PROVING TRIANGLES CONGRUENT

In this unit, you will examine how triangles can be proved congruent. You will explore several postulates related to triangles including the Side-Side-Side (SSS), Side-Angle-Side (SAS) and Angle-Side-Angle (ASA) postulates. You will examine the proof of the Angle-Angle-Side (AAS) Theorem. This unit will conclude with theorems about isosceles and equilateral triangles.

SSS Postulate

SAS Postulate

ASA Postulate

AAS Theorem

Isosceles and Equilateral Triangles

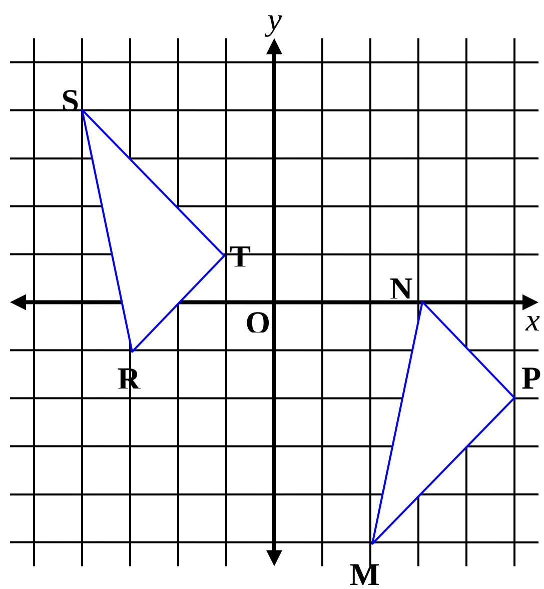
Side–Side–Side Postulate (SSS)

Postulate 13-A **SSS Postulate**	**If the sides of a triangle are congruent to the sides of a second triangle, then the triangles are congruent.**

Let's examine this postulate by looking at two triangles drawn in the coordinate plane. Determine if these triangles are congruent.

Given: $\triangle STR$ with vertices $S(-4,4)$, $T(-1,1)$ and $R(-3,-1)$
$\triangle MPN$ with vertices $M(2,-5)$, $P(5,-2)$ and $N(3,0)$

Determine if $\triangle STR \cong \triangle MPN$.



$\triangle STR$

$$ST = \sqrt{[-4-(-1)]^2 + (4-1)^2} = \sqrt{18}$$

$$TR = \sqrt{[-1-(-3)]^2 + [1-(-1]^2} = \sqrt{8}$$

$$SR = \sqrt{[-4-(-3)]^2 + [4-(-1)]^2} = \sqrt{26}$$

$\triangle MPN$

$$MP = \sqrt{(2-5)^2 + [(-5-(-2)]^2} = \sqrt{18}$$

$$PN = \sqrt{(5-3)^2 + (-2-0)^2} = \sqrt{8}$$

$$MN = \sqrt{(2-3)^2 + (-5-0)^2} = \sqrt{26}$$

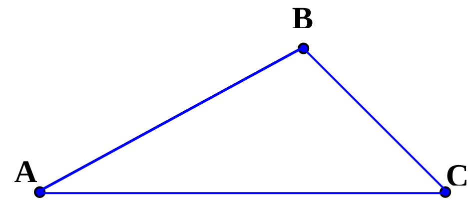
$\overline{ST} \cong \overline{MP}$; $\overline{TR} \cong \overline{PN}$; $\overline{SR} \cong \overline{MN}$, $\therefore \triangle STR \cong \triangle MPN$ by SSS Postulate.

SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.

You can construct a congruent triangle with a compass and a straight edge by applying the SSS postulate.

Given: $\triangle ABC$

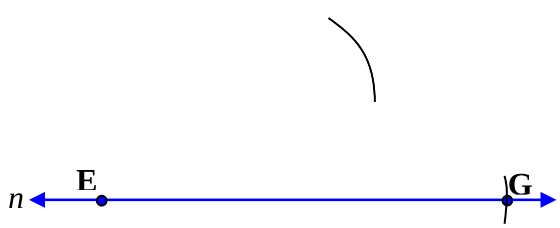
Construct: Congruent $\triangle EFG$



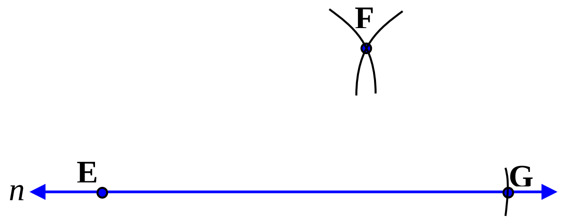
Step 1: Start with drawing a line and placing point E on the line.

Step 2: Move back to the triangle and place the metal point of the compass on point A and adjust the compass so that the pencil point touches point C. The compass will now be set to the length of segment AC. Without changing the setting of the compass, move back to the line. Place the metal point of the compass on point E and make an arc on the line. Name the point of intersection, point G.

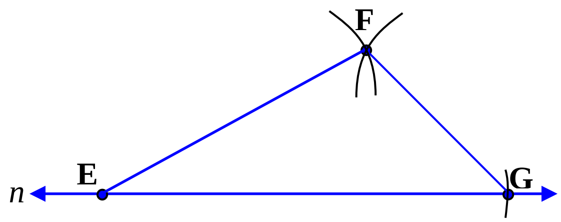
Step 3: Move the metal point of the compass back to point A of the triangle and adjust the compass so that the pencil point touches point B. The compass will now be set to the length of segment AB. Without changing the setting of the compass, move back to the line and place the metal point of the compass on point E. Make an arc above the line.



Step 4: Move the metal point of the compass to point C of the triangle and adjust the compass so that the pencil point touches point B. The compass will now be set to the length of segment BC. Without changing the setting of the compass, move back to the line and place the metal point of the compass on point G. Make an arc above the line that intersects the other arc. Name the point of intersection, point F.



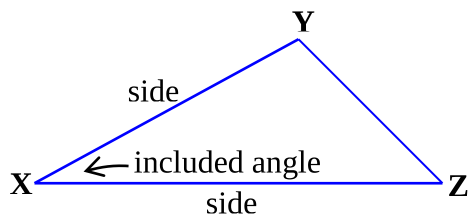
Step 5: Draw segments EF and GF.



$$\triangle ABC \cong \triangle EFG$$

Side–Angle–Side Postulate (SAS)

included angle – An included angle is the angle formed by two adjacent sides in a geometric figure.



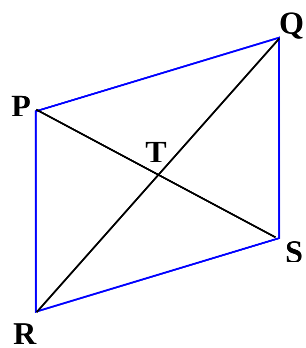
Postulate 13-B **SAS Postulate**	**If two sides and the included angle of a triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.**

In the following proof, we will use the SAS Postulate to make the final conclusion.

Given: Point T is the midpoint of $\overline{PS}$.

Point T is the midpoint of $\overline{RQ}$.

Prove: $\triangle PTQ \cong \triangle STR$



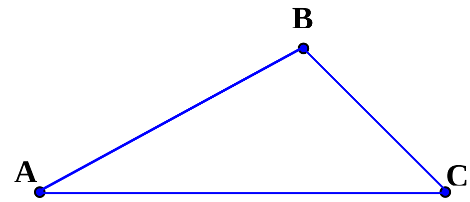
Statements	**Reasons**
1. Point T is midpoint of $\overline{PS}$	Given
2. $\overline{PT} \cong \overline{TS}$	Definition of midpoint
3. Point T is midpoint of $\overline{RQ}$	Given
4. $\overline{RT} \cong \overline{TQ}$	Definition of midpoint
5. $\angle PTQ \cong \angle RTS$	Vertical angles are congruent. (Theorem 10-H)
6. $\triangle PTQ \cong \triangle RTS$	SAS

SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.

You can construct a congruent triangle with a compass and a straight edge by applying the SAS postulate.

Given: $\triangle ABC$

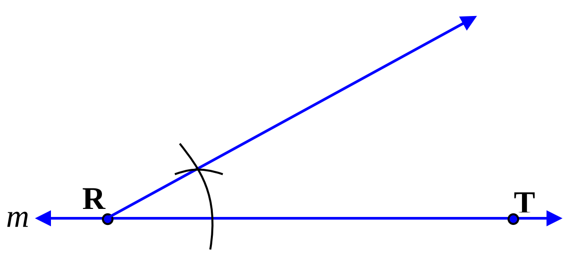
Construct: Congruent $\triangle RST$



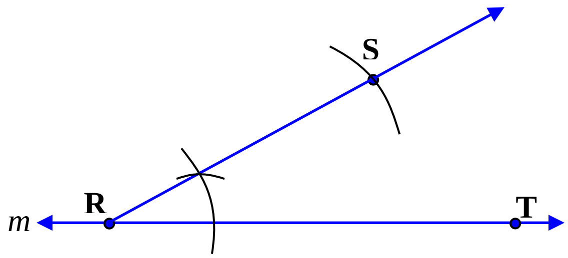
Step 1: Start with drawing a line *n* and placing point R on the line.

Step 2: Move back to the triangle and place the metal point of the compass on point A. Adjust the compass so that the pencil point touches point C. The compass will now be set to the length of segment AC. Without changing the setting on the compass, move back to the line and place the metal point of the compass on point R. Make an arc on the line and name the point of intersection, point T.

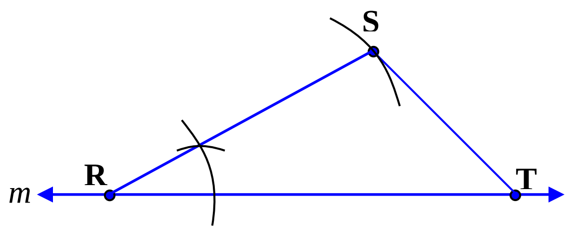
Step 3: Construct an angle at point R that is congruent to ∠A in the triangle. This angle is the **included** angle. (Refer back to a previous unit about “Angle Constructions”.



Step 4: Move back to the triangle and place the metal point of the compass on point A. Adjust the compass so that the pencil point touches point B. The compass will now be set to the length of segment AB. Without changing the setting, move back to line *m* and place the metal point of the compass on point R. Make an arc on the top ray of the angle and name the point of intersection, point S.



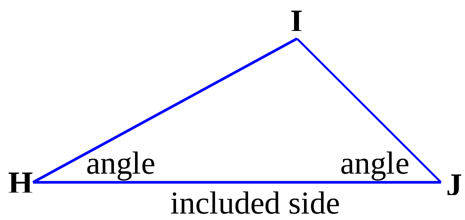
Step 5: In this step no further measuring is needed because all of the necessary constructions have been made that satisfy the SAS postulate. In the final step simply draw a segment to connect points S and T to complete the triangle.



$\triangle ABC \cong \triangle RST$

Angle–Side–Angle Postulate (ASA)

included side – An included side is the side that forms two different angles in a polygon.



Postulate 13-C
ASA Postulate

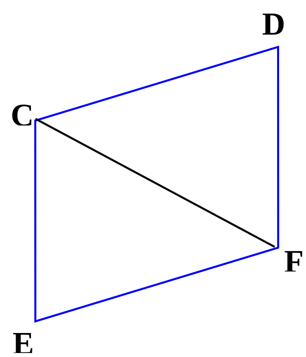
If two angles and the included side of a triangle are congruent to the two angles and included side of a second triangle, then the triangles are congruent.

In the following proof, we will use the ASA Postulate and CPCTC (Corresponding Parts of Congruent Triangles are Congruent) to make the final conclusions.

Given: $\overline{CD} \parallel \overline{EF}$

$\overline{CE} \parallel \overline{DF}$

Prove: $\angle E \cong \angle D$



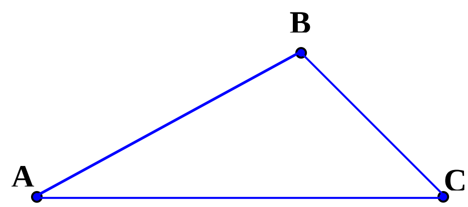
Statements	Reasons
1. $\overline{CD} \parallel \overline{EF}$	Given
2. $\angle DCF \cong \angle CFE$	Alternate interior angles are congruent. (Theorem 10-J)
3. $\overline{CE} \parallel \overline{DF}$	Given
4. $\angle ECF \cong \angle CFD$	Alternate interior angles are congruent. (Theorem 10-J)
5. $\overline{CF} \cong \overline{CF}$	Reflexive Property
6. $\triangle ECF \cong \triangle DFC$	ASA (Postulate 13-C)
7. $\angle E \cong \angle D$	CPCTC

ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.

You can construct a congruent triangle with a compass and a straight edge by applying the ASA postulate.

Given: $\triangle ABC$

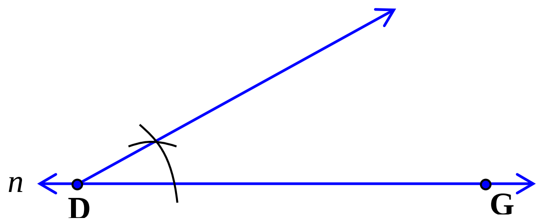
Construct: Congruent $\triangle DEG$



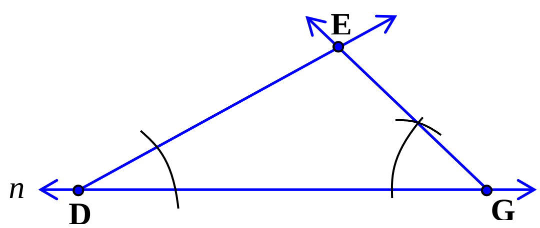
Step 1: Start with drawing a line n and placing point D on the line.

Step 2: Move back to the triangle and place the metal point of the compass on point A. Adjust the compass so that the pencil point touches point C. The compass will now be set to the length of segment AC. Moving back to the line, place the metal point of the compass on point D and make an arc on the line. Name the point of intersection, point G. Segment DG is the **included** side.

Step 3: Construct an angle at point D that is congruent to ∠A in the triangle. (Refer back to a previous unit about "Angle Constructions".)



Step 4: Construct an angle at point G that is congruent to ∠C in the triangle. (Refer back to a previous unit about "Angle Constructions".



Step 5: Name the point where the two rays of the constructed angles intersect, point E. The construction is complete.

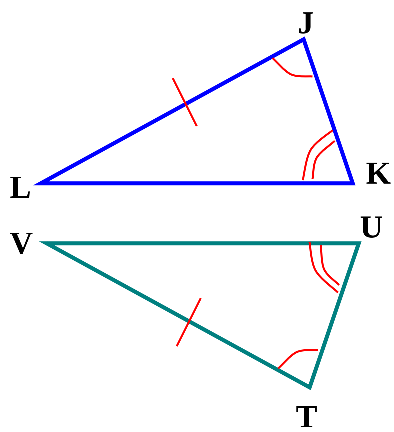
$$\triangle ABC \cong \triangle DEG$$

Angle–Angle–Side Theorem (AAS)

Theorem 13-A
AAS Theorem

If two angles and a non-included side of a triangle are congruent to two angles and a non-included side of a second triangle, then the two triangles are congruent.

Proof of AAS Theorem



Given: $\angle J \cong \angle T$

$\angle K \cong \angle U$

$\overline{JL} \cong \overline{TV}$

Prove: $\triangle JKL \cong \triangle TUV$

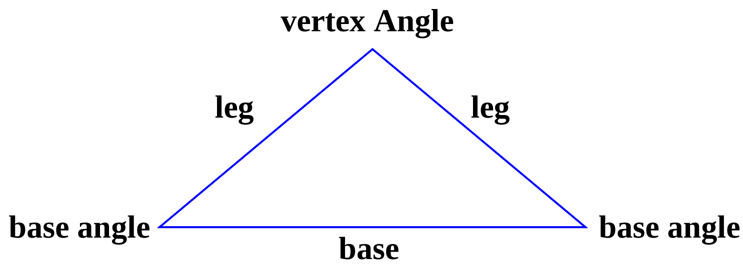
Statements	Reasons
1. $\angle J \cong \angle T$	Given
2. $\angle K \cong \angle U$	Given
3. $\overline{JL} \cong \overline{TV}$	Given
4. $\angle L \cong \angle V$	Third Angle Theorem (Theorem 10-B)
5. $\triangle JKL \cong \triangle TUV$	ASA (Postulate 11-C)

AAS	Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and side of the other triangle.

Isosceles and Equilateral Triangles

Theorem 13-B **Isosceles Triangle Theorem**	**If two sides of a triangle are congruent, then the angles that are opposite those sides are congruent.**
Theorem 13-C	**If two angles of a triangle are congruent, then the sides that are opposite those angles are congruent.**
Corollary 13-B-1	**A triangle is equilateral if and only if it is equiangular.**
Corollary 13-B-2	**Each angle of an equilateral triangle measures 60°.**

Parts of an Isosceles Triangle



vertex angle – The vertex angle of an isosceles triangle is the angle formed by the two congruent sides.

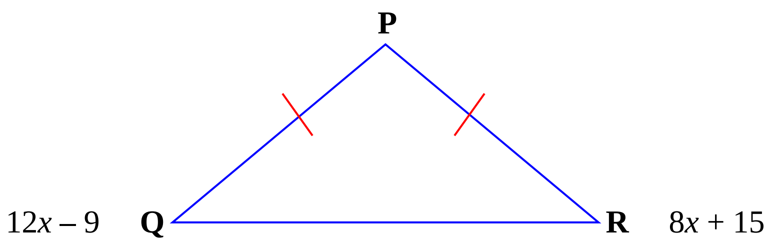
leg – The leg of an isosceles triangle is one of the two congruent sides.

base angle – The base angle of an isosceles triangle is an angle that is opposite one of the congruent sides.

base – The base of an isosceles triangle is the included side of the two congruent angles.

Example: Find the measure of each angle in isosceles triangle PQR. Segment QR is the base, $m\angle Q = 12x - 9$, and $m\angle R = 8x + 15$.

Draw a picture and label the given parts. This will help to solve the problem.



$m\angle Q = m\angle R$	Isosceles Triangle Theorem (Theorem 13-B)
$m\angle Q = 12x - 9$	Given
$m\angle R = 8x + 15$	Given
$12x - 9 = 8x + 15$	Substitution Property
$12x = 8x + 24$	Addition Property
$4x = 24$	Subtraction Property
$x = 6$	Division Property

To find angles *Q* and *R*, use substitution:

$m\angle Q = 12x - 9$	$m\angle R = 8x + 15$
$m\angle Q = 12(6) - 9$	$m\angle R = 8(6) + 15$
$m\angle Q = 63°$	$m\angle R = 63°$

To find angle *P*, use the Angle Sum Theorem:

$m\angle P + m\angle Q + m\angle R = 180$	Angle Sum Theorem (Theorem 10-A)
$m\angle P + 63 + 63 = 180$	Substitution
$m\angle P + 126 = 180$	
$m\angle P = 54°$	Subtraction