GRAPHING EQUATIONS OF LINES

In this unit you will learn how to write equations of lines using the slope-intercept method and the point-slope form. You will also review and apply the properties of slopes, intercepts, and parallel and perpendicular lines. You will then explore scatter plots and investigate the equation properties of “lines of best fit”.

Linear Equations

linear equation – A linear equation is a mathematical description for a line drawn in a coordinate plane.

standard form – The standard form of a linear equation is $Ax + By = C$; where *A*, *B*, and *C* are real numbers and *A* and *B* are not both equal to zero.

Standard Form of a Linear Equation

The standard form of a linear equation, where *A*, *B*, and *C* are real numbers and A and B are not both equal to zero, is

$$Ax + By = C$$

Example 1: Write the following equation in standard form without fractions:

$$\frac{2}{3}y - 1 = -\frac{3}{4}x$$

$\frac{2}{3}y - 1 = -\frac{3}{4}x$	Given Equation
$12(\frac{2}{3}y - 1) = 12(-\frac{3}{4}x)$	Multiply both sides of the equation by 12 (LCD).
$8y - 12 = -9x$	Simplify
$9x + 8y = 2$	Add $9x$ and 12 to both sides of the equation.

The standard form for the given equation is $9x + 8y = 12$.

Example 2: Which equation is a linear equation? Explain why.

a.) $x + y^2 = 5$ b.) $x^3 - 7x = 8$ c.) $x + 2y = 9$ d.) $x^2 - 2x + 1 = 0$

Choice "c" ($x + 2y = 9$) is the linear equation**.** Both variables are raised to the first power.

In linear equations, the variables are raised to the first power. In choice "a", the *y* is raised to the second power; in choice "b", the *x* is raised to the third power; and in choice "d", *x* is raised to the second power.

Slopes and Intercepts

The **slope** of a line refers to the steepness and direction of the line and is found by finding the change in vertical units (y) divided by the corresponding change in horizontal units (x).

There are four (4) types of slopes.

positive **rises right**	**negative** **rises left**	**no slope** **vertical**	**zero slope** **horizontal**

To find the slope of a line given two points (x_1, y_1) and (x_2, y_2),

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{which is the formula for} \frac{\textit{change in } y}{\textit{change in } x}$$

*ratio for slope is $\dfrac{rise}{run}$

Slope of a Line

The slope of a line can be calculated by finding the change in y divided by the change in x.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

*Note: When $x_2 - x_1 = 0$, the line is a vertical line and the slope is “undefined”.

Example 1: Find the slope of a line that passes through the points (3, 5) and (–2, –6).

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Formula for Slope
$m = \dfrac{-6-5}{-2-3}$	Substitute x and y values of points.
	$(x_1 = 3,\ y_1 = 5,\ \ x_2 = -2,\ y_2 = -6)$
$m = \dfrac{-11}{-5}$	Simplify
$m = \dfrac{11}{5}$	Simplify

Thus, the slope of the line is $\dfrac{11}{5}$.

y-intercept – The y-intercept is the point at which the line crosses the y-axis (0, *y*).

x-intercept – The x-intercept is the point at which the line crosses the x-axis (*x*, 0)

slope-intercept form of a line – The slope-intercept form of a line is $y = mx + b$ where m represents the "slope" and *b* represents the "y-intercept".

Theorem 30 **Slope-Intercept Form**	**If an equation of a line is written in the form, *y* = *mx* + *b*, then *m* is the slope of the line and *b* is the y-intercept.**

To graph an equation using the slope and y-intercept

1. Solve the equation for *y* if the equation is not already done so $(y = mx + b)$.

2. Plot the y-intercept which is located at (0, *b*).

3. Use the ratio of $\dfrac{rise}{run}$ to plot more points.

Example 2: Graph: $-3x + y = -4$

Step 1: Solve the equation for y.

$-3x + y = -4$	Given
$y = 3x - 4$	Add $3x$ to both sides.
$y = 3x - 4$	Slope-intercept Form

Step 2: Determine and plot the "y-intercept".

$y = 3x - 4$	Slope-intercept Form
$y = 3(0) - 4$	At the y-intercept of a line, $x = 0$.
$y = -4$	Simplify

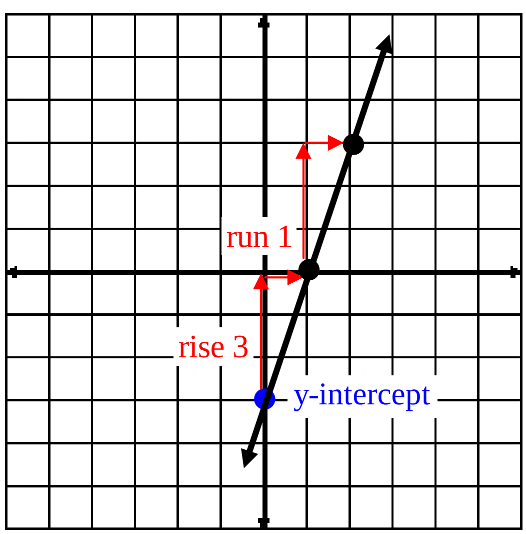
The "y-intercept" is located at $(0, -4)$.

Step 3: Determine the slope as a ratio and plot more points.

$y = 3x - 4$	Slope-intercept Form
$m = \frac{3}{1}$	Write the slope as a ratio.

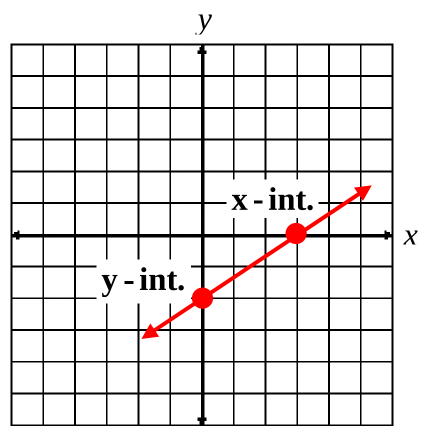
The slope is $\frac{3}{1}$, $\frac{\text{rise}}{\text{run}}$, a rise of 3 (up 3), and a run of 1 (to the right 1).

The graph of $-3x + y = -4$ is shown below.



To graph a line using the x and y-intercepts:

1. Replace x with a zero and solve for y. [this will give you the y-intercept $(0, y)$]
2. Replace y with a zero and solve for x. [this will give you the x-intercept $(x, 0)$]
3. Plot the points.



Example 3: Graph the equation: $3x + 6y = -12$

Step 1: Let $x = 0$.

$3x + 6y = -12$

$3(0) + 6y = -12$

$6y = -12$

$y = -2$

The y-intercept is $(0, -2)$.

Step 2: Let $y = 0$.

$3x + 6y = -12$

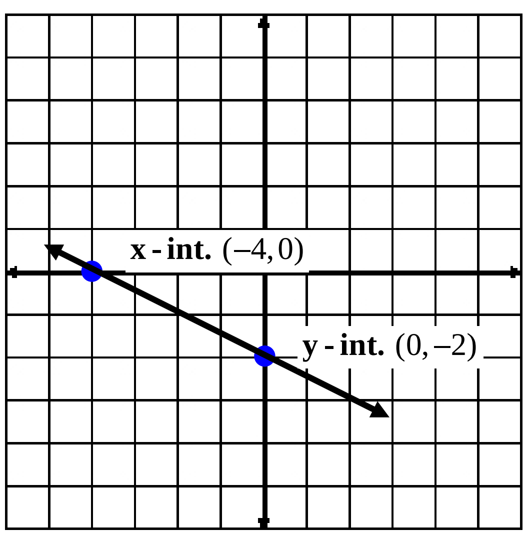
$3x + 6(0) = -12$

$3x = -12$

$x = -4$

The x-intercept is $(-4, 0)$.

Step 3: Plot these two points and connect them with a straight line.



Point-Slope Form

If you are given certain information about a line, you are able to write the equation of the line by using the point-slope form.

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Where x_1 and y_1 represent a point on the line and m represents the slope.

You can use the point-slope form when given the **slope** and a **point** on the line.

Example 1: Write the equation of a line that contains the point (–3, –4) and has a slope of $\frac{2}{3}$.

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - (-4) = m(x - (-3))$	Substitute $(-3, -4)$ in for $(x_1,\ y_1)$ $x_1 = -3,\ y_1 = -4$
$y + 4 = m(x + 3)$	Simplify
$y + 4 = (\frac{2}{3})(x + 3)$	Sustitute $\frac{2}{3}$ for m (Given)
$y + 4 = \frac{2}{3}x + 2$	Distribute $\frac{2}{3}$ times $(x + 3)$
$y = \frac{2}{3}x - 2$	Subtract 4 from both sides.

The equation of the line going through the point $(-3, -4)$ and having a slope of $\frac{2}{3}$ is $y = \frac{2}{3}x - 2$.

You can also use the point-slope form when given **two points** on the line.

Example 2: Write an equation of a line containing the points (2, 1) and (5, 4).

First: Determine the slope using the slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$m = \frac{4-1}{5-2}$ Substitute x and y values of points $(2,1)$ and $(5,4)$

$(x_1 = 2,\ y_1 = 1,\ \ x_2 = 5,\ y_2 = 4)$

$m = \frac{3}{3}$ Simplify

$m = 1$ Simplify

Thus, the slope equals 1.

Now: Determine the equation using the point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope Form

$y - 1 = m(x - 2)$ Substitute (2, 1) in for $(x_1,\ y_1)$.
$x_1 = 2,\ y_1 = 1$ (You may use either point.)

$y - 1 = (1)(x - 2)$ Sustitute 1 for *m.*

$y - 1 = x - 2$ Simplify

$y = x - 1$ Add 1 to both sides.

The equation of the line going through the points (2, 1) and (5, 4) is $y = x - 1$.

Example 3: Determine if these points are collinear: $A(3,4)$, $B(1,-2)$ and $C(-1,-8)$.

To solve this problem, make a plan.

- Use points A and B to determine the slope of the equation.
- Use point A and the slope to determine the equation of the line.
- Check to see if the coordinates of point C satisfy the equation.

Step 1: Determine the slope of the line that passes through points A and B.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Slope Formula
$m = \dfrac{-2-4}{1-3}$	Substitute x and y values of point $A(3,4)$ and point $B(1,-2)$. $(x_1 = 3,\ y_1 = 4,\ x_2 = 1,\ y_2 = -2)$
$m = \dfrac{-6}{-2}$	Simplify
$m = 3$	Simplify

The slope of the line equals 3.

Step 2: Determine the equation of the line that passes through point A and has a slope of three (determined use point A and point B).

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - y_1 = 3(x - x_1)$	Substitute slope $(m = 3)$ in the point-slope form.
$y - 4 = 3(x - 3)$	Substitute point $A(3,\ 4)$ in for $(x_1,\ y_1)$. $x_1 = 3,\ y_1 = 4$
$y - 4 = 3x - 9$	Distribute 3 times $(x - 3)$.
$y = 3x - 5$	Add 4 to both sides.

Step 3: Check to see if the coordinates of point C satisfy the equation. (Note: Since point B was used with point A to determine the slope, it is not necessary to check point B.)

$y = 3x - 5$	Use the equation determined by points A and point B.
$(-8) = 3(-1) - 5$	Substitute the coordinates for point $C(-1, -8)$ into the equation. $x = -1,\ y = -8$
$-8 = -8$	Simplify. Both sides equal the same number; thus, point C satisfies the equation and is part of the line.

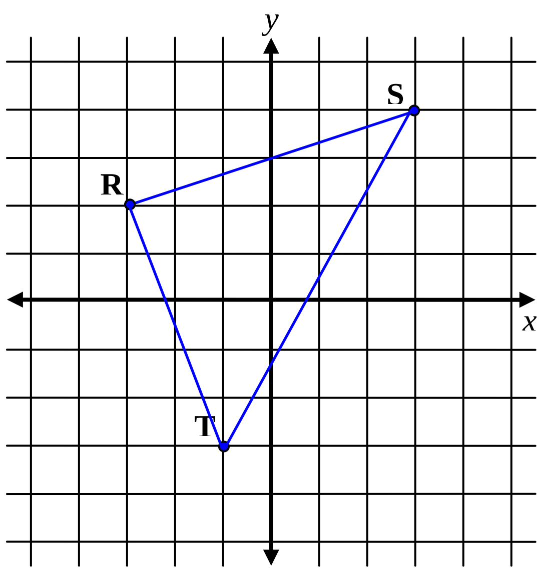
Points A, B, and C are collinear points and they are part of the line represented by the equation $y = 3x - 5$.

Example 4: Find the equation of the line that is a median of triangle RST drawn from Point T to segment RS. The coordinates of the points in triangle RST are R(–3,2), S(3,4), and T (–1,–3).

To solve this problem, make a plan.

- Graph the points.
- Determine the midpoint of segment RS and name it point M. Recall that the median of a triangle is a line segment that connects a vertex and the midpoint of the opposite side .Sketch the median.
- Use the point-slope form to determine the equation of the line that passes through points T and M.

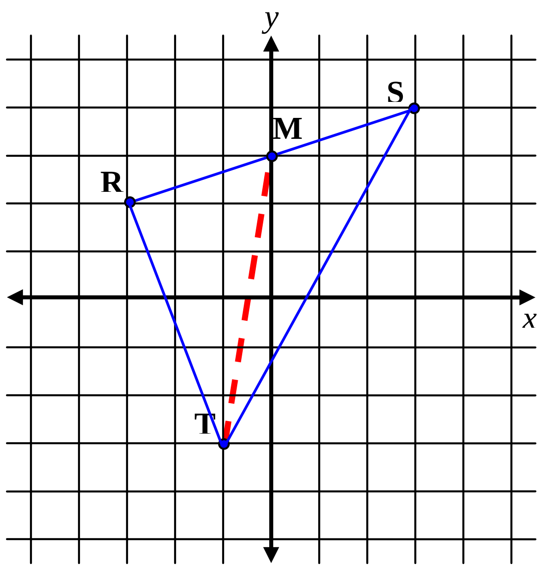
Step 1: Graph the points.



Step 2: Calculate the midpoint (M) and draw the median.

$M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$	Midpoint Formula
$M = (\frac{(-3) + 3}{2}, \frac{2 + 4}{2})$	Substitute values from points $R(-3, 2)$ and $S(3, 4)$. $(x_1 = -3,\ y_1 = 2,\ \ x_2 = 3\ y_2 = 4)$
$M = (\frac{0}{2}, \frac{6}{2})$	Simplify
$M = (0, 3)$	Simplify

The midpoint of segment RS is M(0, 3).



Step 3: Determine the equation of the line (median) that passes through point T and point M.

First: Calculate the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$m = \frac{3-(-3)}{0-(-1)}$$ Substitute x and y values of points T$(-1,-3)$ and M$(0,3)$.

$(x_1 = -1,\ y_1 = -3,\ x_2 = 0\ y_2 = 3)$

$$m = \frac{6}{1} = 6$$ Simplify

$$m = 6$$

Finally: Calculate the equation for the median (segment TM) of the triangle.

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - y_1 = 6(x - x_1)$	Substitue slope in the point-slope form.
$y - 3 = 6(x - 0)$	Substitute (0, 3) in for $(x_1,\ y_1)$. You may use either point M or T.
$y - 3 = 6x$	Simplify
$y = 6x + 3$	Add 3 to both sides.

The equation of the line that is a median from point T to segment RS is $y = 6x + 3$.

Equations of Horizontal and Vertical Lines

Horizontal lines have a **slope of 0**.

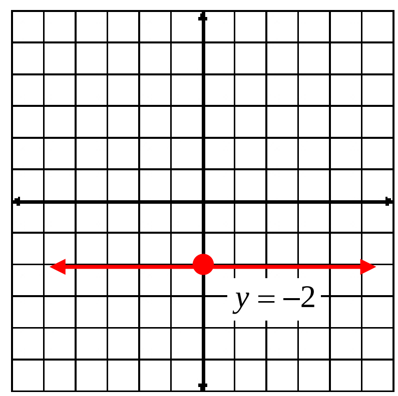
Vertical lines have an **undefined slope** or no slope.

In this section we will discuss the equation of both of these lines and how to graph each.

Horizontal and Vertical lines

The equation of a horizontal line is $\boldsymbol{y = b}$**,** where b is the y-intercept.

The equation of a vertical line is $\boldsymbol{x = a}$, where a is the x-intercept.

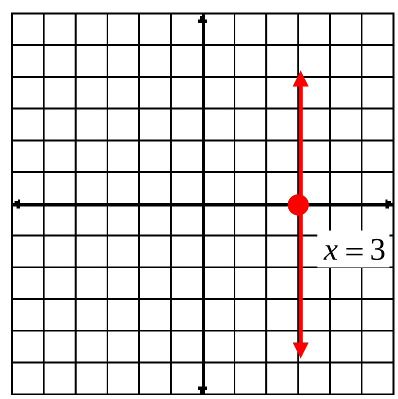


Example 1: Graph the equation, $y = -2$.

This is a horizontal line with a y-intercept of $(0, -2)$.

Plot the point $(0, -2)$ and draw a horizontal line through it.

On this line, all points have a y-value equal to –2.



Example 2: Graph the equation, $x = 3$.

This is a vertical line with an x-intercept of $(3, 0)$.

Plot the point $(3, 0)$ and draw a vertical line through it.

On this line, all points have an x-value equal to 3.

Parallel and Perpendicular Lines

Parallel lines have the **same** slope.

The lines for the equations shown below are parallel because each has a slope of $-\frac{4}{5}$.

$$\left.\begin{array}{l} y=-\frac{4}{5}x-5 \\ \\ y=-\frac{4}{5}x+2 \end{array}\right\} \text{ Equations of Parallel Lines}$$

Perpendicular lines have **opposite reciprocal** slopes.

The lines for the equations shown below are perpendicular because the slopes of the lines are opposite reciprocals, $-\frac{5}{8}$ and $\frac{8}{5}$.

$$\left.\begin{array}{l} y=-\frac{5}{8}x+2 \\ \\ y=\frac{8}{5}x-3 \end{array}\right\} \text{ Equations of Perpendicular Lines}$$

*Note: Recall that the product of the slopes of two perpendicular lines is –1. $(-\frac{\cancel{5}}{\cancel{8}}\times\frac{\cancel{8}}{\cancel{5}}=-1)$.

By knowing this you will be able to write equations of lines that are parallel to or perpendicular to given equations and containing a certain point.

Example 1: Write the equation, in slope-intercept form, of a line that is parallel to $y=2x+3$ and passes through the point (–1, 4).

To solve this problem, make a plan.

- Write the equation in slope-intercept form. Identify the slope of the equation.
- Then, use the point-slope form to determine the equation.

Step 1: Determine the slope of the given equation.

Since $y = 2x + 3$ is in slope-intercept form $(y = mx + b)$, we can determine that the coefficient of x is the slope; thus $m = 2$.

Since we want a line that is parallel to this line, we will use the same slope, $m = 2$, because parallel lines have the same slope.

Therefore, $m = 2$.

Step 2: Use the point-slope form to determine the equation. You will substitute the slope you found above and the given point to determine the equation.

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - y_1 = 2(x - x_1)$	Substitute slope $(m = 2)$ in the point-slope form.
$y - 4 = 2(x - (-1))$	Substitute point $(-1,\ 4)$ in for $(x_1,\ y_1)$. $x_1 = -1,\ y_1 = 4$
$y - 4 = 2(x + 1)$	Simplify
$y - 4 = 2x + 2$	Distribute 2 times $(x + 1)$.
$y = 2x + 6$	Add 4 to both sides.

The equation of the line passing through point $(1, -4)$ and parallel to the equation, $y = 2x + 3$, is $y = 2x + 6$.

Example 2: Write the equation of a line perpendicular to $5x + 2y = 10$ and passing through the point (3, –5).

To solve this problem, make a plan.

- Write the equation in slope intercept form.
- Identify the slope of the equation that would represent a line that is perpendicular to the given equation.
- Then, use the point-slope form to determine the equation.

Step 1: Solve the equation, $5x+2y=10$, for y so that it will be in slope intercept form $(y=mx+b)$.

$5x+2y=10$	Given
$2y=-5x+10$	Subtract $5x$ from both sides.
$y=-\frac{5}{2}x + 5$	Divide both sides by 2.

Therefore, the given equation's slope is $m=-\frac{5}{2}$.

Step 2: Determine the slope of the line that is perpendicular to the line of the given equation.

The slope of the given line is $-\frac{5}{2}$.

The opposite reciprocal is $\frac{2}{5}$.

Therefore, the equation of the line that is perpendicular to the given equation's slope is $m=\frac{2}{5}$.

Step 3: Determine the equation of the line using point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - y_1 = \frac{2}{5}(x - x_1)$	Substitute slope ($m = \frac{2}{5}$) in the point-slope form.
$y - (-5) = \frac{2}{5}(x - 3)$	Substitute point $(3, -5)$ in for $(x_1,\ y_1)$. $x_1 = 3,\ y_1 = -5$
$y + 5 = \frac{2}{5}(x - 3)$	Simplify
$y + 5 = \frac{2}{5}x - \frac{6}{5}$	Distribute $\frac{2}{5}$ times $(x - 3)$.
$y = \frac{2}{5}x - \frac{6}{5} - \frac{25}{5}$	Subtract 5 from both sides. ($5 = \frac{25}{5}$)
$y = \frac{2}{5}x - \frac{31}{5}$	Simplify

The equation of the line passing through the point $(3, -5)$ and perpendicular to the equation, $5x + 2y = 10$, is $y = \frac{2}{5}x - \frac{31}{5}$.

Example 3: Find the equation of the line that is an altitude of triangle RST drawn from Point R to segment TS. The coordinates of the points in triangle RST are R(–3,2), S(3,4), and T (–1,–3).

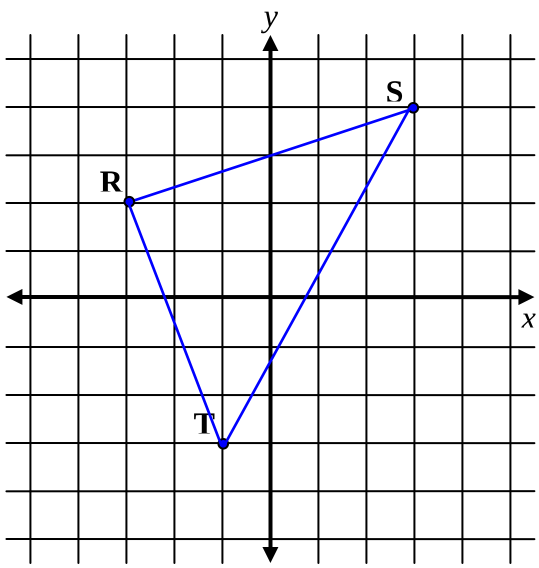
To solve this problem, make a plan.

- Graph the points.
- Determine the slope of segment TS.

- Determine the slope of the line segment that is perpendicular to segment TS (altitude).

- Use the point-slope form to determine the equation of the line that passes through points R and is perpendicular to segment TS.

Step 1: Graph the points.



Step 2: Calculate the slope of segment TS using the slope formula.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$m = \dfrac{4-(-3)}{3-(-1)}$ Substitute x and y values of points $T(-1,-3)$ and $S(3,4)$

$(x_1 = -1,\ y_1 = -3,\ x_2 = 3,\ y_2 = 4)$

$m = \dfrac{7}{4}$ Simplify

The slope of segment TS is $\dfrac{7}{4}$.

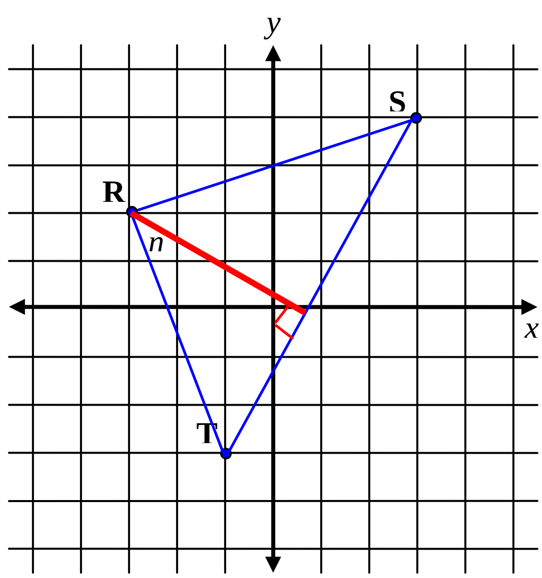
Step 3: Determine the slope of the line segment (altitude) that is perpendicular to segment TS.

The slope of the segment TS is $\frac{7}{4}$.

The opposite reciprocal is $-\frac{4}{7}$.

Therefore, the slope of the altitude is $m = -\frac{4}{7}$.

Step 4: Use the point-slope form to determine the equation if the line that that passes through point R and is perpendicular to segment TS.



Line segment *n* is an altitude of $\triangle$RST.

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - y_1 = -\frac{4}{7}(x - x_1)$	Substitute slope ($m = -\frac{4}{7}$) in the point-slope form.
$y - 2 = -\frac{4}{7}(x - (-3))$	Substitute point $(-3, 2)$ in for $(x_1,\ y_1)$. $x_1 = -3,\ y_1 = 2$
$y - 2 = -\frac{4}{7}(x + 3)$	Simplify
$y - 2 = -\frac{4}{7}x - \frac{12}{7}$	Distribute $-\frac{4}{7}$ times $(x + 3)$.
$y = -\frac{4}{7}x - \frac{12}{7} + \frac{14}{7}$	Add 2 to both sides. $(2 = \frac{14}{7})$
$y = -\frac{4}{7}x + \frac{2}{7}$	Simplify

The equation of the altitude passing through the point $(-3, 2)$ is $y = -\frac{4}{7}x + \frac{2}{7}$.

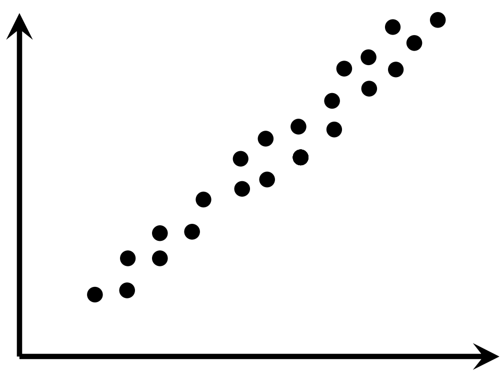
Use, Create, and Interpret Scatter Plots

A **scatter plot** shows the relationship between two sets of data.

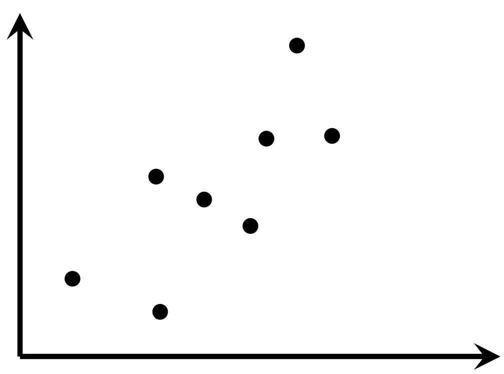
Positive Correlation

If the points of a scatter plot appear to suggest a line that slants upward to the right, then there is a positive relationship between the two sets of data.

In a **positive correlation**, **both** data sets **increase**.



Strong

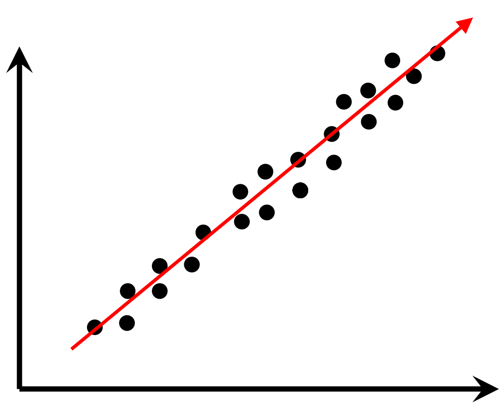


Weak

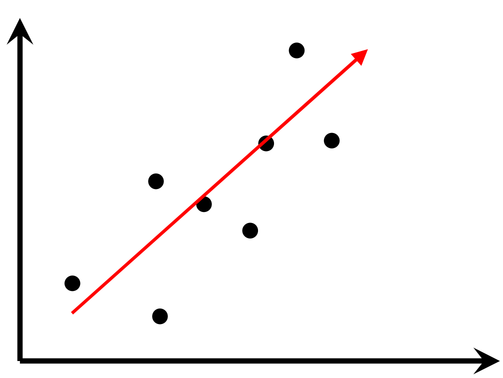
A "line of best fit" is a line drawn so that about the same amount of points occurs above it as below it.

When the "line of best fit" is added to these two scatter plots, the positive correlation becomes more evident. Each line has a positive slope.

*Note: The "line of best fit" does NOT have to pass through every point.



Strong

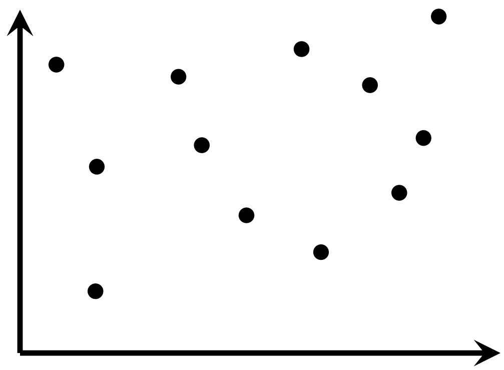


Weak

No Correlation

If the points of a scatter plot seem to be random, then there is **no** relationship them.

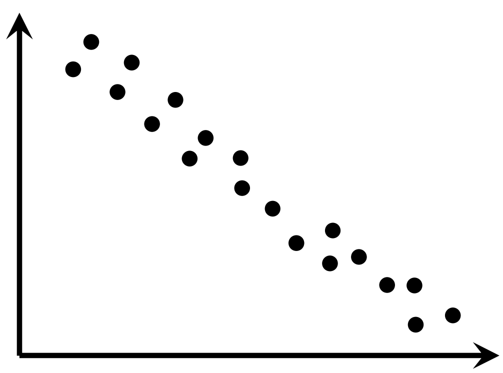
When there is no correlation, the change in one data set does not affect the other data set.



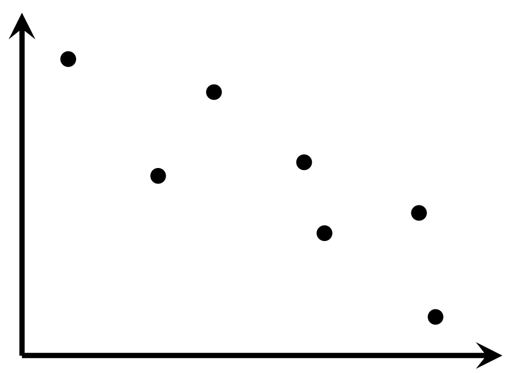
Negative Correlation

If the points of a scatter plot appear to suggest a line that slants downward to the right, then, there is a negative relationship between both sets of data.

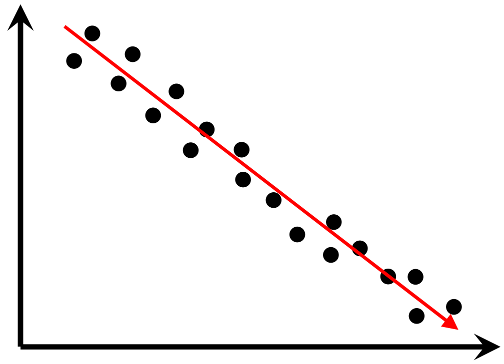
In a **negative correlation**, one set of **data increases** as the other set of **data decreases**.



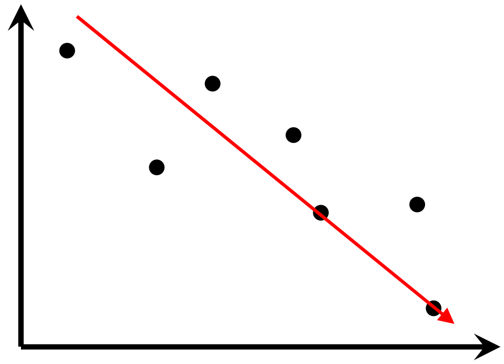
Strong



Weak



Strong



Weak

When the “line of best fit” is added to these two scatter plots, the negative correlation becomes more evident. Each line has a negative slope.

When creating scatter plots and other types of graphs it is necessary to first determine the scope or size of the x-axis and y-axis to fit the values you are presenting. You must take care in selecting values that are clear and will present a graph that is easily read. You must also be consistent with the increments that are labeled so the reader can quickly find the information and understand the graph.

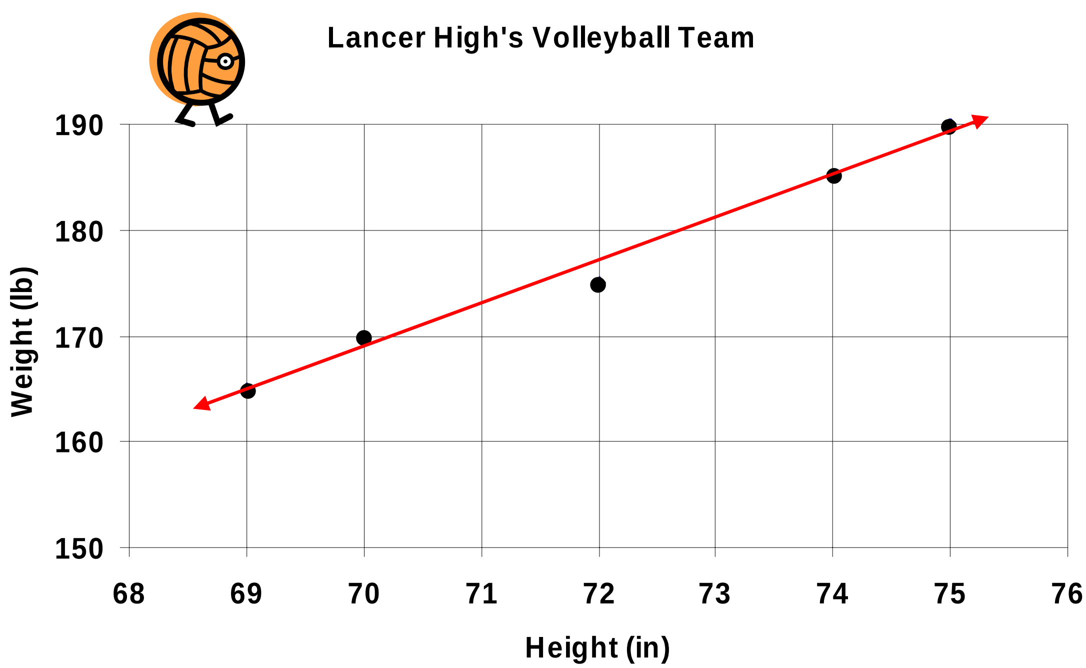
Example: Use the two data sets to make a scatter plot of the weight and height of each member of Lancer's High Volleyball team. Then, determine the equation of a line that could be considered "the line of best fit" for the trend of the data.

Height (in)	70	69	72	74	75
Weight (lb)	170	165	175	185	190

Step 1: Make a scatter plot of the data pairs. The points on the scatter plot are:

(70, 170), (69, 165), (72, 175), (74, 185), and (75, 190).

Step 2: Draw the line that appears to be the "line of best fit" the data points. There should be about the same number of points above the line as below it.



Step 3: Find the slope of the line. We will use two points that fall on the line, (69,165) and (74,185).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$m = \frac{185 - 165}{74 - 69}$$ Substitute x and y values of points (69,165) and (74,185).

$(x_1 = 69,\ y_1 = 165,\ x_2 = 74,\ y_2 = 185)$

$$m = \frac{20}{5}$$ Simplify

$$m = 4$$ Simplify

The slope of the "line of best fit" is 4.

Step 4: Determine the equation of the line that has a slope of 4 and passes through points (69,165) and (74,185).

$y - y_1 = m(x - x_1)$	Point-slope Form
$y - y_1 = 4(x - x_1)$	Substitute slope ($m = 4$) in the point-slope form.
$y - 165 = 4(x - 69)$	Substitute point $(165, 69)$ in for $(x_1,\ y_1)$. $x_1 = 165,\ y_1 = 69$
$y - 165 = 4x - 276$	Distribute 4 times $(x - 69)$.
$y = 4x - 276 + 165$	Add 165 to both sides.
$y = 4x - 111$	Simplify

The equation of the line that relates the height of the players to their heights is $y = 4x - 111$.

Slope of a Line

When graphing lines, the slope of a line must be considered.

slope - The slope of a line describes the steepness of the line. The slope is the ratio of vertical rise to horizontal run.

$$\text{slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

vertical rise - The vertical rise of a line is the change in the *y*-values from one point on the line to another point on the line.

horizontal run – The horizontal run of a line is the change in the *x*-values from one point on a line to another point on the line.

Another way to represent slope is:

$$m = \frac{\triangle y}{\triangle x}$$

$\triangle$ is a short way to represent the word "change".

$\triangle$ is a Greek symbol and can be read as "delta".

For a line, even though the numbers for the vertical rise and horizontal run may change, the slope of the line remains constant.

To find the slope of a line:

-identify a point on the line

-from that point move up or down until you are directly across from the next point

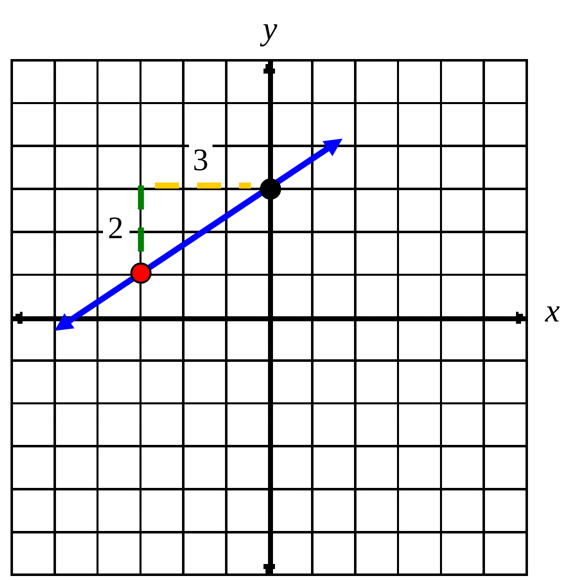
-move left or right to the next point.

Example: Determine the slope of the line shown in the coordinate plane. Each space in the grid represents one unit.

-put your pencil on the red point.

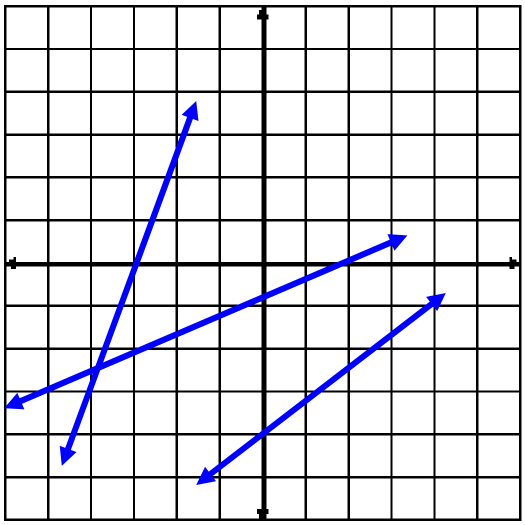
-move straight up (vertical rise) until your pencil is in the same line as the black point, (2 units)

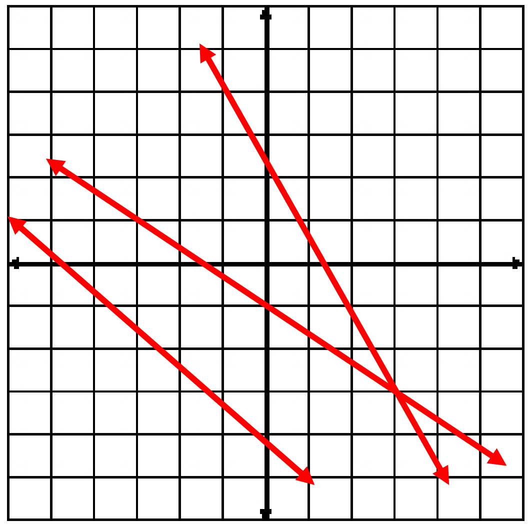
-move right (horizontal run) until you reach the black point. (3 units)



The slope of the line is $\frac{2}{3}$ or $m = \frac{2}{3}$.

On a coordinate plane, there are lines that have positive slopes and lines that have negative slopes.





Lines with positive slopes rise to the **right.**

Lines with negative slopes rise to the **left.**

Finding the Slope of a Line When Given Two Points

The definition of slope states that, given two points (x_1, y_1) and (x_2, y_2), the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

*Notice, this is the vertical change over the horizontal change.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{vertical change}}{\text{horizontal change}} \quad \text{*ratio for slope is } \frac{rise}{run}$$

Example 1: Find the slope of a line going through the points (3, 5) and (–2, –6).

*It may be easier to pick out the *x* and *y* values if you label them as follows. This will help you in setting up the ratio.

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ (3, 5) & (-2, -6) \end{array}$$

$$x_1 = 3 \quad y_1 = 5 \qquad\qquad x_2 = -2 \quad y_2 = -6$$

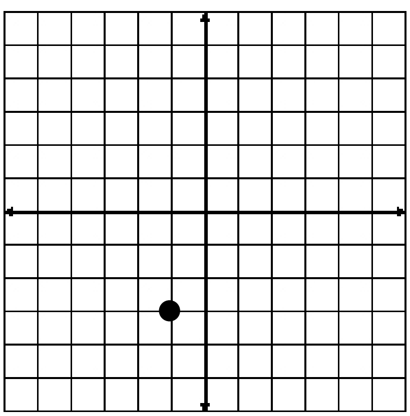
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-5}{-2-3} = \frac{-11}{-5} = \frac{11}{5}$$

Thus, the slope (m) is $\frac{11}{5}$.

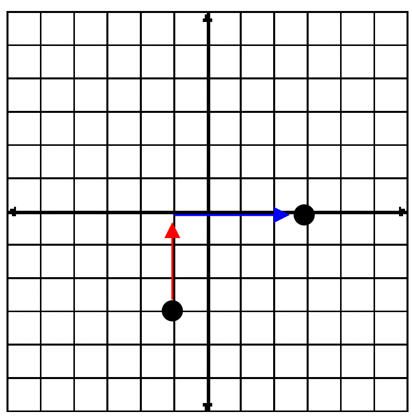
Graphing a Line using the Point-Slope Method

When given the slope of a line and a point (location) on the line, the line can then be graphed.

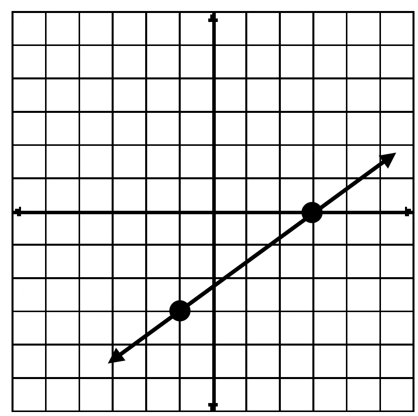
Example 2: Graph the line containing the point (–1, –3) and having a slope of $m = \frac{3}{4}$.



1. Plot the point (–1, –3)



2. Use the rise (3 units) over run (4 units) ratio for slope to plot a second point.

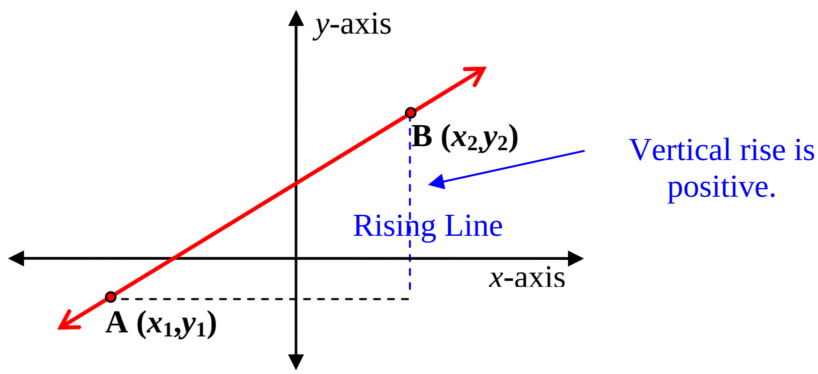


3. Draw a line through the points with a straightedge.

Four Types of Slopes

When working with slopes of lines in a coordinate plane, there are four types of slopes.

Positive Slope

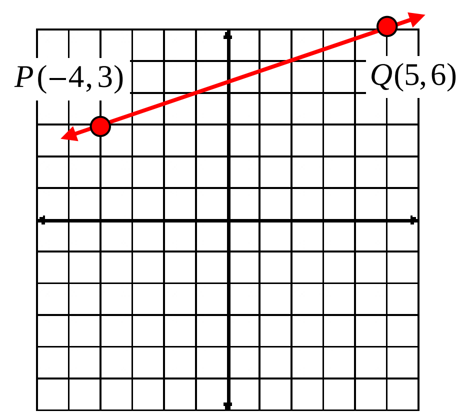


Example 1: Find the slope of the line containing the point $P(–4, 3)$ and $Q(5, 6)$.

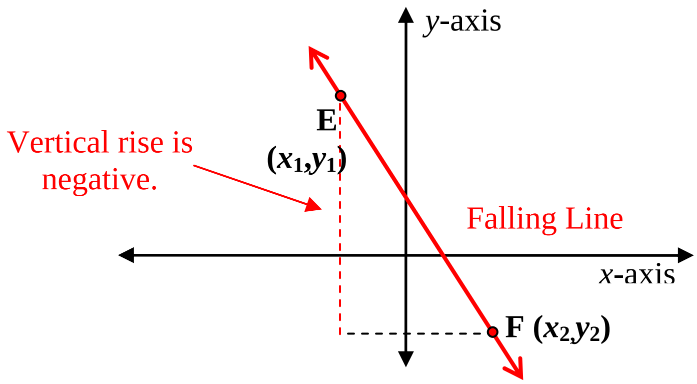
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6-3}{5-(-4)}$$

$$m = \frac{3}{9} \, or \, \frac{1}{3}$$



Negative Slope

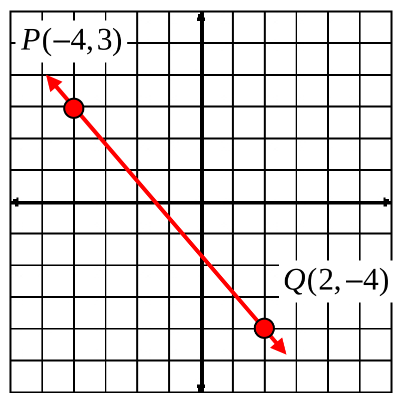


Example 2: Find the slope of the line containing the point *P*(–4, 3) and *Q*(2, –4).

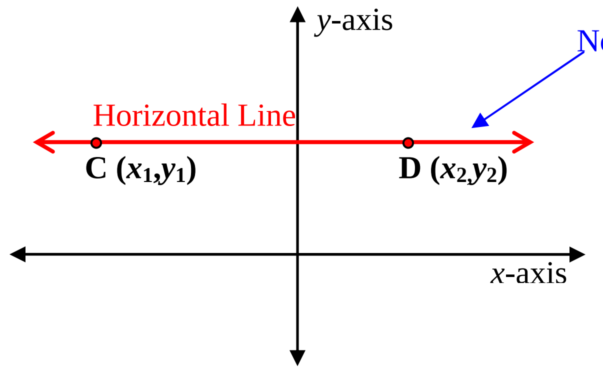
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-4-3}{2-(-4)}$$

$$m = \frac{-7}{6} \text{ or } -\frac{7}{6}$$

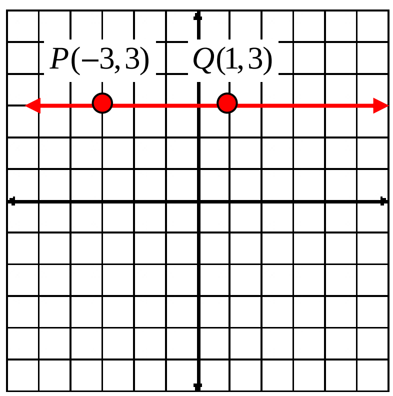


Zero Slope



$$\text{slope} = \frac{0}{\text{horizontal run}} = 0$$

Example 3: Find the slope of the line containing the point $P(-3, 3)$ and $Q(1, 3)$.



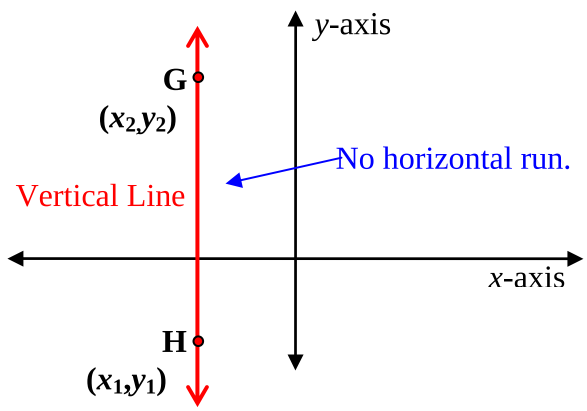
The *y*-coordinate for every point on a horizontal line is the same.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-3}{-3-1} = \frac{0}{-4} = 0$$

***Note: The slope of EVERY horizontal line in a coordinate plane is zero (0).**

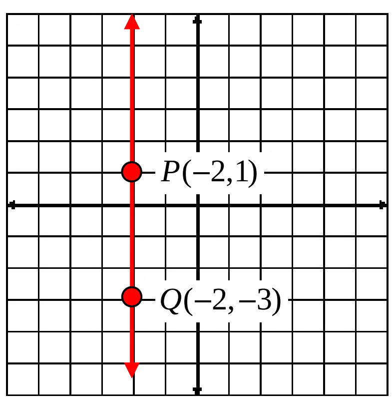
No Slope



$$\text{slope} = \frac{\text{vertical rise}}{0} = \text{undefined}$$

**Division by 0 is undefined.*

Example 4: Find the slope of the line containing the point $P(-2,1)$ and $Q(-2,-3)$.



The *x*-coordinate for every point on a vertical line is the same.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-(-3)}{-2-(-2)} = \frac{4}{0} = \text{undefined}$$

$$m = \text{undefined or "no" slope}$$

***Note: The slope of EVERY vertical line in a coordinate plane is "undefined".**

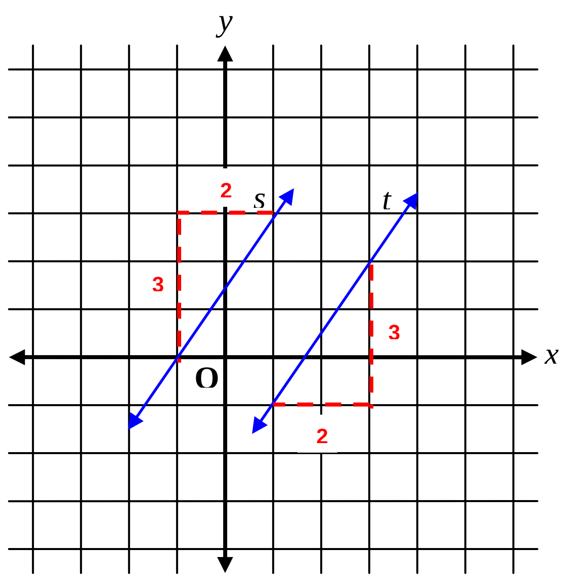
Slopes of Parallel and Perpendicular Lines

Slope of Parallel Lines

Lines have the same slope if they are parallel. Conversely, if lines are parallel, they have the same slope. Since both the original statement and its converse are true, we can condense the statement into one statement using the phrase "if and only if".

Postulate 8-A	**Two non-vertical lines have the same slope if and only if they are parallel.**

Example 1: Are lines *s* and *t* parallel?



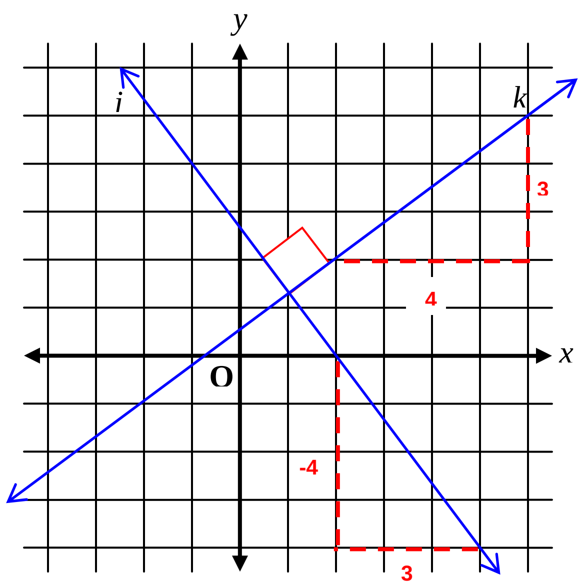
The slope of line *s* is $\frac{2}{3}$. The slope of line *t* is $\frac{2}{3}$. Since the lines have the same slope, they are parallel.

Slope of Perpendicular Lines

If two lines are perpendicular, then the product of their slopes is –1. Conversely, if the product of the slopes of two lines is –1, then the lines are perpendicular. Since both the original statement and its converse are true, we can condense the statement into one statement using the phrase "if and only if".

Postulate 8-B **Two non-vertical lines are perpendicular if and only if the product of their slopes is –1.**

Example 2: Are lines *j* and *k* perpendicular?



The slope of line *j* is $-\frac{4}{3}$ and the slope of line *k* is $\frac{3}{4}$.

The product of $-\frac{4}{3}\times\frac{3}{4}=-1$; therefore, lines *j* and *k* are perpendicular.