ANGLES

In this unit, you will examine angles and learn how connect various angle relationships. You will also learn how to make geometric constructions.
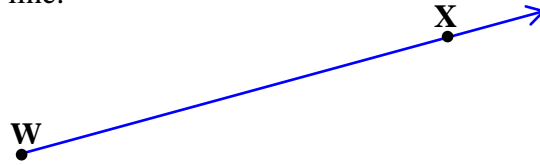
Angles

Measuring Angles

Angle Constructions

Angle Relationships

Angles

Ray – A ray is a half line.

The name of this ray is $\overrightarrow{WX}$ or Ray WX. Since the ray starts at point W, W must be the first letter of its name.

**Vertex** – A vertex is the point where two rays meet.

Angle – An angle is formed when two rays meet at a common point. The measure of an angle is the amount of circular rotation about a point starting with a ray and ending with a second ray.
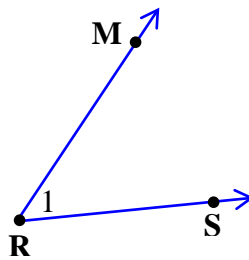
Angle 1, also denoted as $\angle 1$, may be named $\angle MRS$ (Angle MRS). Point R is the vertex, $\overrightarrow{RS}$ and $\overrightarrow{RM}$ are the rays.

* When using three letters to name an angle, be sure to make the vertex letter the center letter of the name.
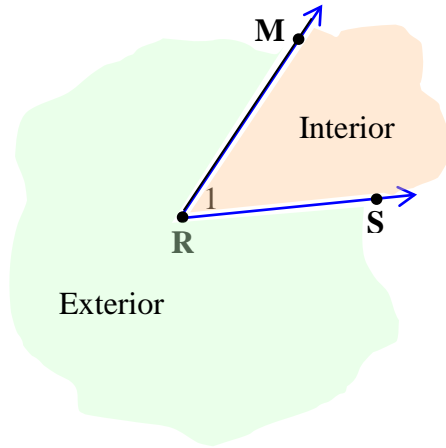
Opposite Rays – Opposite rays are two half lines that are formed at a point on a line.
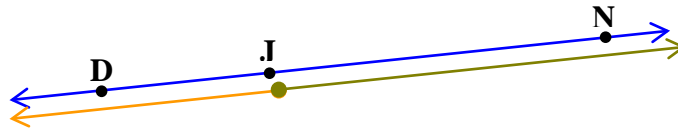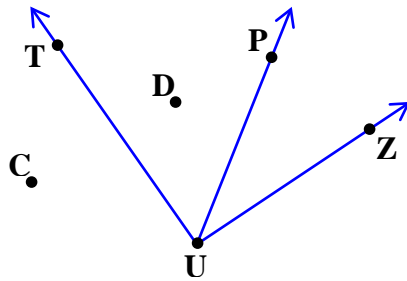
The opposite rays in this diagram are $\overrightarrow{JD}$ and $\overrightarrow{JN}$.

Interior – The interior of an angle is the area within the two rays.

Exterior – The exterior of an angle is the area outside the two rays.

M

Interior

1

R

S

Exterior

Example 1: Refer to the diagram to answer the questions.

T

P

D

C

Z

U

a) Name a vertex.

Point U is the vertex.

b) Name a ray.

There are three rays. They are $\overrightarrow{UT}, \overrightarrow{UP}, \overrightarrow{UZ}$.

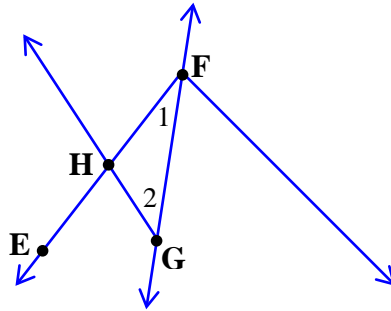c) Name three angles.

They are $\angle TUP, \angle PUZ, TUZ$.

d) Name a point that lies in the interior of $\angle TUZ$.

Point D or Point P

e) Name a point that lies in the exterior of ∠*TUZ* .

    Point C

Example 2: Refer to the diagram to answer the questions.

a) What are other names for ∠1 ?

    ∠*HFG* and ∠*GFH* (The vertex letter may be used to name an angle but
    should not be used when it can also be used to name other angles, so
    ∠*F* would not be a good choice.)

b) What is the name of the vertex for ∠*FHG* ?

    Point H

c) What is the name of the vertex for ∠2 ?

    Point G

d) Name two different rays on $\overleftrightarrow{FG}$.

 $\overrightarrow{FG}$ starting at Point F extending downward and $\overrightarrow{GF}$ starting at Point G and
    extending upward.

Measuring Angles

Postulate - A postulate is a mathematical statement that is accepted as true. It is based on the fundamentals of mathematical belief.

Measurement of an angle – Angles are measured in degrees.

Degree – A degree is a unit of rotation around a point that may be used to measure angles. One degree is $\frac{1}{360}th$ of a rotation around a point.
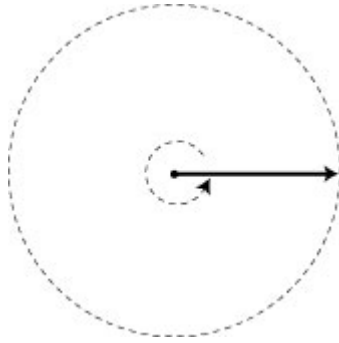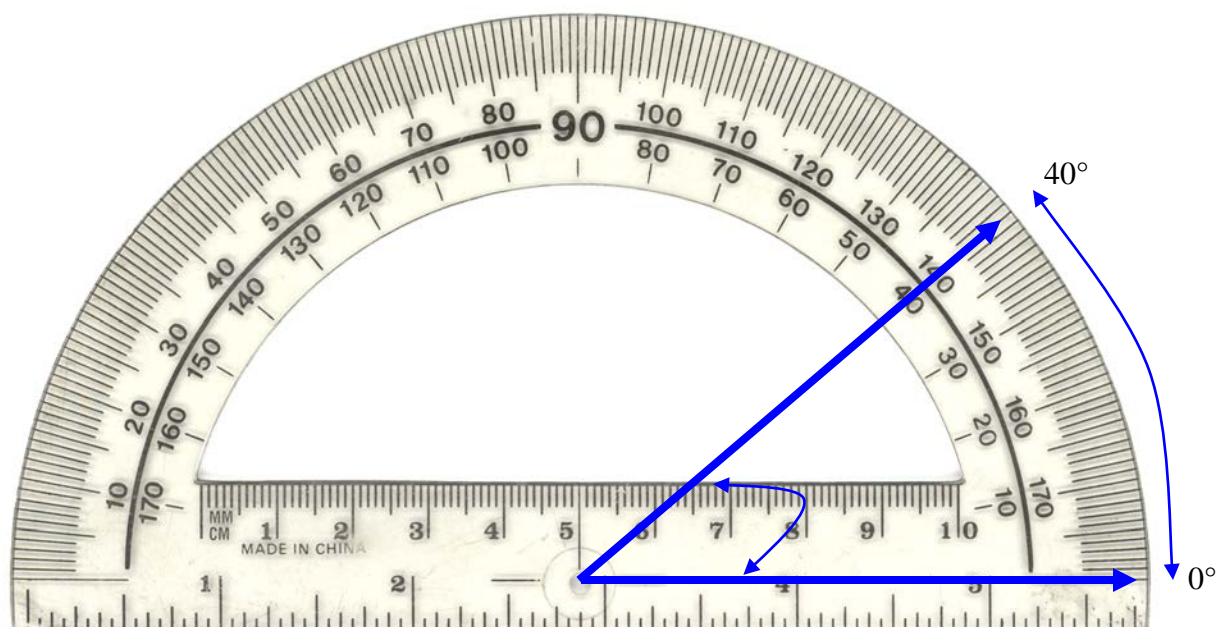
**Protractor** – A protractor is a measurement tool used to measure angles in degrees.
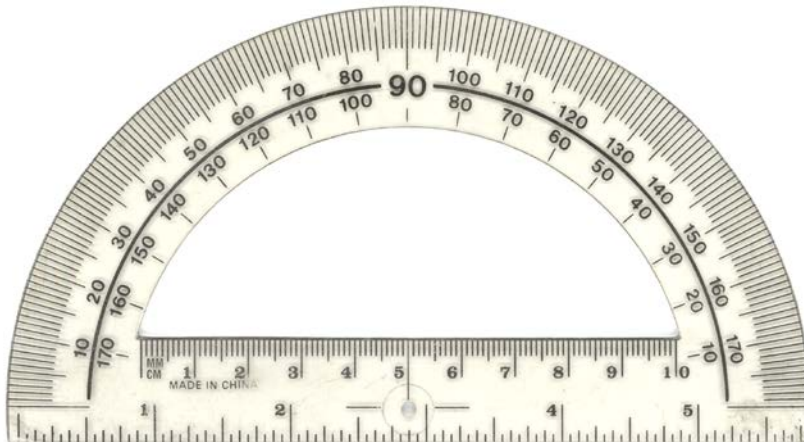
Postulate 2-A Protractor Postulate	Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.

The starting ray, in this case, the bottom ray, is at 0°. Read the other ray. The ray is passing through both 40 and 140. You must decide which number makes sense. Think about a right angle – it measures 90°. This angle is not as open as a right angle; thus, you would read the smaller number. This angle measures 40°. Notice the numbers near the bottom ray, the lower set of numbers start at 0, then 10, 20, etc. That is the set of numbers that is used to read this angle. The starting ray starts at 0°. This is an **acute angle** because it measures more than 0° and less than 90°.
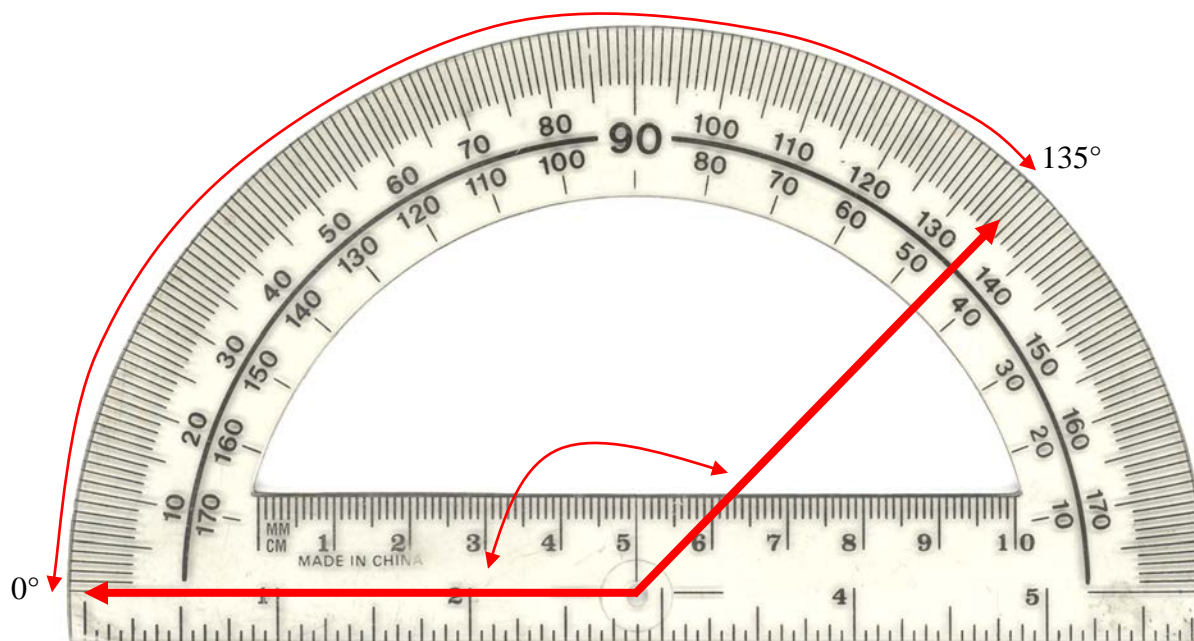

The starting ray, in this case, the bottom ray, is at 0°. Read the other ray. The ray is passing through both 45 and 135. You must decide which number makes sense. Think about a right angle – it measures 90°. This angle is open wider than a right angle; thus, you would read the larger number. This angle measures 135°. Notice the numbers near the bottom ray, the upper set of numbers start at 0, then 10, 20, etc. That is the set of numbers that is used to read this angle. The starting ray starts at 0°. This is an **obtuse angle** because it measure more than 90° and less than 180°.

Angles are Degrees of Rotation

The measure of an angle is the amount of **circular rotation** about a point starting with a ray and ending with a second ray. All of these angles measure **45°**.
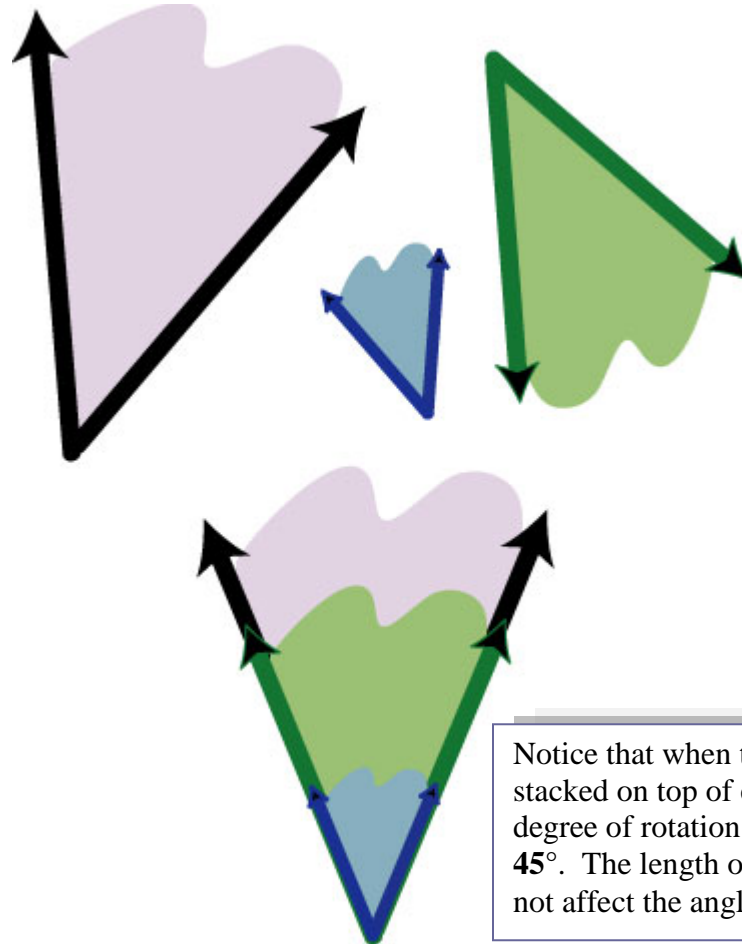
Notice that when the angles are stacked on top of each other, the degree of rotation is the same, **45°**. The length of the rays does not affect the angle of rotation.

Definition of Right, Acute and Obtuse Angles	$\angle A$ is a right angle if $m\angle A$ is 90.

$\angle A$ is a right angle if $m\angle A$ is 90.
$\angle A$ is an acute angle if $m\angle A$ is less than 90.
$\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180.

Angle Constructions

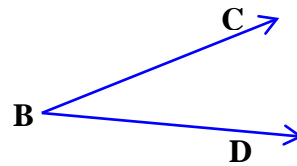**Compass** – A compass is a measurement tool used to draw arcs and circles.

Arc – An arc is a portion of a circle.

Congruent figures – geometric figures that have the same size and shape.

We will now look at using a compass to make some geometric constructions.

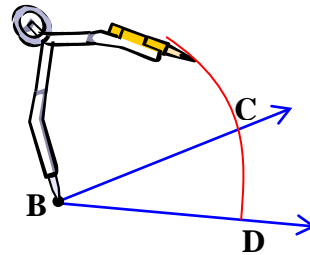   *Example 1*: Construct $\angle TNR$ to be equal in measure to $\angle B$.
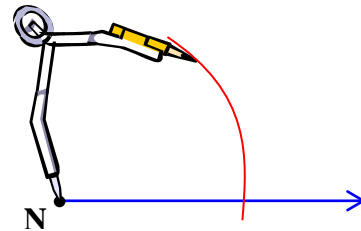
   Draw $\angle B$.

   Draw $\overrightarrow{NT}$

   Place the metal point of the compass at
   vertex B and draw an arc passing through
   both rays of $\angle B$.

   Move to $\overrightarrow{NT}$ and without changing the
   setting on the compass, place the metal
   point of the compass on point N and
   draw the same arc across the ray.

   Place the metal point of the compass at point D
   and the pencil point at point C, where the arc
   crosses $\overrightarrow{BD}$. You will have to adjust the settings
   of the compass to do this.
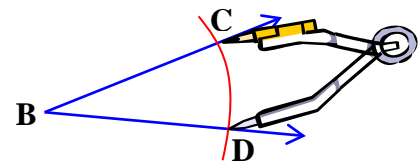
Make sure the compass has not changed settings.

On $\overrightarrow{NT}$ place the metal point of the compass at the point of intersection for the ray and the arc. Use the pencil point to make a small arc that crosses the larger arc.
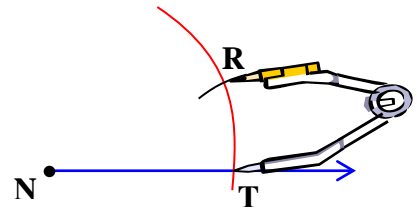
Draw $\overrightarrow{NR}$ starting at point N and passing through Point R, the point of intersection between the two arcs.
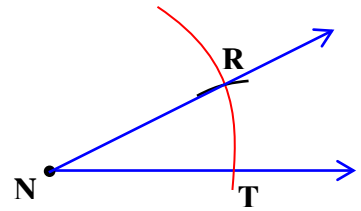
Angle RNT is the same size as angle CBD.

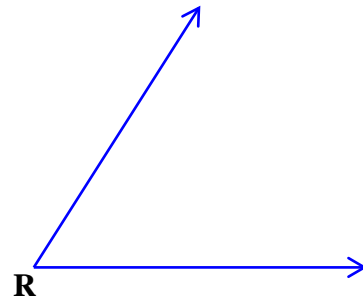This can be represented as follows:

$m\angle RNT = m\angle CBD$

We can also say that angle RNT is congruent to angle CBD.

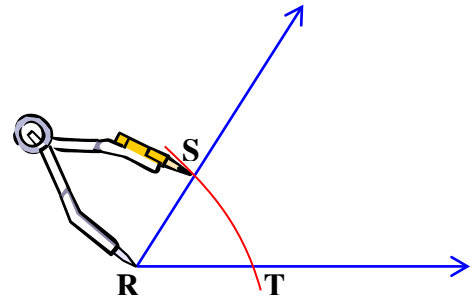The symbol for congruence is $\cong$. Thus,

$\angle RNT \cong \angle CBD$

Example 2: Find a ray that divides the angle R into two congruent parts by constructing an angle bisector.
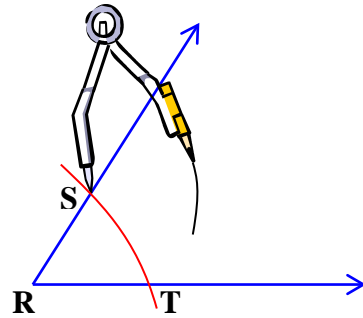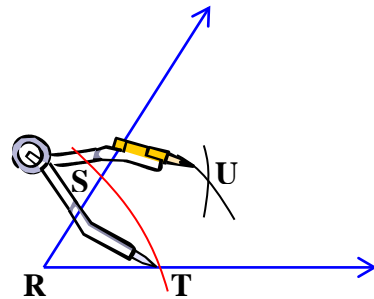
Draw $\angle R$.

Place the metal point of the compass at point R and draw an arc through the angle rays, naming the points of intersection, S and T.

Adjust the compass settings a little wider and place the metal point of the compass at point S. Draw an arc in the interior of the angle.

Keep the compass setting the same and place the metal point of the compass at point T. Draw a second arc in the interior of the angle letting it cross the other arc.

Draw $\overrightarrow{RU}$ so that it starts at the vertex R and extends though the intersection of the two arcs, point U.
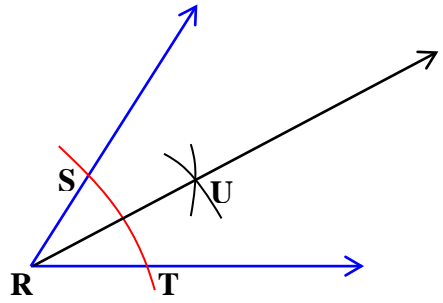
$\overrightarrow{RU}$ is the bisector of $\angle SRT$, thus $m\angle SRU = m\angle URT$ and $\angle SRU \cong \angle URT$.

Angle Relationships

Right Angle - A right angle measures 90˚.

Straight Angle – A straight angle measures 180˚.

Supplementary angles – Supplementary angles are angles that total 180˚.

**Complementary angles** – Complementary angles form a right angle.

Linear pair – A linear pair is a pair of adjacent angles whose sum forms a straight angle.

Vertical angles – Vertical angles are the opposite congruent angles formed when two lines intersect.

Postulate 2-B **Angle Addition**	**If** R **is in the interior of** $\angle PQS$ **, then** $m\angle PQR + m\angle RQS = m\angle PQS$. **If** $m\angle PQR + m\angle RQS = m\angle PQS$, **then** R **is in the interior of** $\angle PQS$.

Example 1: Use the diagram to answer the questions.
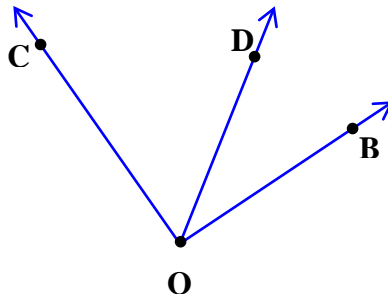
a) $m\angle COD + m\angle DOB = ?$

By the Angle Addition Postulate, the sum of the two angles equals $m\angle COB$.

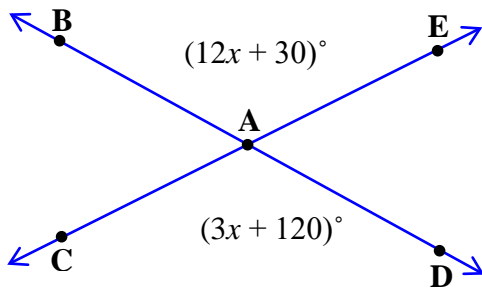b) If $m\angle COD$ equals 50˚ and $m\angle COB$ equals 85˚, what is $m\angle DOB$?

$$m\angle COD + m\angle DOB = m\angle COB$$

$$50˚ + x = 85˚$$
$$x = 35˚$$

<p style="text-align:center; background-color:#90EE90;">**Vertical angles are congruent.**</p>

Example 2: Find $m\angle BAE$.

Since $\angle BAE$ and $\angle CAD$ are vertical angles, they are congruent and their measures are equal.
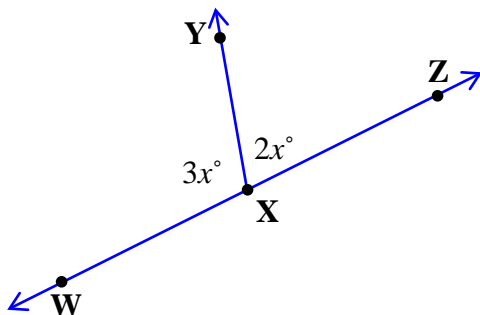
$$12x + 30 = 3x + 120$$
$$9x = 90$$
$$x = 10$$
$$m\angle BAE = 12x + 30$$
$$m\angle BAE = 12(10) + 30$$
$$m\angle BAE = 150°$$

Diagram labels: B, E, A, C, D with $(12x + 30)°$ and $(3x + 120)°$

<p style="text-align:center; background-color:#90EE90;">**The sum of the measures of the angles in a linear pair is 180˚.**</p>

Example 3: Find the measure of $\angle WXY$ and $\angle ZXY$.

Since $\angle WXY$ and $\angle ZXY$ are a linear pair, the sum of their measures is 180 degrees.
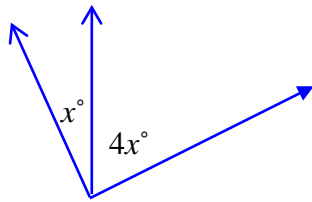
$$m\angle WXY + m\angle ZXY = 180$$
$$3x + 2x = 180$$
$$5x = 180$$
$$x = 36$$

Diagram labels: Y, Z, X, W with $3x°$ and $2x°$

$m\angle WXY = 3x$	$m\angle ZXY = 2x$
$m\angle WXY = 3(36)$	$m\angle ZXY = 2(36)$
$m\angle WXY = 108°$	$m\angle ZXY = 72°$

Check: Linear pairs total $180°$. $72° + 108° = 180°$

The sum of the measures of complementary angles is 90˚.

Example 4: A pair of angles is complementary. One of the angles is 4 times larger than the other angle. How large are each of the angles?

Draw a picture.

Let x represent the smaller angle. x

Let $4x$ represent the larger angle. $4x$

Complementary angles total 90. first angle + second angle = 90

Write an equation and solve. $x + 4x = 90$

$$5x = 90$$

$$x = 18$$

The first angle (x) is 18˚; the second angle ($4x$) equals 4(18) which is 72˚.

Check: Complementary angles total 90˚. $18˚ + 72˚ =$ $90˚$