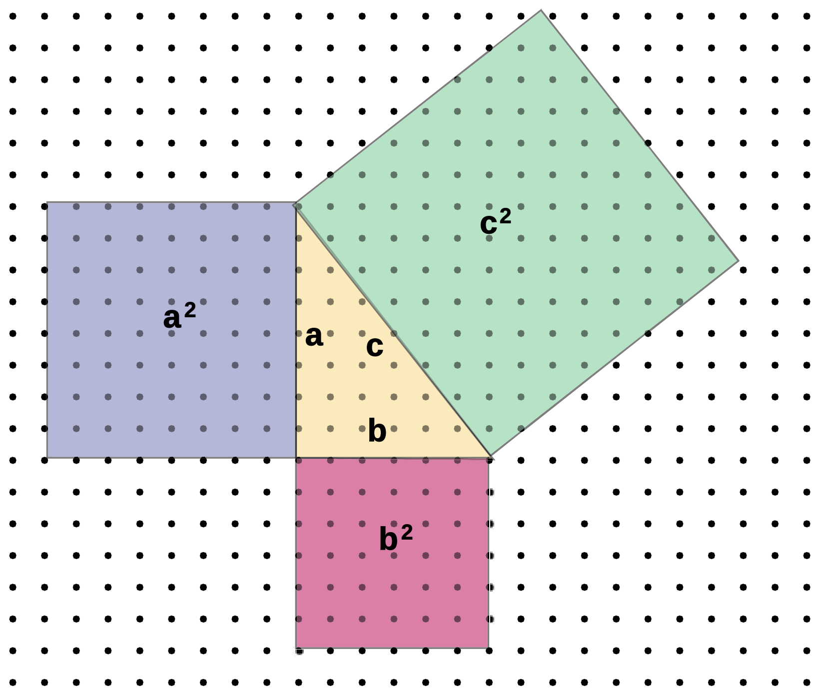
COORDINATE GEOMETRY

In this unit, you will first review the Pythagorean Theorem and apply it to the distance formula to find the length of line segments in the coordinate plane. You will then examine the midpoint formula to find the midpoint for both line segments and segments in the coordinate plane.

Pythagorean Theorem and Distance Formula

Theorem - A theorem is a mathematical statement that must be proven before it is accepted as being true.

Pythagorean Theorem - The Pythagorean Theorem is a relationship between the three sides of a right triangle. The sum of the squares of the two sides of the right triangle that make up the right angle are equal to the square of the third side, the hypotenuse which is the side opposite the right angle.



Special names are given to the sides of a right triangle. The two sides that make up the right triangle are called "**legs**" and the side opposite the right angle is called the "**hypotenuse**".

A special relationship exists between the sides of a right triangle. The sum of the squares of the two legs equals the square of the hypotenuse.

$$c^2 = a^2 + b^2$$

In the example above, the legs measure 6 and 8 units. What does the diagonal measure?

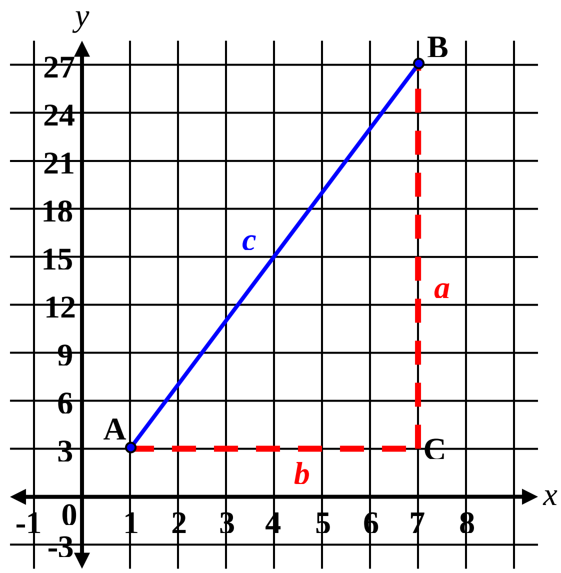
$c^2 = 6^2 + 8^2$
$c^2 = 36 + 64$
$c^2 = 100$
$c = \sqrt{100}$
$c = 10$

Theorem 4-A Pythagorean Theorem

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Example 1: Use the Pythagorean Theorem to find the distance from A to B.

*Note: Each space on the *x*-axis equals one unit. Each space on the *y*-axis equals 3 units.



Point A (1, 3) Point B (7, 27) Point C (7, 3)

Leg *a*: Find the length of $\overline{BC}$.

Point C (7, 3) to Point B (7, 27)

To find the distance, look at the change in the y-coordinates.

a = $\overline{BC} = |\,3 - 27| = |-24| = 24$

Leg *b*: Find the length of $\overline{AC}$.

Point A (1, 3) to Point C (7, 3)

To find the distance, look at the change in the *x*-coordinates.

$b = \overline{AC} = |\,1 - 7| = |-6| = 6$

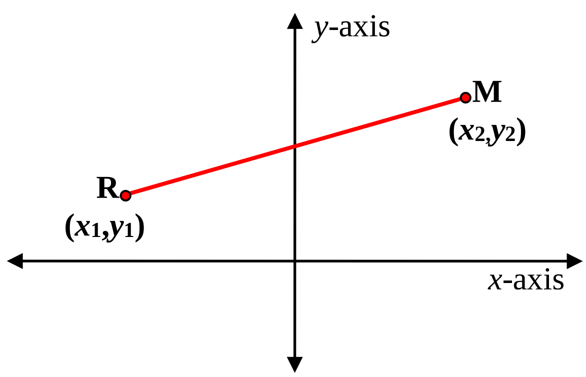
$$\begin{aligned}
a^2 + \ b^2 &= c^2 \\
24^2 + 6^2 &= c^2 \\
576 + 36 &= c^2 \\
612 &= c^2 \\
\pm\sqrt{612} &= c \qquad\qquad \text{*612 is an irrational number} \\
24.7 &\approx c
\end{aligned}$$

*612 is an irrational number and its root extends on forever and never develops into a repeating pattern. For this course, round irrational answers as directed. The symbol for approximately equal is $\approx$. In geometry, the negative root is often ignored because the problems are mostly about distance as is true in this problem.

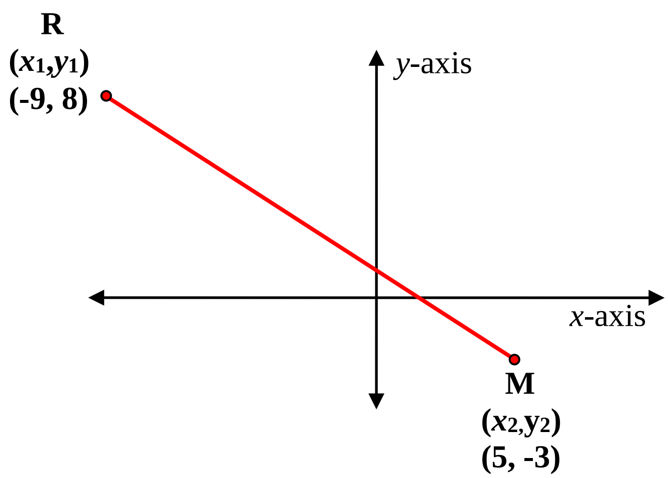
In coordinate geometry, the Pythagorean Theorem can be adapted to the Distance Formula.

Distance Formula

The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **.**



Example 2: Find the length of $\overline{RM}$ for R(–9, 8) and M(5, –3).



Let R be Point 1 and represented by (x_1,y_1). $(x_1,y_1) = (–9, 8)$

Let M be Point 2 and represented by (x_2,y_2). $(x_2,y_2) = (5, –3)$

Thus,

$x_1 = –9$ $y_1 = 8$ $x_2 = 5$ $y_2 = –3$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(5-(-9))^2 + (-3-8)^2}$$
$$d = \sqrt{(14)^2 + (-11)^2}$$
$$d = \sqrt{196+121}$$
$$d = \sqrt{317}$$
$$d \approx 17.8$$

Midpoint Formula

Midpoint – The midpoint of a line segment is the point that is halfway between the two end points.

Midpoint Definition

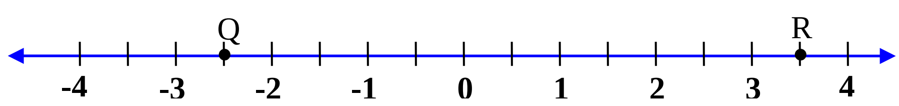
The midpoint, M, of $\overline{AB}$ is the point between A and B such that AM = MB.

To find the midpoint between two points on a number line, find the average distance between the two points.

Midpoint Formula Number Line

With endpoints of A and B on a number line, the midpoint of $\overline{AB}$ is $\frac{A+B}{2}$.

Example 1: Find the midpoint of Q and R.



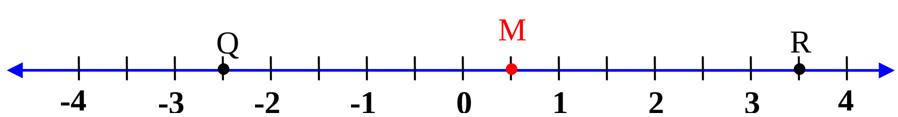
Let A represent point Q at –2.5.

A= –2.5

Let B represent point R at 3.5.

B= 3.5

$$M = \frac{A+B}{2} = \frac{-2.5+3.5}{2} = \frac{1.0}{2} = 0.5$$

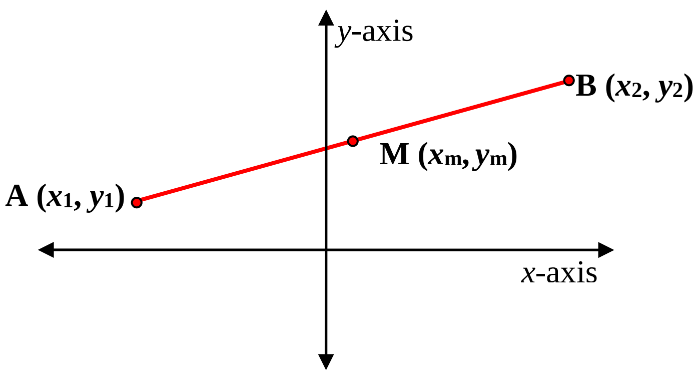


Midpoint Formula Coordinate Plane

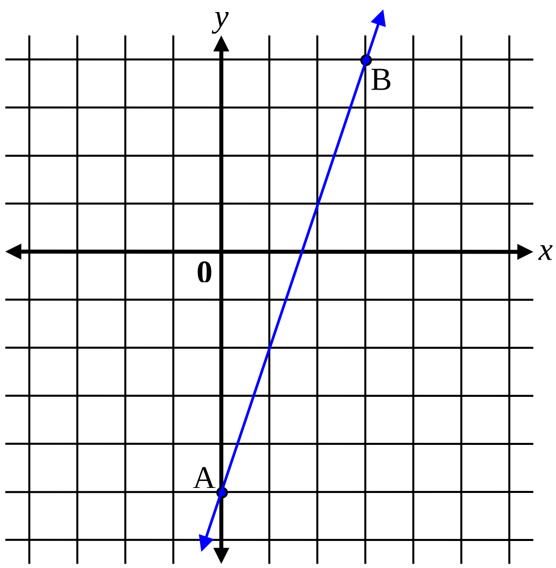
In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$**.**

To find the midpoint between two points in the coordinate plane, find the average distance between the two points. Study this formula and diagram.

$$\mathbf{M} = (x_m, y_m) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



Example 2: Find the midpoint (M) of $\overline{AB}$.



Point A (x_1, y_1) Point B (x_2, y_2)

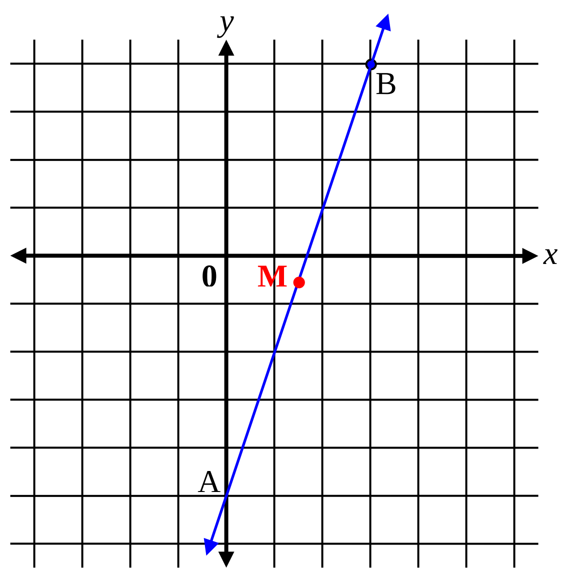
Point A (0, –5) Point B (3, 4)

$$M = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = (x_m, y_m) = \left(\frac{0+3}{2}, \frac{-5+4}{2}\right)$$

$$M = (x_m,, y_m) = \left(\frac{3}{2}, \frac{-1}{2}\right)$$

$$M = (x_m, y_m) = \left(1\frac{1}{2}, -\frac{1}{2}\right)$$



Theorem 4-B
Midpoint Theorem

If M is the midpoint of $\overline{PQ}$, then $\overline{PM} \cong \overline{MQ}$.