**

$$C = \pi d \qquad \text{or} \qquad C = 2\pi r$$

**The sum of the central angles in a circle is 360°.**

**Definition of Arc Measure**

**The measure of a minor arc is the same as the measure of its central angle.**

**The measure of a major arc is the 360° minus the measure of its central angle.**

**The measure of a semicircle is 180°.**

**Postulate 24–A**

**When an arc is formed by two adjacent arcs, the measure of the arc is the sum of the measures of the two adjacent arcs.**

**Theorem 24-A**

**In a circle or in congruent circles, two minor arcs are congruent, if and only if, their corresponding chords are congruent.**

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| **Theorem 24-B** | **In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.** |
| **Theorem 24-C** | **In a circle or in congruent circles, two chords are congruent, if and only if, they are equidistant from the center.** |