RIGHT TRIANGLE TRIGONOMETRY

In this unit, you will investigate how basic trigonometry is derived from the angles and sides of right triangles. You will then apply trigonometry to solve problems that involve angles of elevation and angles of depression. In the last part of the unit, you will explore the Law of Sines and the Law of Cosines to solve any type of triangle. The Law of Sines is used when two angles and a side are known or two sides and an angle opposite one of these sides is known. The Law of Cosines is used when two sides and the included angle are known or all three sides are known.

Trigonometric Ratios

trigonometric ratio - A trigonometric ratio is a ratio between the sides of a right triangle.

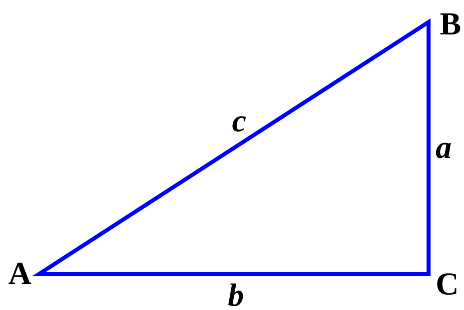
The "trig" (short for trigonometry) ratios, *sine*, *cosine*, and *tangent* are based on properties of right triangles. These three ratios are the most common trig ratios.

$$\text{sine } \angle x = \frac{\text{side opposite } \angle x}{\text{hypotenuse}} = \sin x \text{ (abbreviated)}$$

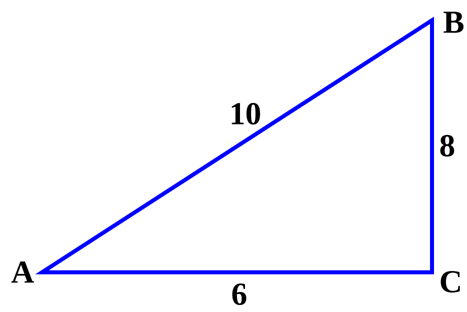
$$\text{cosine } \angle x = \frac{\text{side adjacent } \angle x}{\text{hypotenuse}} = \cos x \text{ (abbreviated)}$$

$$\text{tangent } \angle x = \frac{\text{side opposite } \angle x}{\text{side adjacent } \angle x} = \tan x \text{ (abbreviated)}$$

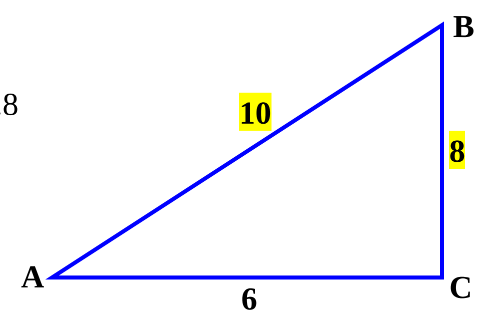
In the figure below, sine $A = \frac{a}{c}$, cosine $A = \frac{b}{c}$, and tangent $A = \frac{a}{b}$.



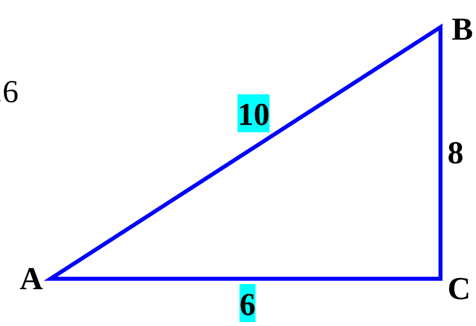
Example 1: Find the sin, cos, and tan of $\angle A$ in the triangle below.



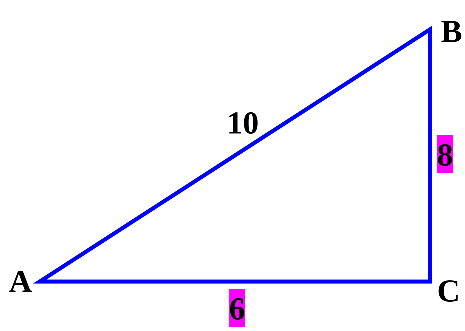
$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{8}{10} = 0.8$$



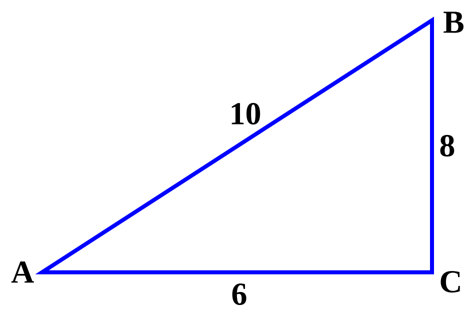
$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{6}{10} = 0.6$$



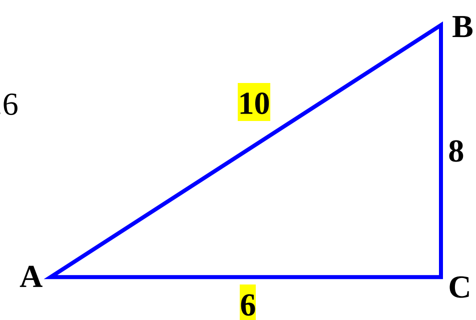
$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{8}{6} \approx 1.3$$



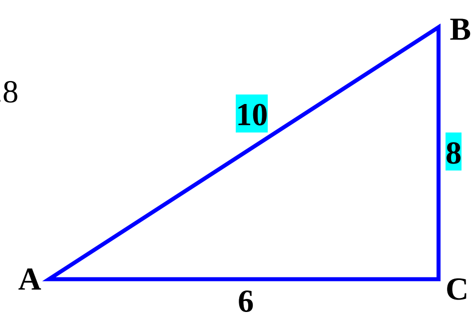
Example 2: Find the trigonometric ratios of sin, cos, and tan for the same triangle, but for $\angle B$.



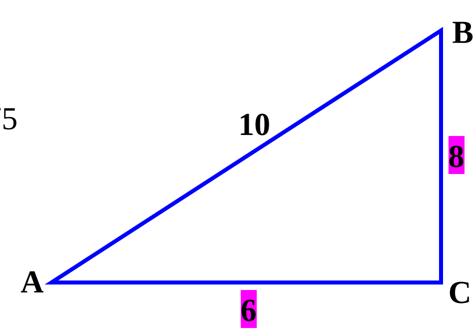
$$\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = 0.6$$



$$\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = 0.8$$



$$\tan B = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = 0.75$$



You may use a table of trigonometric ratios to find the sin, cos, and tan if you know the angle measure. You may also use a scientific calculator or a graphing calculator to find the ratios.

Example 3: Use the "Table of Trigonometric Ratios" to evaluate. Go back to the overview page for a link to the "Table of Trigonometric Ratios". (The table includes other trig ratios that you will study in a more advanced math class.) Once you have determined the ratios, check using the computer's scientific caclulator.

a. $\sin 35°$

Go to the table and go down to the 35-degree angle row, and then move over to the right to the *sin* column and find the value, 0.573576. (It is highlighted.)

$\sin 35° \approx 0.573576$

b. $\cos 58°$

Since the table only goes down to 45 degrees, you must go to the bottom of the table and read up the right side to find the 58-degree row, and then move left across to the *cos* column and find the value, 0.529919.

$\cos 58° \approx 0.529919$

***NOTE: The headings for the columns will be at the bottom of the table for angles greater than 45 degrees. These headings are different than the headings at the top of the table, so pay close attention to them.**

c. $\tan 61°$

Again, since the table only goes down to 45 degrees, you must go to the bottom of the table and read up the right side to find 61-degree row, and then move across to the left to the *tan* column and find the value, 1.804048.

$\tan 61° \approx 1.804048$

The trig values of these three angles are highlighted in the unit link's table.

We will now find the same values using the computer's scientific calculator. Make sure you open the calculator program (usually find in the Accessories Menu) and display it in scientific view. (View/Scientific) Point and click in the angle measure and then select the correct trig button.

a. $\sin 35°$ 35 sin   0.57357643635104609610803191282616

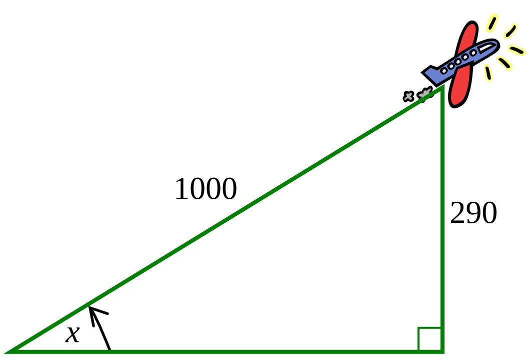
b. $\cos 58°$ 58 cos   0.52991926423320495404678115181609

c. $\tan 61°$ 61 tan   1.8040477552714239373817847482370

*To copy the results from the computer's calculator, select Edit/Copy, and then select Edit/Paste to paste the value into the document of your choice.

**You may also use a scientific or graphing calculator to determine the trig values.

Example 4: An airplane is taking off from an airport and has traveled 1000 ft climbing steadily at the same angle. The plane is 290 feet from the ground. At what angle has the plane been ascending for its flight?



To solve this problem, determine which trig function would be most useful. In this problem, we will use sin *x*.

$$\sin x = \frac{290}{1000} \qquad \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin x = 0.29 \qquad 290 \div 1000$$

Using the table of trig values...

Look up the value 0.29 in the *sin* column of table of trig ratios, and then look to the left to find the angle which is closest to the value.

The closest angle to 0.29 is 17°.

Using the computer's scientific calculator....

Select the checkbox before INV for inverse function, point and click in the decimal value, and then click on the sin button.

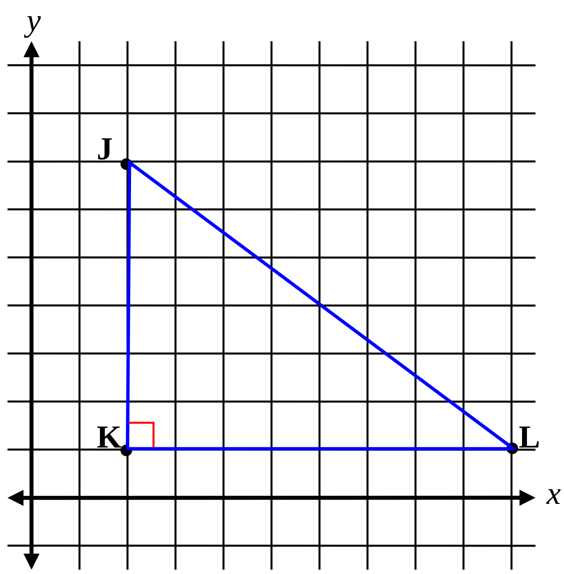
The angle is approximately 16.86°.

Using a graphing calculator....

Make sure the calculator is in degree mode, and then press on $\boxed{\text{2nd}}$ $\boxed{\text{sin}}$ 0.29.

The angle is approximately 16.85795602°.

Example 5: Find the measurement of angle J in right $\triangle$JKL with vertices of J(2,7), K(2,1) and L(10,1). Angle K is a right angle.



We can find the measure of angle J by determining sin, cos, or tan of angle J.

We will find cos J.

We can determine the length of the hypotenuse and the adjacent side by using the distance formula.

$$JL = \sqrt{(2-10)^2 + (7-1)^2}$$
$$JL = \sqrt{(-8)^2 + (6)^2}$$
$$JL = \sqrt{100}$$
$$JL = 10$$

$$JK = \sqrt{(2-2)^2 + (7-1)^2}$$
$$JK = \sqrt{0+36}$$
$$JK = \sqrt{36}$$
$$JK = 6$$

$$\cos\angle J = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{JK}{JL}$$
$$\cos\angle J = \frac{6}{10}$$
$$\cos\angle J = 0.6$$
$$\angle J \approx 53.1^\circ$$

Table of Trigonometric Ratios

Deg	Sin	Cos	Tan	Csc	Sec	Cot	Deg
0	0	1	0	Infinity	1	Infinity	90
1	0.017452	0.999848	0.017455	57.298688	1.000152	57.289962	89
2	0.034899	0.999391	0.034921	28.653708	1.00061	28.636253	88
3	0.052336	0.99863	0.052408	19.107323	1.001372	19.081137	87
4	0.069756	0.997564	0.069927	14.335587	1.002442	14.300666	86
5	0.087156	0.996195	0.087489	11.473713	1.00382	11.430052	85
6	0.104528	0.994522	0.105104	9.566772	1.005508	9.514364	84
7	0.121869	0.992546	0.122785	8.205509	1.00751	8.144346	83
8	0.139173	0.990268	0.140541	7.185297	1.009828	7.11537	82
9	0.156434	0.987688	0.158384	6.392453	1.012465	6.313752	81
10	0.173648	0.984808	0.176327	5.75877	1.015427	5.671282	80
11	0.190809	0.981627	0.19438	5.240843	1.018717	5.144554	79
12	0.207912	0.978148	0.212557	4.809734	1.022341	4.70463	78
13	0.224951	0.97437	0.230868	4.445411	1.026304	4.331476	77
14	0.241922	0.970296	0.249328	4.133565	1.030614	4.010781	76
15	0.258819	0.965926	0.267949	3.863703	1.035276	3.732051	75
16	0.275637	0.961262	0.286745	3.627955	1.040299	3.487414	74
17	0.292372	0.956305	0.305731	3.420304	1.045692	3.270853	73
18	0.309017	0.951057	0.32492	3.236068	1.051462	3.077684	72
19	0.325568	0.945519	0.344328	3.071553	1.057621	2.904211	71
20	0.34202	0.939693	0.36397	2.923804	1.064178	2.747477	70
21	0.358368	0.93358	0.383864	2.790428	1.071145	2.605089	69
22	0.374607	0.927184	0.404026	2.669467	1.078535	2.475087	68
23	0.390731	0.920505	0.424475	2.559305	1.08636	2.355852	67
24	0.406737	0.913545	0.445229	2.458593	1.094636	2.246037	66
25	0.422618	0.906308	0.466308	2.366202	1.103378	2.144507	65
26	0.438371	0.898794	0.487733	2.281172	1.112602	2.050304	64
27	0.45399	0.891007	0.509525	2.202689	1.122326	1.962611	63
28	0.469472	0.882948	0.531709	2.130054	1.13257	1.880726	62
29	0.48481	0.87462	0.554309	2.062665	1.143354	1.804048	61
30	0.5	0.866025	0.57735	2	1.154701	1.732051	60
31	0.515038	0.857167	0.600861	1.941604	1.166633	1.664279	59
32	0.529919	0.848048	0.624869	1.88708	1.179178	1.600335	58
33	0.544639	0.838671	0.649408	1.836078	1.192363	1.539865	57
34	0.559193	0.829038	0.674509	1.788292	1.206218	1.482561	56
35	0.573576	0.819152	0.700208	1.743447	1.220775	1.428148	55
36	0.587785	0.809017	0.726543	1.701302	1.236068	1.376382	54
37	0.601815	0.798636	0.753554	1.66164	1.252136	1.327045	53
38	0.615661	0.788011	0.781286	1.624269	1.269018	1.279942	52
39	0.62932	0.777146	0.809784	1.589016	1.28676	1.234897	51
40	0.642788	0.766044	0.8391	1.555724	1.305407	1.191754	50
41	0.656059	0.75471	0.869287	1.524253	1.325013	1.150368	49
42	0.669131	0.743145	0.900404	1.494477	1.345633	1.110613	48
43	0.681998	0.731354	0.932515	1.466279	1.367327	1.072369	47
44	0.694658	0.71934	0.965689	1.439557	1.390164	1.03553	46
45	0.707107	0.707107	1	1.414214	1.414214	1	45
Deg	**Cos**	**Sin**	**Cot**	**Sec**	**Csc**	**Tan**	**Deg**

sin 17°

tan 61°

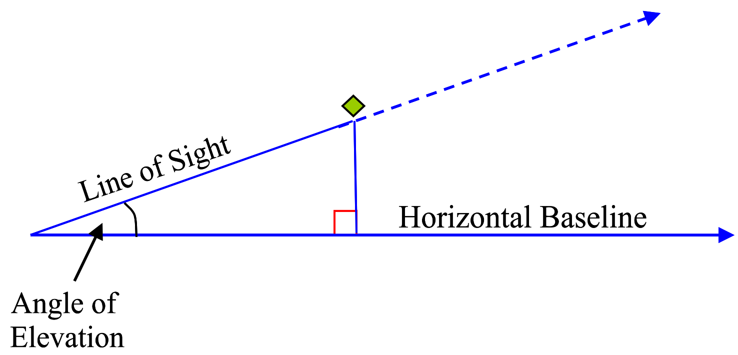
sin 35°

cos 58°

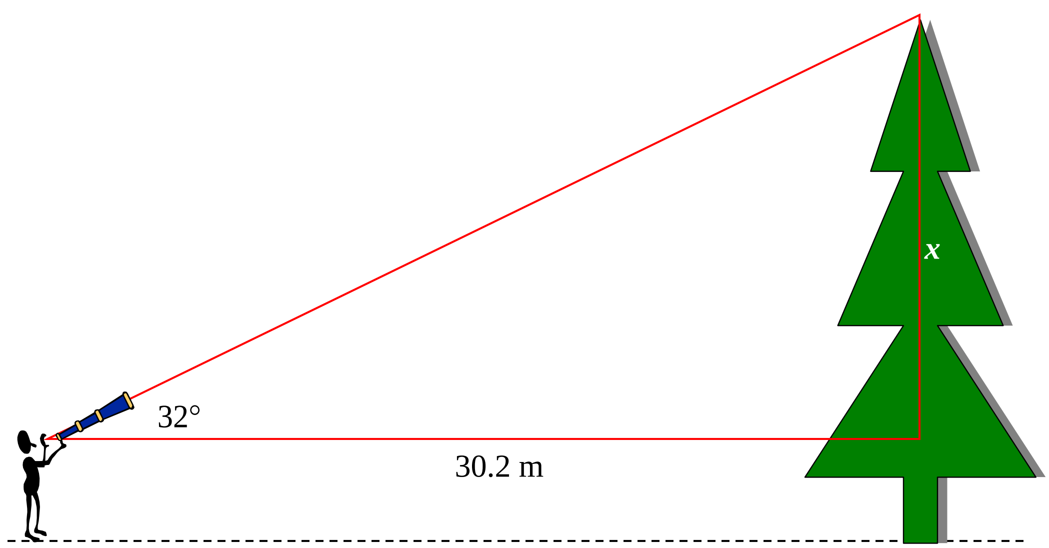
Angles of Elevation and Depression

Angle of Elevation

The angle between the line of sight and the horizontal line along a baseline is called the **angle of elevation**. The line of sight is "elevated" above the horizontal baseline. An angle of elevation is formed when a person looks at an object **above** his or her location.



Example 1: The angle of elevation from the line of sight of a person to the top of a tree is 32 degrees. The tree is 30.2 meters away on level ground. What is the height of the tree from the line of sight to the top of the tree?



In the diagram, it is obvious that a right triangle and trigonometry may be used to solve this problem.

An angle ($32°$) and an "adjacent side" (30.2 m) is given.

An "opposite side" (x) is to be determined.

*Note: In these types of problem, often times the Greek symbol, theta (θ), is used to represent the unknown angle.

Thus, we can apply the tangent ratio, $\tan(\theta) = \dfrac{opp}{adj}$.

$\tan(\theta) = \dfrac{opp}{adj}$	Tangent ratio
$\tan(32) = \dfrac{x}{30.2}$	Substitute
$\dfrac{\tan(32)}{1} = \dfrac{x}{30.2}$	Write as a proportion.
$x = \left[\tan(32)\right](30.2)$	Cross-multiply
$x \approx 18.9$	

The height of the tree from the line of sight to the top of the tree is approximately 18.9 meters.

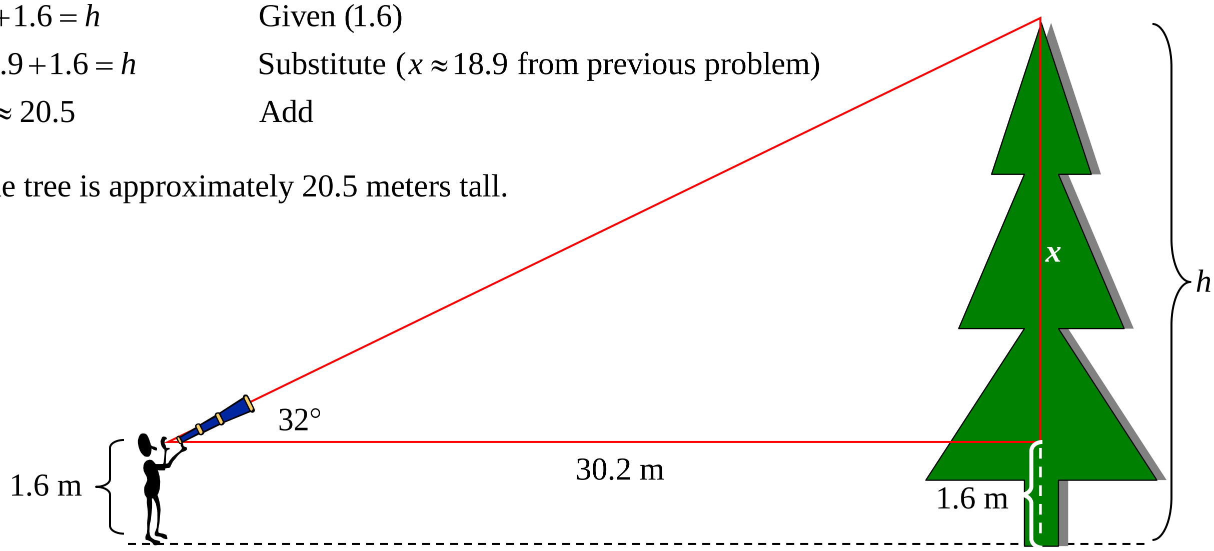
Example 2: Now, let's determine the true height of the tree in the previous problem. If the height of the person sighting in the top of the tree is 1.6 meters from the ground to his line of sight, what is the actual height of the tree?

In the previous problem, we determine that *x* is approximately 18.9 meters. Thus, to determine *y*, we will simply add.

Let *h* represent the height of the tree.

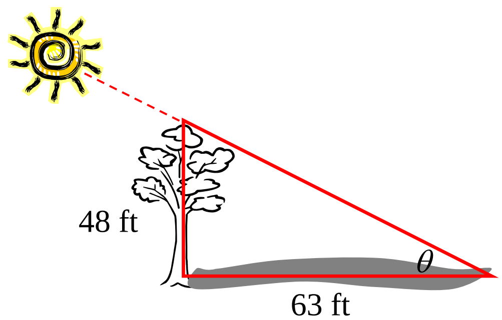
$x + 1.6 = h$	Given (1.6)
$18.9 + 1.6 = h$	Substitute ($x \approx 18.9$ from previous problem)
$h \approx 20.5$	Add

The tree is approximately 20.5 meters tall.



Now, let's take a look at a problem where we must determine the angle of elevation.

Example 3: Suppose a tree that is 48 feet high casts a shadow of 63 feet. What is the measure of the angle of elevation that is created by the sun and the end of the shadow of the tree?



To determine the measure of θ, we will find $\tan(\theta)$.

$$\tan(\theta) = \frac{opp}{adj} = \frac{48}{63} \approx 0.7619$$

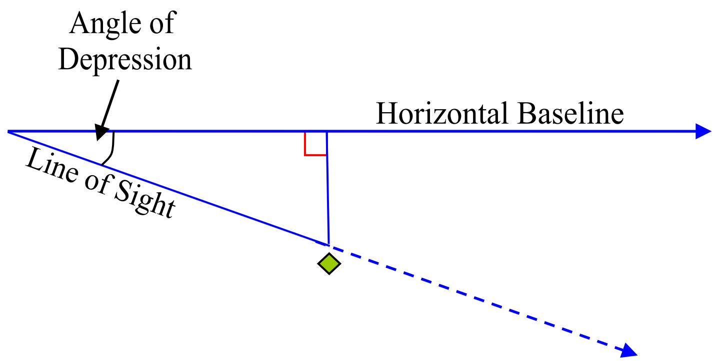
$$\theta \approx 37.3°$$

The angle of elevation is approximately 37.3°.

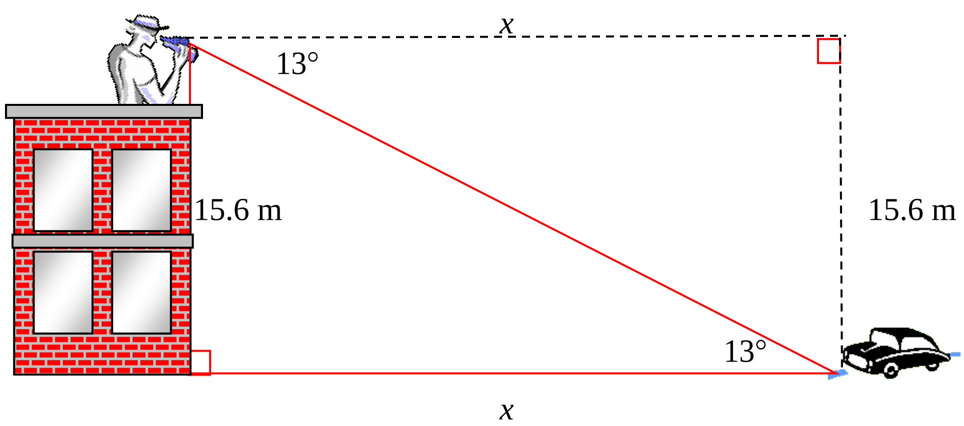
*Reminder: If using a calculator to determine the angle measure, use $\boxed{\text{TAN}^{-1}}$ and make sure the calculator mode is set to "degrees".

Angle of Depression

The angle between the line of sight and the horizontal line along a baseline is called the **angle of depression**. The line of sight is "depressed" below the horizontal baseline. An angle of depression is formed when a person looks at an object **below** his or her location.



Example 4: A person is positioned 15.6 meters above ground level on top of a building and is looking down at a car. The angle of depression to the car is 13 degrees. How far is the car from the base of the building?



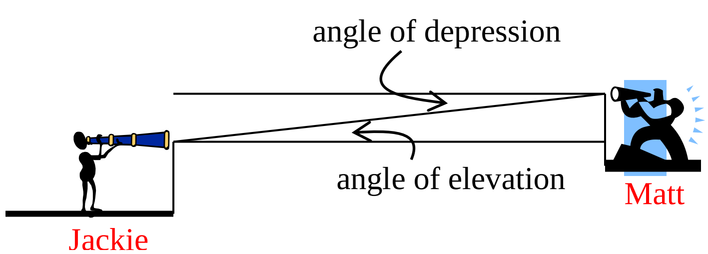
Since we have a right triangle formation, trigonometry can be used to solve for the angle of depression. Use either right triangle to solve the problem.

$\tan(\theta) = \dfrac{opp}{adj}$	Tangent ratio
$\tan(13) = \dfrac{15.6}{x}$	Substitute
$\dfrac{\tan(13)}{1} = \dfrac{15.6}{x}$	Write as a proportion.
$\left[\tan(13)\right]x = 15.6$	Cross-multiply
$x = \dfrac{15.6}{\tan(13)}$	Divide
$x \approx 67.6$	

The car is approximately 67.6 feet from the base of the building.

Study the figure below.

From Jackie's view point, she is looking up to Matt; and thus, an angle of elevation is formed. From Matt's view point, he is looking down to Jackie and an angle of depression is formed. The angle of elevation from Jackie's view point is congruent to the angle of depression from Matt's view point. Notice parallel lines are formed when extending straight perpendicular lines across from one person to the other person; and thus, the angles of depression and elevation are congruent alternate interior angles.



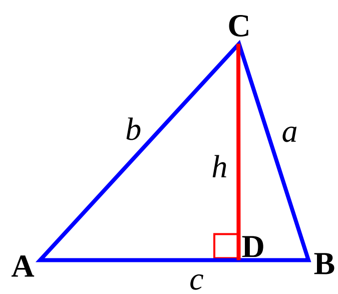
Law of Sines

Law of Sines

When given any triangle, ABC, with sides named *a*, *b*, and *c* representing the measures of the sides opposite the angles with measures A, B, and C, respectively; the following ratios exist:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Let's begin the proof of this law.



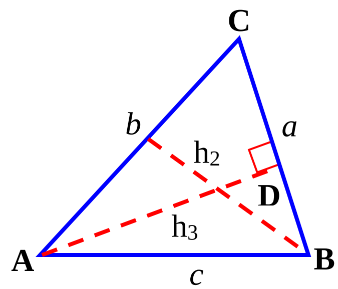
Given: Triangle ABC with altitude *h* that intersects segment AB at point D.

Sides *a*, *b*, and *c* represent lengths opposite angles named A, B, and C, respectively.

Statement	Reason
$\angle ADC$ is a right angle.	Definition of altitude.
$\triangle ACD$ is a right triangle.	Definition of right triangle.
$\sin A = \frac{h}{b}$	Definition of sine.
$b \sin A = h$	Multiplication Property (both sides by *b*)
$\angle BDC$ is a right angle.	Definition of altitude.
$\triangle BCD$ is a right triangle.	Definition of right triangle.
$\sin B = \frac{h}{a}$	Definition of sine.
$a \sin B = h$	Multiplication Property (both sides by *a*)
$b \sin A = a \sin B$	Substitution Property (both equal *h*)
$\frac{b \sin A}{ab} = \frac{a \sin B}{ab}$	Division Property (both sides by *ab*)
$\frac{\sin A}{a} = \frac{\sin B}{b}$	Simplify

The rest of the Law of Sines can be proven using a different altitude.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



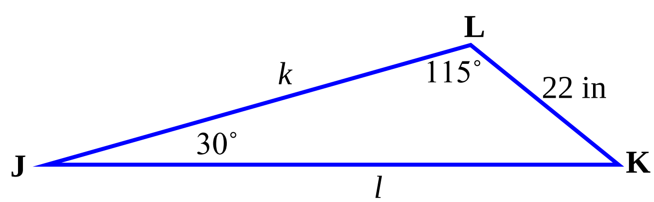
solving a triangle – Solving a triangle is the process of finding the lengths of all the sides of any triangle and the measures of all its angles.

The Law of Sines can be used to solve a triangle when the following conditions are met:

Case I: Two angles and a side are given. (The third angle can be found using the Angle Sum Theorem.)

Case II: Two sides and an angle opposite one of these sides is given.

Example 1: Solve triangle JKL when $m\angle J = 30°$, $m\angle L = 115°$, and $j = 22$ inches. This is an example of *Case I* in which two angles and a side are given.



*Note: the variables change, but the Law of Sines is the same: $\frac{\sin J}{j} = \frac{\sin K}{k} = \frac{\sin L}{l}$.

Find l:

$\frac{\sin J}{j} = \frac{\sin L}{l}$ Law of Sines (just the part that applies)

$\frac{\sin 30}{22} = \frac{\sin 115}{l}$ Write a proportion with one unknown.

$l \sin 30 = 22(\sin 115)$ Cross Multiply

$\frac{l \sin 30}{\sin 30} = \frac{22(\sin 115)}{\sin 30}$ Division Property (Divide both sides by Sin 30°)

$l = \frac{22(\sin 115)}{\sin 30}$ Simplify

$l = \frac{22\ (0.9063)}{0.5}$ Substitute

$l \approx\ 39.9\ in$ Simplify

*Note: Make sure your calculator is in "degree" mode.

Find $m\angle K$

$30 + 115 + m\angle K = 180$ Triangle Sum Theorem

$145 + m\angle K = 180$ Simplify

$m\angle K = 35°$ Subtraction Property

Find k:

$$\frac{\sin J}{j} = \frac{\sin K}{k}$$ Law of Sines

$$\frac{\sin 30}{22} = \frac{\sin 35}{k}$$ Write a proportion with one unknown.

$$k \sin 30 = 22(\sin 35)$$ Cross Multiply

$$\frac{k \sin 30}{\sin 30} = \frac{22(\sin 35)}{\sin 30}$$ Division Property (Divide both sides by Sin 30°)

$$k = \frac{22(\sin 35)}{\sin 30}$$ Simplify

$$k = \frac{22\ (0.5736)}{0.5}$$ Substitute

$$k \approx 25.2 \text{ in}$$ Simplify

The lengths of the three sides of $\triangle JKL$ are 22 inches, 25.2 inches, and 39.9 inches.

The measures of the three angles of $\triangle JKL$ are 30°, 35°, and 115°.

It is always nice to know a way to check answers. The Law of Sines is just three equivalent ratios. A good check is to see of all the ratios are equivalent or very close.

To check the answers, check for the equivalence of each of the ratios.

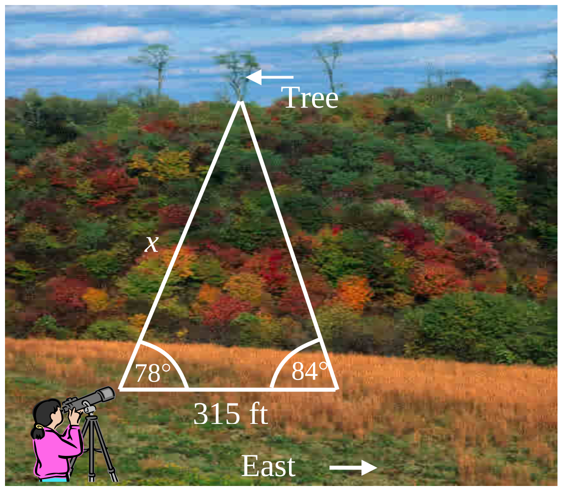
$$\frac{\sin J}{j} = \frac{\sin K}{k} = \frac{\sin M}{m}$$

$$\frac{\sin 30}{22} = \frac{\sin 35}{25.2} = \frac{\sin 115}{39.9}$$

$$0.0227 \approx 0.0228 \approx 0.0227$$

*Note: The ratios are not exactly the same because rounding is used in the computations.

Example 2: Ms. Smith sighted in a tree on the other side of the valley. She walked due East 315 feet and sighted the tree in again. Her first sighting was at an angle of 78 degrees and the second sighting was at an angle of 84 degrees. What is the distance across the valley from her original sighting of the tree?



This is an example of *Case I* because we know two angles and a side; however to use the Law of Sines, we will have to determine the size of the angle opposite the side that measures 315 feet.

Let t represent the third angle.

$78 + 84 + t = 180$	Triangle Sum Theorem
$162 + t = 180$	Simplify
$t = 18°$	Subtraction Property

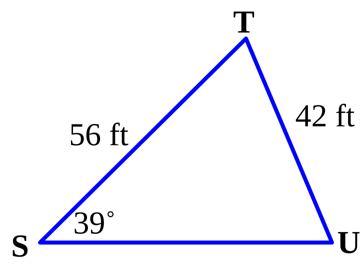
Now, we have enough information to solve this problem using the Law of Sines.

$\frac{\sin 18°}{315} = \frac{\sin 84°}{x}$	Write a proportion using the Law of Sines.
$x \sin 18° = 315(\sin 84°)$	Cross Multiply
$\frac{x \sin 18°}{\sin 18°} = \frac{315(\sin 84°)}{\sin 18°}$	Division Property (Divide both sides by Sin 30°)
$x = \frac{315(\sin 84°)}{\sin 18°}$	Substitute
$m = \frac{315(0.9945)}{0.3090} \approx 1013.8 \text{ f}t$	Simplify

The distance across the valley from Ms. Smith to the tree is approximately 1013.8 feet.

Example 3: Solve triangle STU when $m\angle S = 39°$, $s = 42$ feet, and $u = 56$ feet.

This is an example of *Case II* where two sides and an angle opposite one of the sides are given.



*Note: the variables change, but the Law of Sines is the same:

$$\frac{\sin S}{s} = \frac{\sin U}{u} = \frac{\sin T}{t}$$

We'll substitute the given information into the formula.

$$\frac{\sin 39°}{42} = \frac{\sin U}{56} = \frac{\sin T}{t}$$

So, to solve the triangle we must find the measures of angle U and angle T and the length of side *t*.

First, we'll find measure of angle U.

*Note: Make sure your calculator is in **degree mode** before making the calculations.

$\dfrac{\sin S}{s} = \dfrac{\sin U}{u}$	Begin using this part of the Law of Sines to write a proportion.
$\dfrac{\sin 39°}{42} = \dfrac{x}{56}$	Write the proportion. $S = 39°$, $s = 42$, $u = 56$, $\sin U = x$
$42x = 56(\sin 39°)$	Cross Multiply
$\dfrac{42x}{42} = \dfrac{56(\sin 39°)}{42}$	Division Property (Divide both sides by 42)
$x = \dfrac{56(0.6293)}{42}$	Substitute
$x = 0.8391$	Simplify

Now, we must find the angle knowing that the sin U = 0.8391.

Using a calculator, find $\sin^{-1} U$.

$m\angle U = \sin^{-1}(0.8391) = 57°$ (This answer is rounded to the nearest degree.)

Next, we'll find the measure of angle T based on the Angle Sum Theorem.

$m\angle S + m\angle U + m\angle T = 180°$	Angle Sum Theorem
$39 + 57 + x = 180$	Substitute: $m\angle S = 39$, $m\angle U = 57$, $m\angle T = x$
$96 + x = 180$	Simplify
$x = 84$	Subtract

$m\angle T = 84°$

Finally, we'll find the length of side *t*.

$\dfrac{\sin S}{s} = \dfrac{\sin T}{t}$	Begin using this part of the Law of Sines to write a proportion.
$\dfrac{\sin 39°}{42} = \dfrac{\sin 84°}{t}$	Write the proportion. $S = 39°,\ s = 42,\ T = 84°$
$t \sin 39° = 42(\sin 84°)$	Cross Multiply
$\dfrac{t \sin 39°}{\sin 39°} = \dfrac{42(\sin 84°)}{\sin 39°}$	Division Property (Divide both sides by Sin 39°.)
$t = \dfrac{42(0.9945)}{0.6293}$	Substitute
$t = 66.4$	Simplify

The length of *t* is approximately 66.4 rounded to the nearest tenth.

Check to see if all the ratios are about equal to the same number. Since there is rounding throughout this problem, the numbers may not come out to be exactly the same.

$$\frac{\sin S}{s} = \frac{\sin U}{u} = \frac{\sin T}{t} \quad \to \quad \frac{\sin 39}{42} = \frac{\sin 57}{56} = \frac{\sin 84}{66.4}$$

$$\frac{\sin 39}{42} \approx 0.01498 \qquad \frac{\sin 57}{56} \approx 0.01498 \qquad \frac{\sin 84}{66.4} \approx 0.01498$$

The lengths of the three sides of $\triangle STU$ are 56 feet, 42 feet, and 66.4 feet.

The measures of the three angles of $\triangle STU$ are 39°, 57°, and 84°.

Law of Cosines

If the Law of Sines does not help to solve a triangle, then we use the Law of Cosines.

Law of Cosines

When given any triangle, ABC, with sides named *a*, *b*, and *c* representing the measures of the sides opposite the angles with measures A, B, and C, respectively; the following is true:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

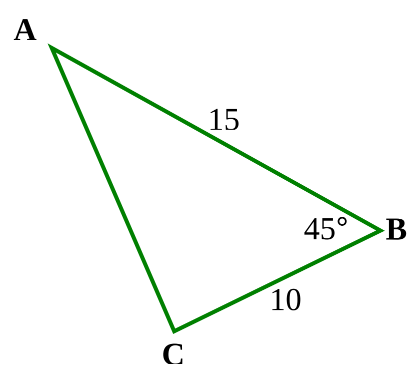
$$c^2 = a^2 + b^2 - 2ab \cos C$$

As in the Law of Sines, there are specific cases that apply to the Law of Cosines.

The Law of Cosines may be used in the following cases:

Case I: Two sides and the included angle are given.

Case II: All three sides are given



Example 1: Solve $\triangle ABC$. AB measures 15 millimeters, BC measures 10 millimeters, and angle B measures 45 degrees.

This problem is an example of *Case I*.

We will start by solving for the length of AC (b), the side opposite angle B.

$a = 10$ $c = 15$ $m\angle B = 45°$

$b^2 = a^2 + c^2 - 2ac\cos B$	Law of Cosines
$b^2 = 10^2 + 15^2 - 2(10)(15)\cos 45°$	Substitute
$b = \sqrt{10^2 + 15^2 - 2(10)(15)\cos\ 45°}$	Take the square root of both sides of the equation.
$b = \sqrt{100 + 225 - 212.13}$	Simplify
$b \approx 10.62$	Simplify
$AC \approx 10.62$ mm	

Next, we will now find the measure of angle A.

$a = 10$ $b = 10.62$ $c = 15$

$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$10^2 = 10.62^2 + 15^2 - 2(10.62)(15)\cos A$	Substitution
$100 = 112.7844 + 225 - 318.6\cos A$	Simplify
$100 = 337.7844 - 318.6\cos A$	Simplify
$100 - 337.7844 = -318.6\cos A$	Subtraction Property
$-237.7844 = -318.6\cos A$	Simplify
$\frac{-237.7844}{-318.6} = \cos A$	Division Property
$0.7463 = \cos A$	Simplify
$m\angle A \approx 41.7°$	Find $\cos^{-1} A$.

Finally, we will find the measure of the third angle using the Triangle Sum Theorem

$m\angle A + m\angle B + m\angle C = 180$	Triangle Sum Theorem
$41.7 + 45 + m\angle C = 180$	Substitution
$86.7 + m\angle C = 180$	Simplify
$m\angle C = 93.3°$	Subtraction Property

To check the answers, use the Law of Sines and check for the equivalence of each of the ratios.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 41.7}{10} = \frac{\sin 45}{10.62} = \frac{\sin 93.3}{15}$$

$$0.0665 \approx 0.0666 \approx 0.0666$$

*Note: The ratios are not exactly the same because rounding is used in the computations.

Example 2: Given triangle EFG with sides of length 8, 12, and 15. Determine the three angles of the triangle.

This is an example of Case II in which the length of all three sides is known.

First we'll determine the measurement of angle E.

Let $e = 8,\ f = 12,$ and $g = 15.$

$e^2 = f^2 + g^2 - 2fg\cos E$	Write the Law of Cosines as it applies to $\triangle$EFG.
$8^2 = 12^2 + 15^2 - 2(12)(15)\cos E$	Substitution
$64 - 144 - 225 = -2(12)(15)\cos E$	Subtraction Property and Simplify
$\frac{-305}{-360} = \cos\ E$	Division Property and Simplify
$0.8472 \approx \cos E$	Simplify
$m\angle E \approx 32.1°$	Find $\cos^{-1} E$.

Next we'll determine the measurement of angle F.

Let $e = 8,\ f = 12,$ and $g = 15.$

$f^2 = e^2 + g^2 - 2eg\cos F$	Write the Law of Cosines as it applies to □EFG.
$12^2 = 8^2 + 15^2 - 2(8)(15)\cos F$	Substitution
$144 - 64 - 225 = -2(8)(15)\cos F$	Subtraction Property and Simplify
$\dfrac{-145}{-240} = \cos F$	Division Property and Simplify
$0.6042 \approx \cos F$	Simplify
$m\angle F \approx 52.8^\circ$	Find $\cos^{-1} F$.

We will now find the measure of the third angle by using the Triangle Sum Theorem.

$m\angle A + m\angle B + m\angle C = 180$	Triangle Sum Theorem
$32.1 + 52.8 + m\angle C = 180$	Substitution
$86.7 + m\angle C = 180$	Simplify
$m\angle C \approx 95.1^\circ$	Subtraction Property

To check the answers, use the Law of Sines and check for the equivalence of each of the ratios.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 32.1}{8} = \frac{\sin 52.8}{12} = \frac{\sin 95.1}{15}$$

$$0.0664 = 0.0664 = \ 0.0664$$