ANGLES, TRANSVERSALS, AND PARALLEL LINES

In this unit, you will examine and prove theorems about supplementary, complementary, vertical, congruent, and right angles. You will learn terms that are used with lines cut by a transversal. You will then apply the characteristics of these lines and angles to prove theorems about parallel lines cut by a transversal.

Angle Relationships

In this section you will examine several theorems about angle relationships. Some of these theorems will be examined closes through proofs. Please note that theorems can be used as reasons for statements in a proof.

Theorem 10-A	**Congruence of angles is reflexive, symmetric, and transitive.**
Theorem 10-B	**If two angles form a linear pair, then they are supplementary angles.**
Theorem 10-C	**Angles supplementary to the same angle are congruent.**

Let's examine the proof to this theorem. Using the theorem's information, we can make general statements about what is given and what is to be proved.

In the following proof, we will use three general angles and make statements about them based on the proof.

Given: $\angle 1$ and $\angle 3$ are supplementary
$\angle 2$ and $\angle 3$ are supplementary

[Angles supplementary to the same angle
$\angle 1$, $\angle 2$ $\angle 3$]

Prove: $\angle 1 \cong \angle 2$

[are congruent.]

Statements	**Reasons**
1. $\angle 1$ and $\angle 3$ are supplementary	Given
2. $\angle 2$ and $\angle 3$ are supplementary	Given
3. $m\angle 1 + m\angle 3 = 180$	Definition of supplementary angles
4. $m\angle 2 + m\angle 3 = 180$	Definition of supplementary angles
5. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	Substitution*
6. $m\angle 1 = m\angle 2$	Subtraction Property
7. $\therefore \angle 1 \cong \angle 2$	Definition of Congruence

*In statement #5, the 180 in statement #3 is replaced with $m\angle 2 + m\angle 3$ from statement #4.

Theorem 10-D

Angles supplementary to congruent angles are congruent.

Given: $\angle 4 \cong \angle 5$
$\angle 4$ and $\angle 6$ are supplementary
$\angle 5$ and $\angle 7$ are supplementary

$\left[\begin{array}{ll}\text{Angles supplementary to} & \text{congruent angles} \\ \angle 6, \angle 7 & \angle 4, \angle 5\end{array}\right]$

Prove: $\angle 6 \cong \angle 7$

[are congruent.]

Statements	Reasons
1. $\angle 4 \cong \angle 5$	Given
2. $\angle 4$ and $\angle 6$ are supplementary	Given
3. $\angle 5$ and $\angle 6$ are supplementary	Substitution*
4. $\angle 5$ and $\angle 7$ are supplementary	Given
5. $\therefore \angle 6 \cong \angle 7$	Theorem 10C**

*$\angle 5$ from Step #1 is substituted for $\angle 4$ in Step #2.
**Theorem 10C: Angles supplementary to the same angle are congruent.
In Steps #3 and #4, $\angle 5$ is shown to be supplementary to both $\angle 6$ and $\angle 7$.

Theorem 10-E

Angles complementary to the same angle are congruent.

Theorem 10-F

Angles complementary to congruent angles are congruent.

Theorem 10-G

Right angles are congruent.

Theorem 10-H	**Vertical angles are congruent.**
Theorem 10-I	**Perpendicular lines intersect to form right angles.**

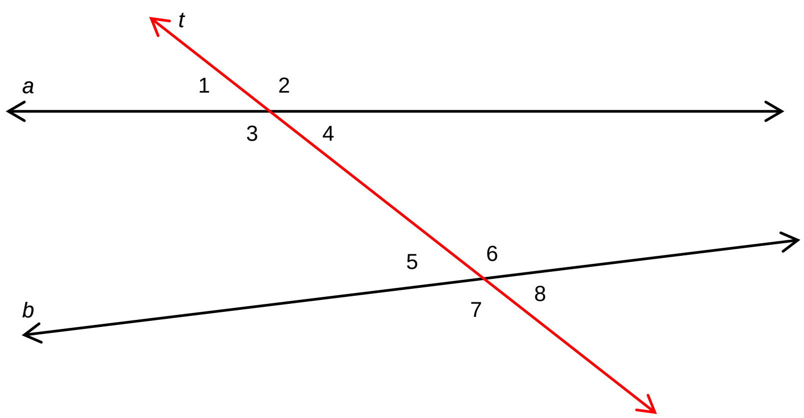
Lines Cut by a Transversal

In the given drawing two lines, *a* and *b,* are cut by a third line, *t*, called a transversal.

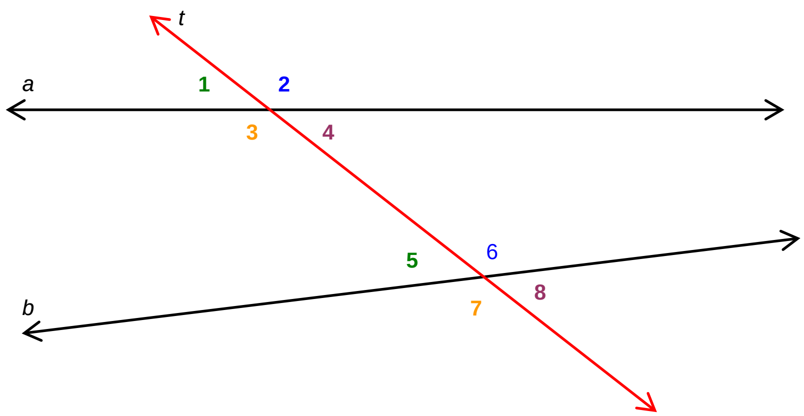
transversal – A transversal is a line that crosses two or more lines at different points.

Many angles are formed when a transversal crosses over two lines. Some of these angles are given special names and/or definitions based on their positions relative to the transversal and the lines.

The definitions in this section relate two lines cut by a transversal.



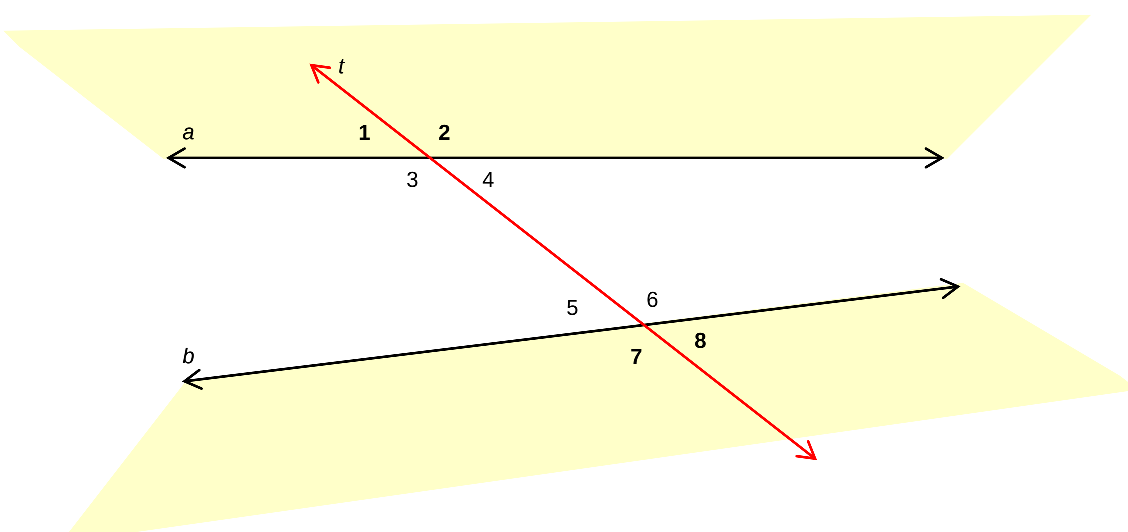
corresponding angles – When two lines are cut by a transversal, the corresponding angles are the angles that are located on the same side of the transversal and their position is corresponding relative to the lines.



Corresponding Angles:

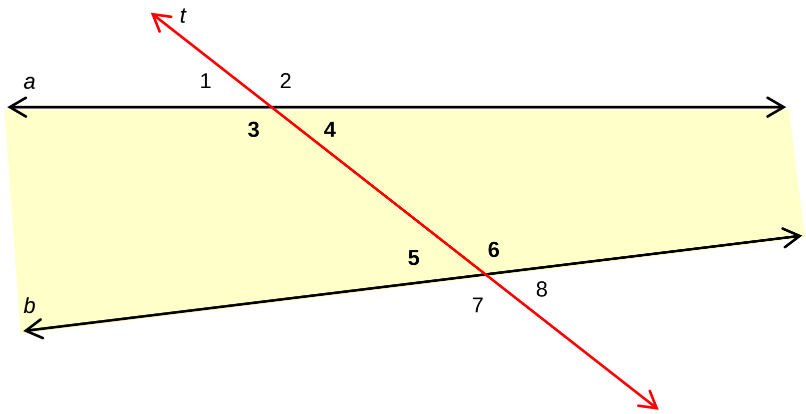
$\angle 1$ and $\angle 5$	Location: left of the transversal; above the lines
$\angle 2$ and $\angle 6$	Location: right of the transversal; above the lines
$\angle 3$ and $\angle 7$	Location: left of the transversal; below the lines
$\angle 4$ and $\angle 8$	Location: right of the transversal; below the lines

exterior angles – When two lines are cut by a transversal, the exterior angles are the angles that are located within the exterior of the lines.



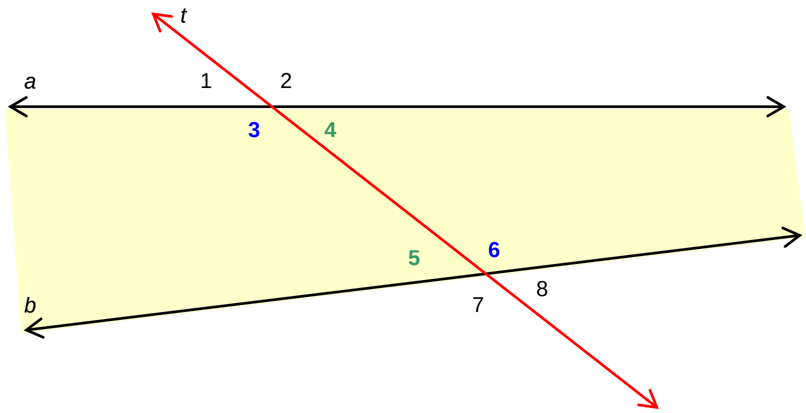
Exterior Angles: $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$

interior angles – When two lines are cut by a transversal, the interior angles are the angles that are located within the interior of the lines.



Interior Angles: $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$

alternate interior angles – When two lines are cut by a transversal, the alternate interior angles are the angles that are located within the interior of the lines and on opposite sides of the transversal.

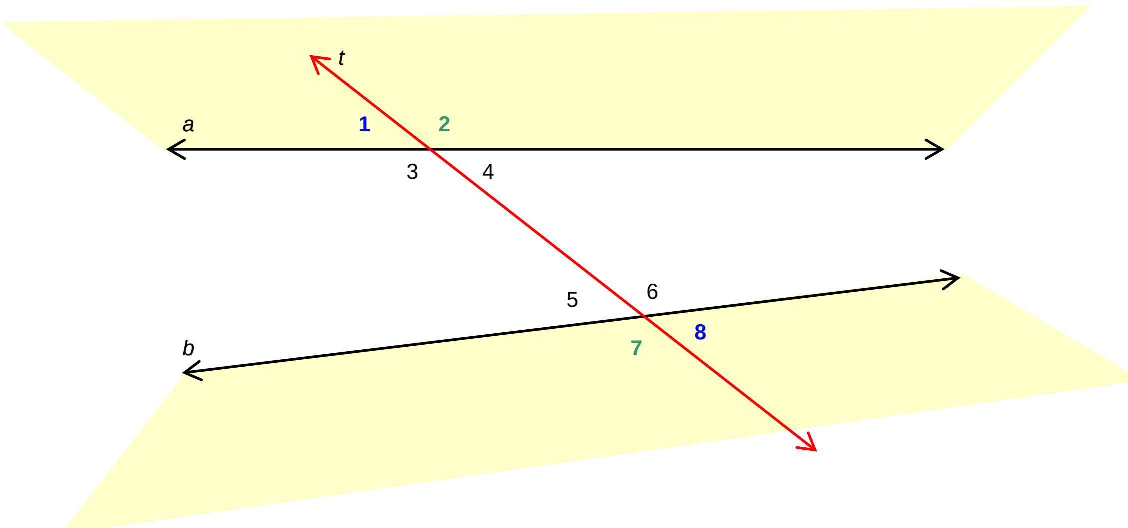


Alternate Interior Angles:

$\angle 3$ & $\angle 6$ Location: interior; opposite sides of transversal

$\angle 4$ & $\angle 5$ Location: interior; opposite sides of transversal

alternate exterior angles – When two lines are cut by a transversal, the alternate exterior angles are the angles that are located within the exterior of the lines and on opposite sides of the transversals.

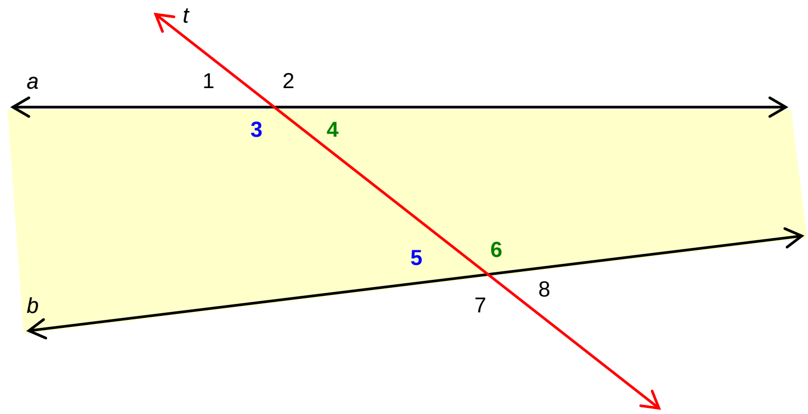


Alternate Exterior Angles:

∠1 & ∠8 Location: exterior; opposite sides of transversal

∠2 & ∠7 Location: exterior; opposite sides of transversal

consecutive interior angles – When two lines are cut by a transversal, the consecutive interior angles are the angles that are located within the interior and on the same side of the transversal.



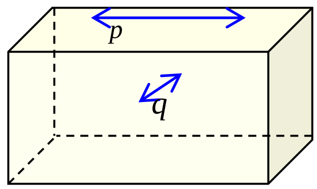
Consecutive Interior Angles:

$\angle 3$ & $\angle 5$ Location: interior; same side of transversal

$\angle 4$ & $\angle 6$ Location: interior; same side of transversal

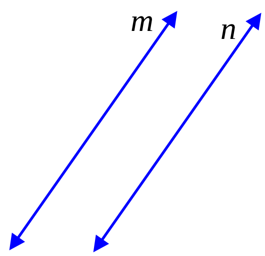
Special Lines and Planes

skew lines – Skew lines are lines that do not intersect and DO NOT lie in the same plane.



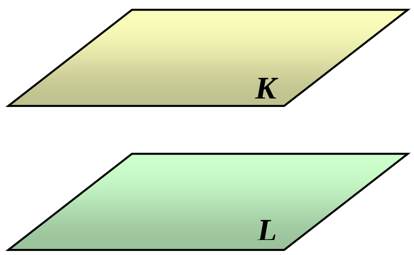
Lines *p* and *q* are skew lines.

parallel lines – Parallel lines are lines that do not intersect and do lie in the same plane.
A short notation for “parallel to” is□.



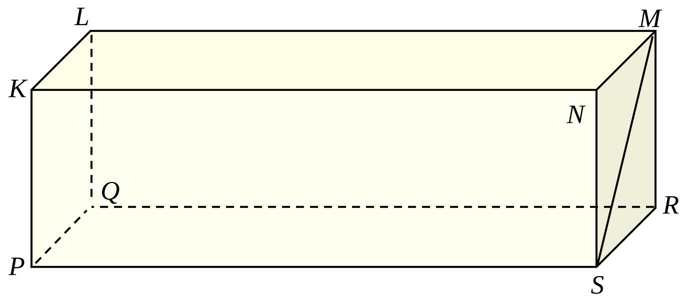
Lines *m* and *n* are parallel lines.

parallel planes – Parallel planes are planes that DO NOT intersect.



Planes *K* and *L* are parallel planes.

Refer to the figure of the rectangular prism below to answer the questions in the examples below.



Example 1: Name all line segments that are parallel to $\overline{KL}$.

$\overline{PQ}, \overline{MN}, \overline{SR}$

*To understand that $\overline{SR} \parallel \overline{KL}$, imagine a slanted plane slicing through the prism that has both $\overline{SR}$ & $\overline{KL}$ lying in it.

Example 2: Name a pair of parallel planes.

Planes KLQ and NMR are parallel planes. Can you find the other parallel planes?

Example 3: Name a pair of skew line segments.

$\overline{PQ}$ and $\overline{KN}$ are skew line segments. Can you find other skew line segments?

Example 4: Name a line segment that intersects with $\overline{SM}$.

$\overline{RS}$ intersects with $\overline{SM}$ at point S. Can you find others?

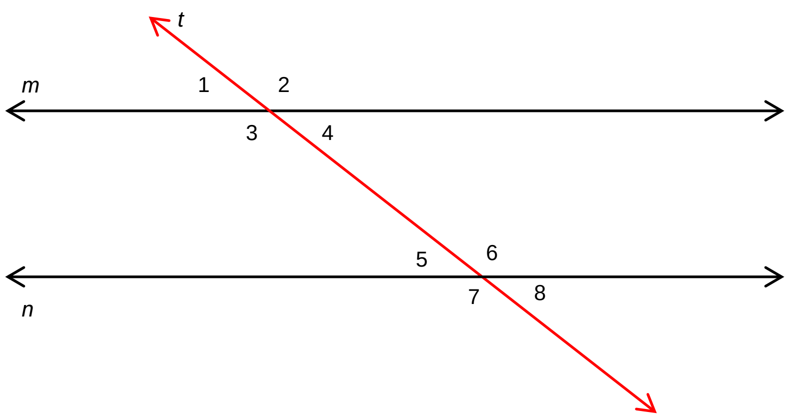
Parallel Lines Cut by a Transversal

When parallel lines are cut by a transversal, the following types of angles are **congruent**: corresponding angles, alternate exterior, and alternate interior angles. Consecutive interior angles are **supplementary**.

Now we'll take a look at a postulate and some theorems about angles that are formed when parallel lines are cut by a transversal.

Postulate 10-A

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.



Activity: Use your notebook paper and the lines on it to draw a pair of parallel lines. Then, using a ruler, draw a third line (a transversal) that intersects the parallel lines. Label the parallel lines as lines *m* and *n*. Label the transversal as *t*.

Check for congruence!

(1) Using a very thin paper, trace over the angle located to the left of the transversal and above the top parallel line. Slide the "tracing" paper down the transversal and check to see if its corresponding angle is congruent. You should find that it is indeed congruent!

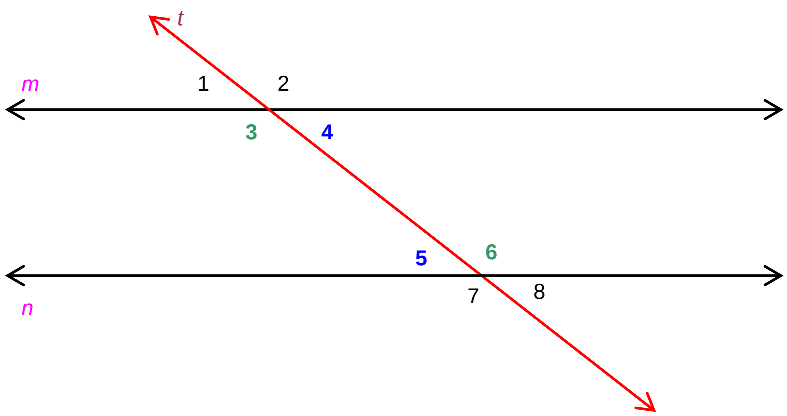
(2) Using the same procedure with thin paper, trace over the angle located to the right of the transversal and above the top parallel line. Slide the "tracing" paper down the transversal and check to see if its corresponding angle is congruent. You should find that it is indeed congruent!

(3) Using the same procedure with thin paper, trace over the angle located to the left of the transversal and below the top parallel line. Slide the “tracing” paper down the transversal and check to see if its corresponding angle is congruent. You should find that it is indeed congruent!

(4) Using the same procedure with thin paper, trace over the angle located to the right of the transversal and below the top parallel line. Slide the “tracing” paper down the transversal and check to see if its corresponding angle is congruent. You should find that it is indeed congruent!

Theorem 10-J

If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.



Given: $m \parallel n$

[If two parallel lines are cut by a transversal,
m, n t]

Prove: $\angle 3 \cong \angle 6$, $\angle 4 \cong \angle 5$

[then each of the pair of alternate interior angles is congruent.]

Statements	Reasons
1. $m \parallel n$	Given
Part 1	
2. $\angle 3 \cong \angle 7$	Corresponding angles are congruent. (Postulate 10-A)
3. $\angle 7 \cong \angle 6$	Vertical angles are congruent. (Theorem 10-H)
4. $\therefore \angle 3 \cong \angle 6$	Transitive Property (Theorem 10-A)

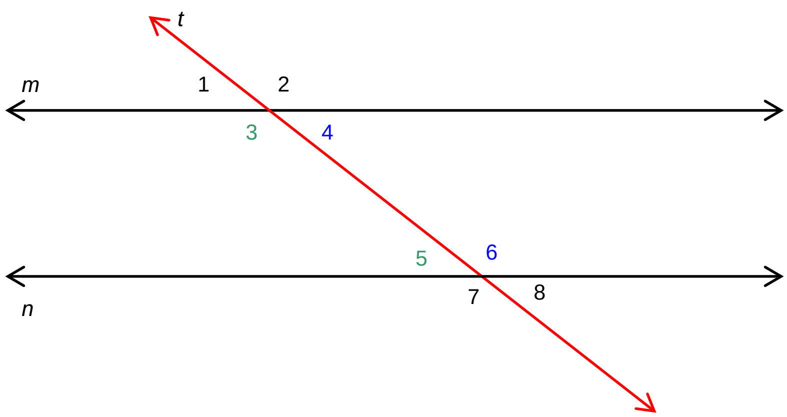
Part 2 (This will be left for you to prove in the problem set.)

Theorem 10-K

If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

Theorem 10-L

If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.



Given: $m \parallel n$

[If two parallel lines are cut by a transversal,
m, n t]

Prove: $\angle 3$ & $\angle 5$ are supplementary
$\angle 4$ & $\angle 6$ are supplementary

[then each pair of consecutive interior angles
is supplementary.]

Statements	**Reasons**
1. $m \parallel n$	Given
Part 1	
2. $\angle 1 \cong \angle 5$	Corresponding angles are congruent. (Postulate 10-A)
3. $m\angle 1 = m\angle 5$	Definition of congruent angles.
4. $m\angle 1 + m\angle 3 = 180$	Definition of linear pair.
5. $m\angle 5 + m\angle 3 = 180$	Substitution*
6. $\therefore \angle 5$ and $\angle 3$ are supplementary.	Definition of supplementary angles.

*Replace $m\angle 1$ in step#4 with $m\angle 5$ from step #3.

Part 2 (This will be left for you to prove in the problem set.)

Theorem 10-M

If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.