

SURFACE AREA

In this unit you will create and visualize three-dimensional shapes and figures. You will be given various views of a figure and determine its 3-D model. You will examine the properties of various solids including common 3-D shapes and the Platonic solids. You will calculate various problems that are related to determining the surface area of a variety of 3-D shapes.

Visualize Models

Isometric Dot Paper

Three-Dimensional Figures

Platonic Solids

Surface Area and Nets

Surface Area of a Prism

Surface Area of a Cylinder

Surface Area of a Pyramid

Surface Area of a Cone

Surface Area of a Sphere

Surface Area Formulas

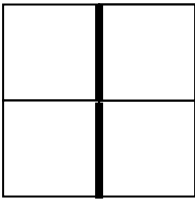
Visualizing Models

Models may be constructed when given a diagram of different views of a three-dimensional figure.

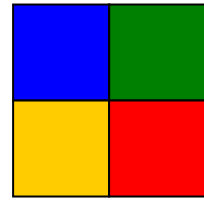
We will examine a three dimensional model of cubes drawn from two-dimensional views.

Given below are five two-dimensional views of the same three-dimensional model built from cubes. The heavy segments between the squares represent columns of different heights.

Top View

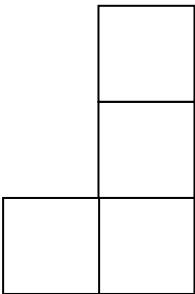


As the model is viewed from the top, it shows the **tops** of four columns. The heavy line segments represent that the columns are different heights.

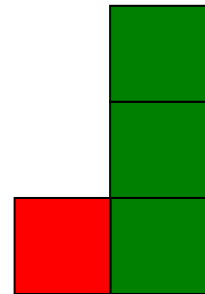


- tops of four columns
- columns are different heights

View of Left Side

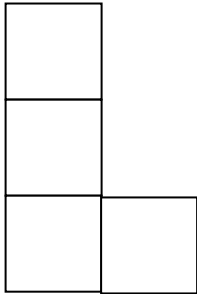


The left view reveals that the figure is **two blocks wide** with one column being 3 blocks high and the other column being 1 block high. All surfaces are flush to the side (no heavy lines).

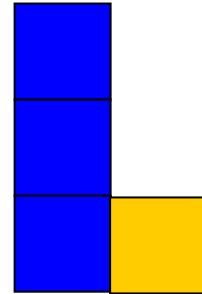


- side view from left
- two columns
- columns are different heights

View of Right Side

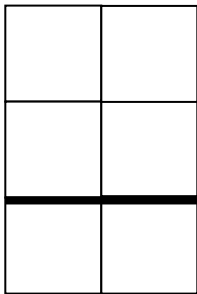


The right view reveals that the figure is **two blocks wide** with one column being 1 block high and the other column being 3 blocks high. All surfaces are flush to the side (no heavy lines).

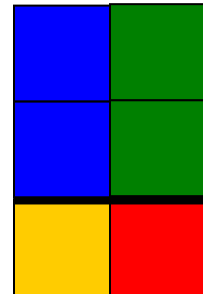


- side view from right
- two columns
- columns are different heights

Front View

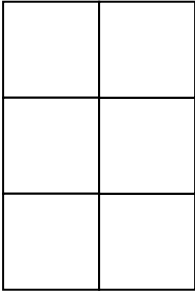


The front view reveals that there are **three rows of two blocks** and that the bottom row is only one block high.

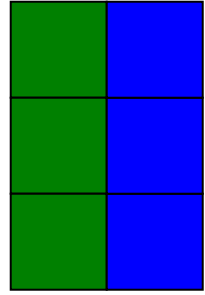


- front view
- three rows, two columns
- bottom row is different height than other rows

Back View

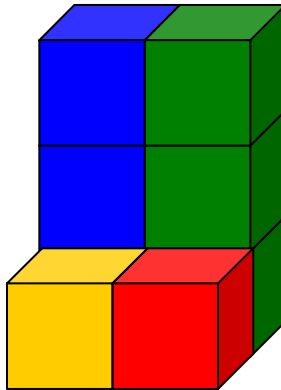


The back view reveals that the figure is **three blocks high** and **two blocks wide** with all surfaces flush to the side (no heavy lines).



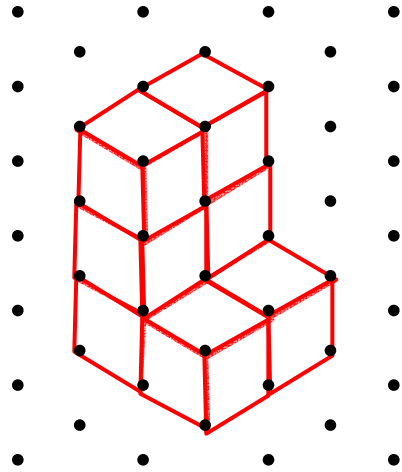
- back view
- two columns, three rows
- all the same height and width

Putting all the views together, the 3-D figure shown below is the figure described in the various views above.

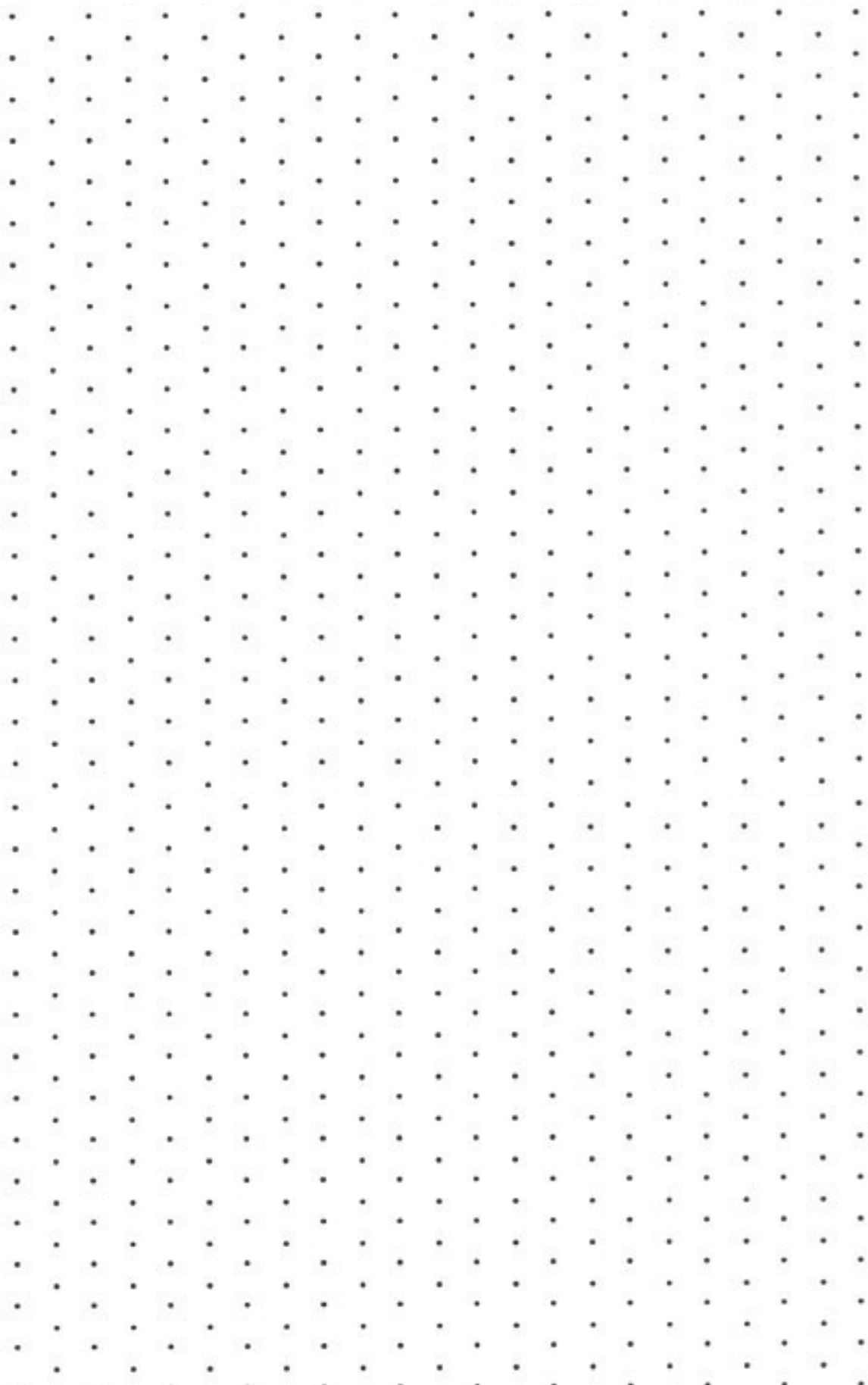


corner view (perspective view) – A corner view is the view from a corner. Isometric dot paper may be used to draw the figure from a corner view.

The figure shown above is drawn on isometric dot paper from a different perspective.



Isometric Dot Paper

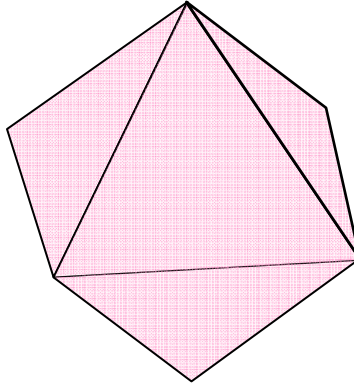


Three-Dimensional Figures

Polyhedra

polyhedron – A polyhedron is a solid with all flat faces that enclose a single region of space.

Polyhedron

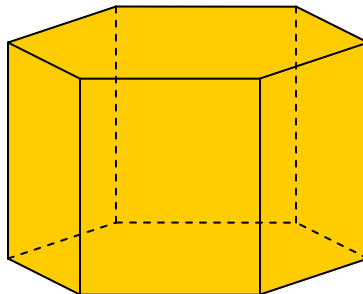


polyhedra – Polyhedra is the plural of polyhedron.

Prisms

prism – A prism is a polyhedron with two congruent faces that are polygons contained in parallel planes.

Prism



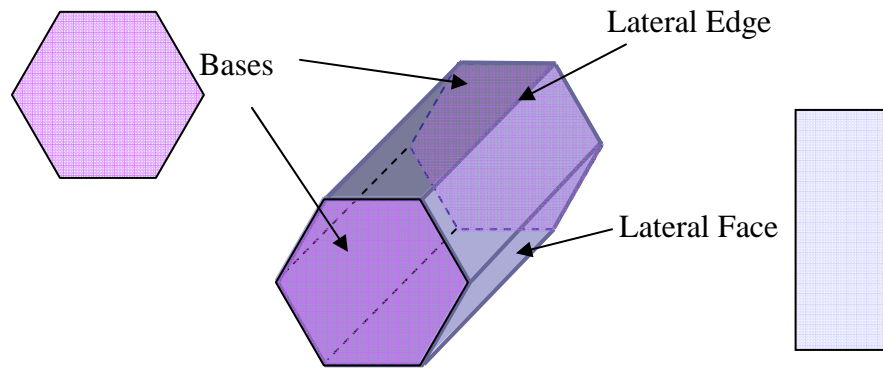
bases of a prism – The bases of a prism are its congruent faces that are polygons contained in parallel planes.

lateral faces of a prism – The lateral faces of a prism are the faces shaped like parallelograms that connect the edges of the bases of the prism.

lateral edges – Lateral edges are the line segments formed at the intersection of lateral faces. Lateral edges are parallel line segments.

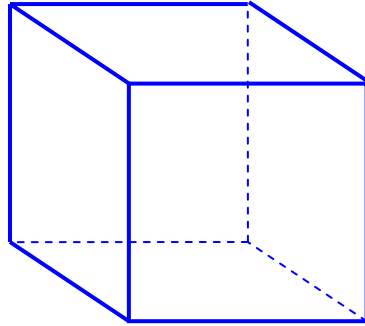
regular prism – A regular prism is a prism whose bases are regular polygons.

Regular Prism

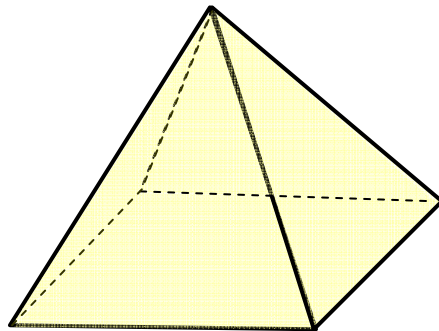


Common Three-Dimensional Shapes

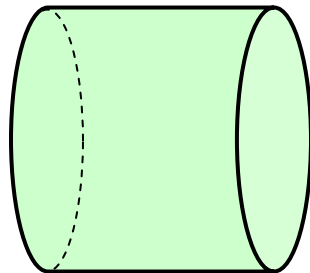
cube – A cube is a prism in which all of its faces are congruent squares.



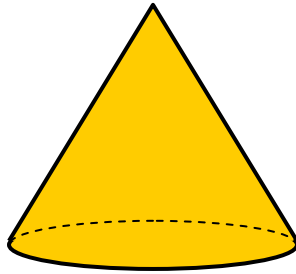
pyramid - A pyramid is a polyhedron in which all of its faces, except one, meet at a point.



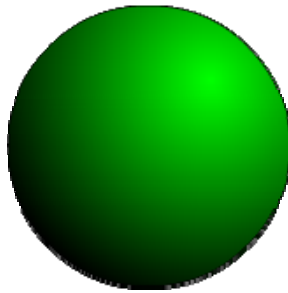
cylinder – A cylinder is a solid with congruent circular bases and a curved rectangle as its lateral face.



cone – A cone is a solid that has one circular base, a vertex point that does not lie in the same plane as the base, and a curved lateral surface area that is made up of all points that lie on segments connecting the vertex and the edge of the base.

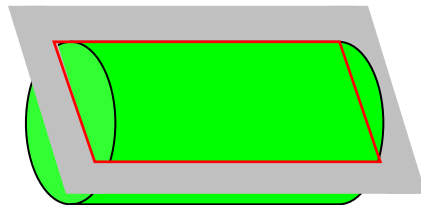


sphere – A sphere is a set of points in space that are equidistant from a given point.



slice – A slice is the shape formed when a plane intersects a solid.

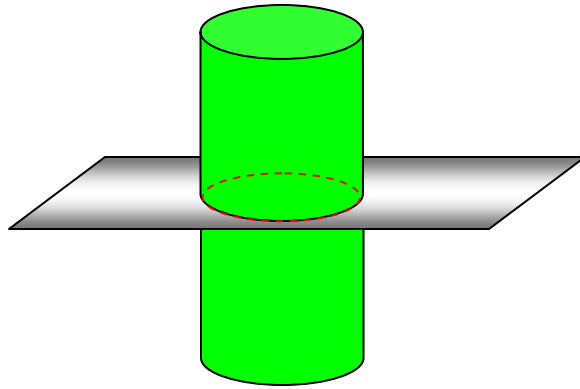
Example 1: What is the shape of the slice that is formed when a plane intersects the body of a cylinder laterally as shown below?



The shape of the slice is a rectangle. (The slice is outlined in red.)

cross section – A cross section of a solid is a slice made by a plane that is parallel to the base or bases of the solid.

Example 2: What is the shape of the cross-section that is formed by the intersection of a cylinder and a plane that is parallel to the cylinder's bases?

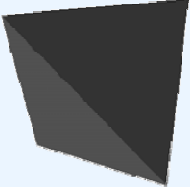
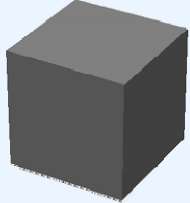
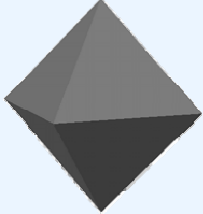
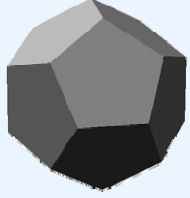
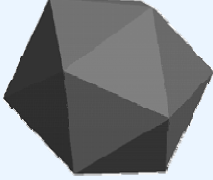


The shape of the cross-section is a circle.

Platonic Solids

regular polyhedron – A regular polyhedron is a polyhedron in which all of its faces are congruent regular polygons. All edges are also congruent.

There are exactly five types of regular polyhedra. They are called the Platonic Solids.

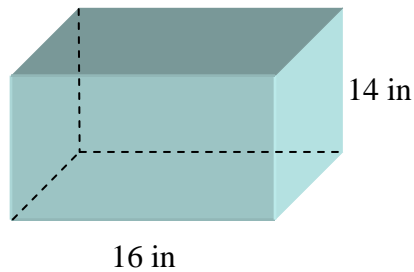
Platonic Solids		
Name	Number and Shape of Faces	Shape
Tetrahedron	4 equilateral triangles	
Hexahedron	6 squares	
Octahedron	8 equilateral triangles	
Dodecahedron	12 regular pentagons	
Icosahedron	20 equilateral triangles	

Surface Area and Nets

surface area – Surface area is the sum of all the areas of a solid's outer surfaces.

net - A net is a two-dimensional representation of a solid. The surface area of a solid is equal to the area of its net.

Example: Find the surface area of rectangular prism that measures 16 inches by 10 inches by 14 inches.



Method 1:

Use the formula $A = lw$ to find the areas of the surfaces.

$$\text{Front and Back:} \quad (16 \times 14) \times 2 = 448$$

$$\text{Top and Bottom:} \quad (16 \times 10) \times 2 = 320$$

$$\text{Two Sides:} \quad (10 \times 14) \times 2 = 280$$

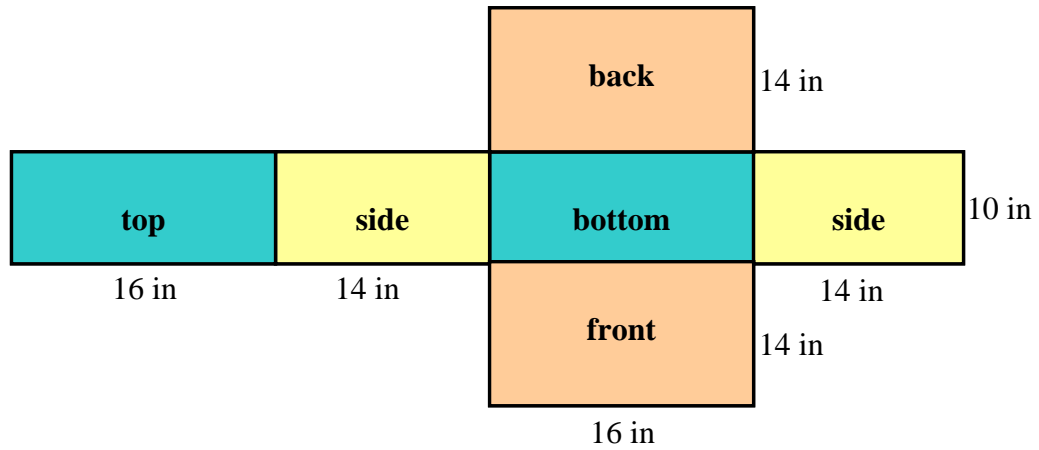
$$\text{Add to find the total surface area: } 448 + 320 + 280 = 1048$$

$$SA = 1048 \text{ in}^2$$

The surface area of a 16 by 10 by 14 inch rectangular prism is 1048 square inches.

Method 2:

Draw a net for the rectangular prism and label the dimensions of each face. Find the area of each face, and then add to find the total surface area.



Side: $14 \times 10 = 140$

Bottom: $16 \times 10 = 160$

Side: $14 \times 10 = 140$

Top: $16 \times 10 = 160$

Front: $16 \times 14 = 224$

Back: $16 \times 14 = 224$

Total: $140 + 160 + 140 + 160 + 224 + 224 = 1048$

The surface area of the rectangular prism is 1048 in^2 .

Surface Area of a Prism

prism – A prism is any figure that has two parallel and congruent bases in the shape of polygons and the other faces are all parallelograms.

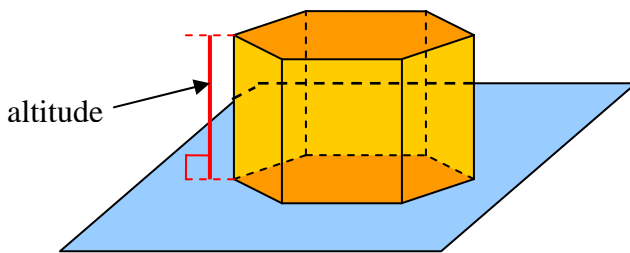
altitude – An altitude of a prism is a segment that is perpendicular to the two planes that contain the two bases of the prism.

height – The height of a prism is the length of the altitude.

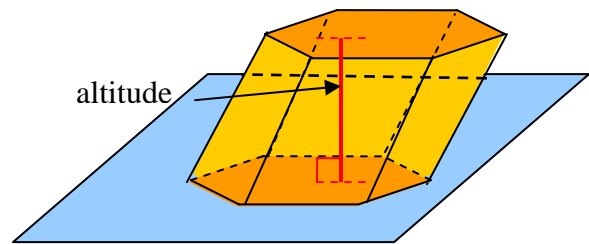
right prism – A right prism is a prism in which its lateral edges are also altitudes.

oblique prism – An oblique prism is a prism that is not a right prism. The lateral edges are not altitudes.

Right Prism



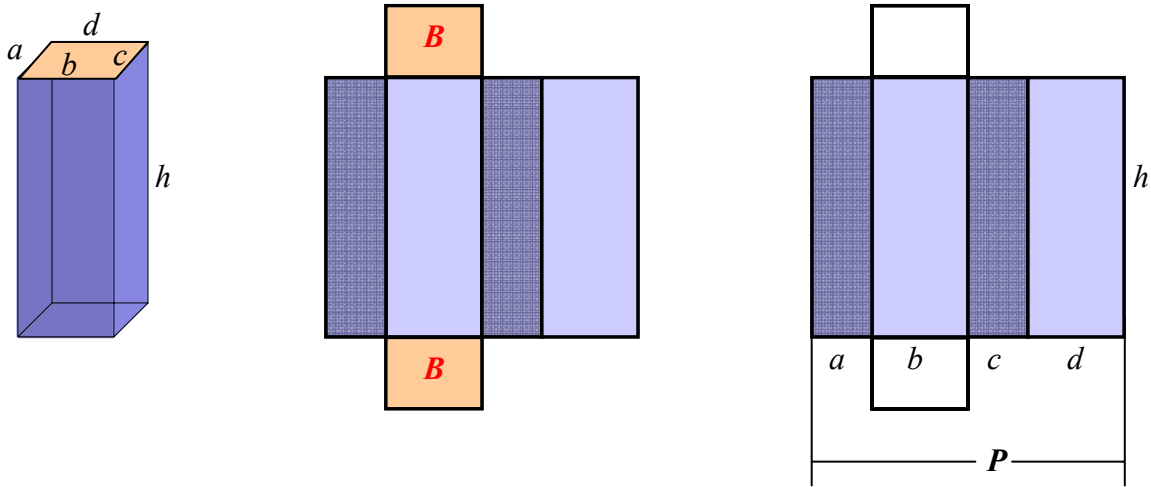
Oblique Prism



lateral faces – The lateral faces of a prism are the parallelograms that connect its bases.

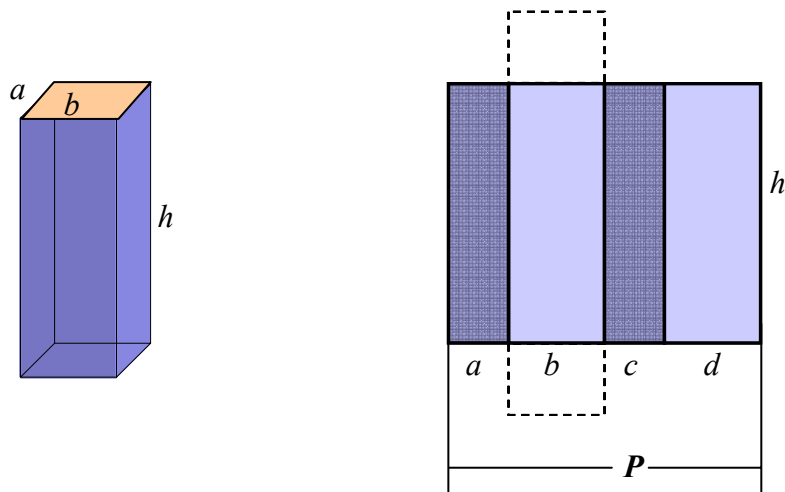
A right rectangular prism and its net are drawn below to explore the formulas for the lateral and the surface area of prisms. We will use a rectangular prism to develop the formula.

The length of each edge of a base is represented by a , b , c , and d , respectively. The height of the prism is represented by h and the base area is represented by B . Perimeter is represented by P .



Lateral Area

lateral area – The lateral area of a prism is the sum of the areas of the lateral faces.



$$L = ah + bh + ch + dh$$

Sum of the Areas of Each Lateral Face

$$L = h(a + b + c + d)$$

Distributive Property

$$P = a + b + c + d$$

Definition of Perimeter

$$L = h(P)$$

Substitution

$$L = Ph$$

Commutative Property

Lateral Area of a Right Prism

The lateral area L of a right prism is the product of the perimeter P of its base and the height h of the prism.

$$L = Ph$$

*It can be shown that this formula applies to all right prisms such as the triangular prism, pentagonal prism, and so on.

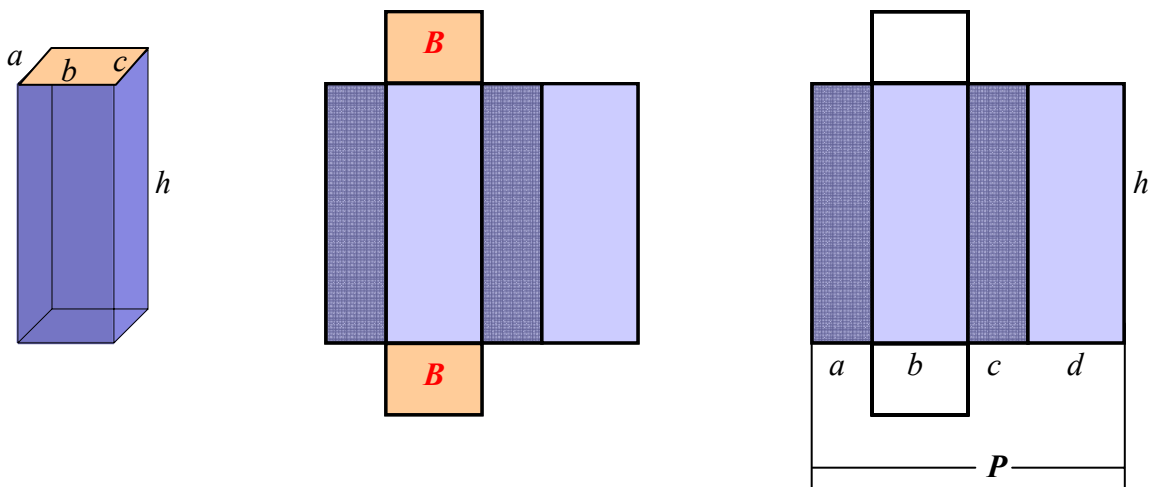
Surface Area

surface area – The surface area of a prism is the sum of the areas of its bases and the lateral area.

The bases of a right prism are congruent; thus, the total surface area is found by adding the lateral area and the area of the two bases.

$$\text{Surface area} = 2 \times \text{Base Area} + \text{Lateral Area}$$

$$T = 2B + Ph$$



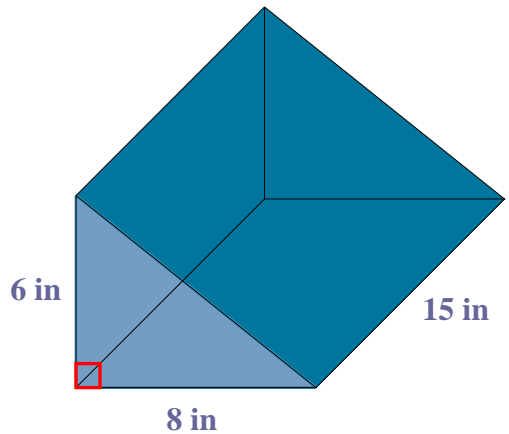
Surface Area of a Right Prism

The surface area T of a right prism is the sum of twice the base area B and its lateral area (Ph).

$$T = 2B + Ph$$

Example: Find the lateral area and surface area of a prism with a height of 15 inches and a base which is a right triangle. The dimensions of the right triangle are a base measuring 6 inches and a height measuring 8 inches.

The parallel bases of the prism are shaped like triangles and the faces are perpendicular to the bases. Therefore, this prism is called a **right triangular prism**.



Plan:

- (1) Find the length of the hypotenuse of the right triangle, so that the perimeter of the right triangle can be calculated.
- (2) Find the lateral area of the prism.
- (3) Find the area of one base of the prism.
- (4) Find the surface area.

Step 1: Determine the length of the hypotenuse and the perimeter of the right triangle.

Find the length of the third side of the triangle by using the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

Pythagorean Theorem

$$6^2 + 8^2 = c^2$$

Substitution ($a = 6$, $b = 8$)

$$100 = c^2$$

Simplify

$$c = 10$$

Take the square root of both sides.

The length of the hypotenuse of the right triangle is 10 inches.

Find the perimeter of the base (right triangle).

$$P = 6 + 8 + 10 \quad \text{Definition of Perimeter}$$

$$P = 24 \quad \text{Simplify}$$

The perimeter of the base is 24 inches.

Step 2: Find the lateral area (area of all the lateral faces).

$$L = Ph \quad \text{Formula for determining the lateral area of right prisms.}$$

$$L = 24(15) \quad \text{Substitution } (P = 24, h = 15)$$

$$L = 360 \quad \text{Simplify}$$

The lateral area of the right triangular prism is 360 square inches.

Step 3: Find the area of the triangular bases.

$$A = \frac{1}{2}bh \quad \text{Area formula for a Triangle}$$

$$A = \frac{1}{2}(8)(6) \quad \text{Substitution } (b = 8, h = 6)$$

$$A = 24 \quad \text{Simplify}$$

The area of one triangle base is 24 square inches.

Step 4: Find the surface area (total area of all faces)

$$T = 2B + Ph \quad \text{Formula for Total Surface Area of Right Prisms}$$

$$T = 2(24) + 360 \quad \text{Substitution}$$

$$T = 408$$

The surface area is 408 square inches.

An alternate way of find the surface area is as follows:

$$T = 2B + L$$

$$L = 15(6) + 15(8) + 15(10) \quad \text{Sum of the Areas of Each Lateral Face}$$

$$L = 15(6 + 8 + 10) \quad \text{Distributive Property}$$

$$L = 15(24) \quad \text{Simplify}$$

$$L = 360 \quad \text{Simplify}$$

$$T = 2B + L \quad \text{Formula for Surface Area of Right Prisms}$$

$$T = 2(24) + 360 \quad \text{Substitution } (B = 24, L = 360)$$

$$T = 408 \quad \text{Simplify}$$

Surface Area of a Cylinder

cylinder – A cylinder is a three-dimensional shape that has two parallel, congruent bases which are usually circles and a curved side connecting the bases.

axis – The axis of a cylinder is the segment with endpoints at the centers of its two circular bases.

altitude – The altitude of a cylinder is a segment that is perpendicular to the planes containing the circular bases.

right cylinder – A right cylinder is a cylinder in which the axis and the altitude are the same segment.

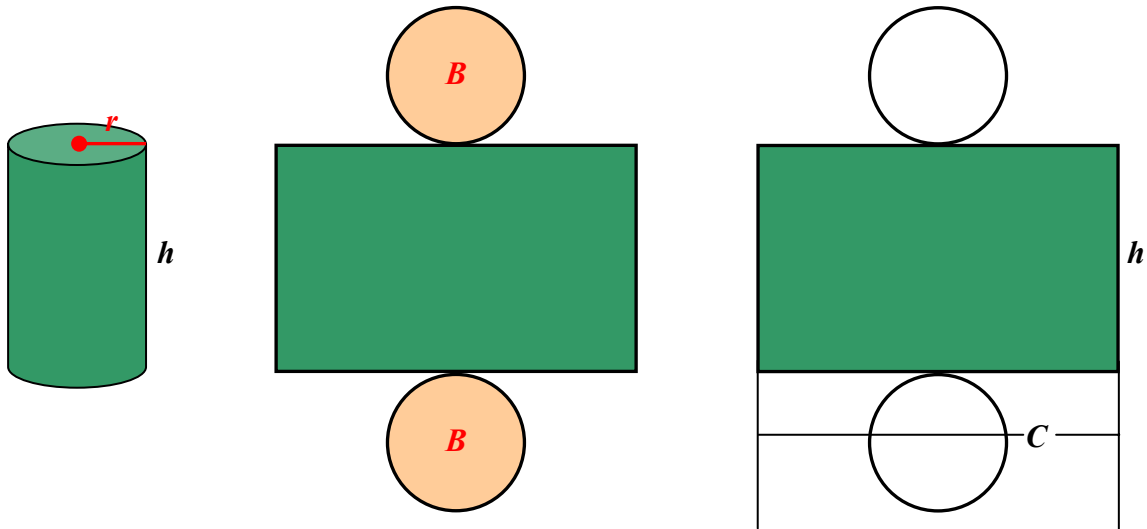
oblique cylinder – An oblique cylinder is a cylinder that is not a right cylinder. The axis and the altitude are different segments.



lateral surface – The lateral surface of a cylinder is the curved surface that connects the circular bases.

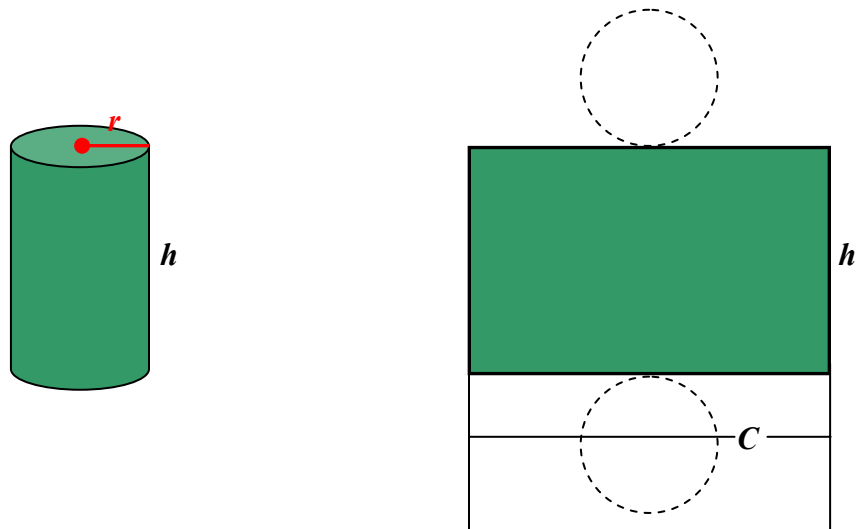
A cylinder and its net are drawn below to explore the formulas for the lateral and the surface area of cylinders. We will use a right cylinder to develop the formula.

The length of the radius is represented by r , the height of the cylinder is represented by h , and the base area is represented by B .



Lateral Area

lateral area – The lateral area of a cylinder is the area of the lateral surface.



The lateral area of a cylinder is the area of the curved surface of the cylinder. Similar to a right prism, the lateral area of a cylinder may be found using $L = Ph$; BUT in a circle the perimeter is called the circumference.

$L = Ph$	Formula for Lateral Area
$L = (2\pi r)h$	Substitution (Perimeter = Circumference = $2\pi r$)
$L = 2\pi rh$	Simplify

Lateral Area of a Right Cylinder

The lateral area L of a right cylinder is the product of the circumference ($2\pi r$) of its base and the height h of the cylinder.

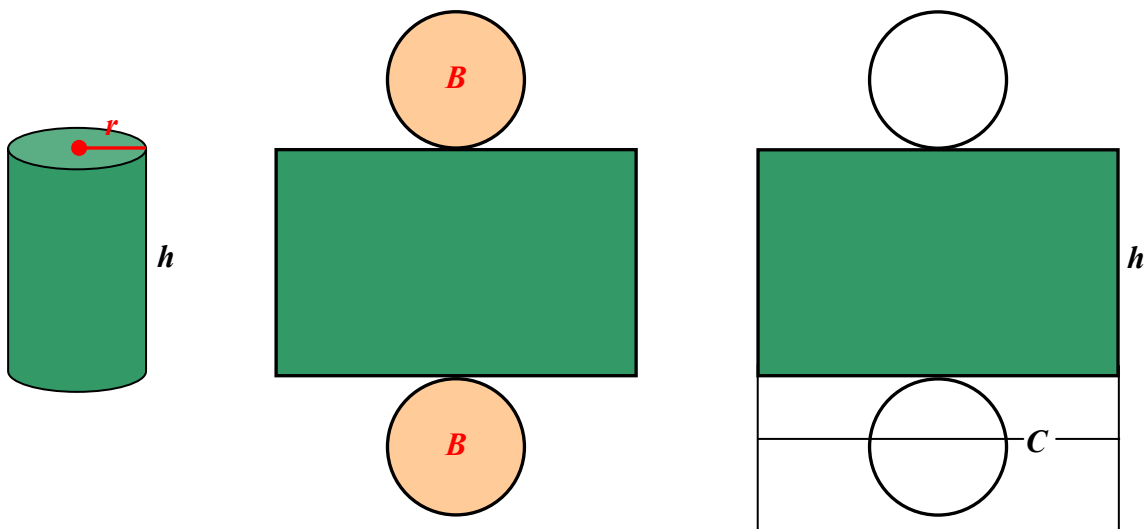
$$L = 2\pi rh$$

Surface Area

surface area – The surface area of a cylinder is the sum of the areas of the bases and the product of the circumference of the base and the height of the cylinder.

The bases of a right cylinder are congruent and have an area of $2(\pi r^2)$; thus, the total surface area is found by adding the lateral area and the area of the two bases of the cylinder.

Surface area	=	$2 \times$ Base Area	+	Lateral Area
T	=	$2B$	+	L



The surface area T of a cylinder may be determined as follows:

$$T = 2B + L$$

Definition of the Surface Area of a Cylinder

$$T = 2(\pi r^2) + 2\pi rh$$

Substitution ($B = \pi r^2$, $L = 2\pi rh$)

Surface Area of a Right Cylinder

The surface area T of a right cylinder is the sum of twice the area of its circular base and the product of the circumference ($2\pi r$) of its base and the height h of the cylinder.

$$T = 2\pi r^2 + 2\pi rh$$

Example 1: Find the surface area of the cylinder with a radius of 3 inches and a height of 8 inches.

$$T = 2\pi r^2 + 2\pi rh$$

Formula for Surface Area of Cylinder

$$T = 2\pi(3^2) + 2\pi(3)(8)$$

Substitution

$$T = 18\pi + 48\pi$$

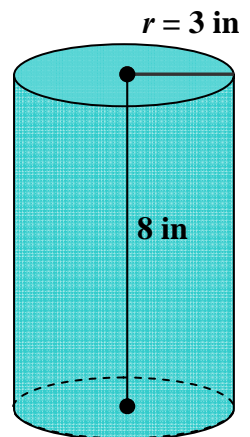
Simplify

$$T = 66\pi$$

Collect Like Terms

$$T = 207.24$$

Simplify



The surface area of the cylinder is 207.24 square inches.

Example 2: The surface area of a right cylinder is 400 square centimeters. If the height of the cylinder is 12 centimeters, what is the radius of the base? Round the answer to nearest hundredth.

$$T = 2\pi r^2 + 2\pi rh$$

Formula for Surface Area of Cylinder

$$400 = 2\pi(r^2) + 2\pi(r)(12)$$

Substitution ($T = 400$, $h = 12$)

$$400 = 6.28r^2 + 75.36r$$

Simplify

$$0 = 6.28r^2 + 75.36r - 400$$

Write the quadratic equation in standard form to solve by the quadratic formula.

$$6.28r^2 + 75.36r - 400 = 0$$

Property of Symmetry

Solve using the quadratic formula.

$$A = 6.28, B = 75.36, C = -400$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$r = \frac{-75.36 \pm \sqrt{(75.36)^2 - 4(6.28)(-400)}}{2(6.28)}$$

Substitution

$$r = \frac{-75.36 \pm \sqrt{15727.1296}}{12.56}$$

Simplify

$$r = \frac{-75.36 \pm 125.41}{12.56}$$

Simplify

$$r = \frac{-75.36 + 125.41}{12.56}; \quad r = \frac{-75.36 - 125.41}{12.56}$$

Simplify

$$r = 3.98 \text{ or } -15.98$$

Since a negative radius has no meaning in this problem, the radius of the cylinder is 3.98 centimeters.

Surface Area of a Pyramid

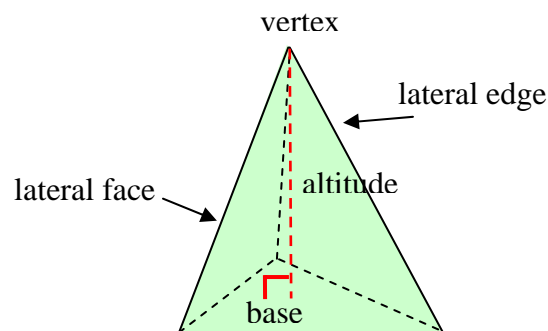
pyramid – A pyramid is a three-dimensional figure which has a base that is a polygon and triangles for its sides that all meet at a common vertex.

base – The base of a pyramid is the face that does not intersect the other faces at the vertex.

vertex – The vertex of a pyramid is the point at which all the faces, except the base, intersect.

lateral faces – The lateral faces of a pyramid are the triangular faces that intersect at the vertex.

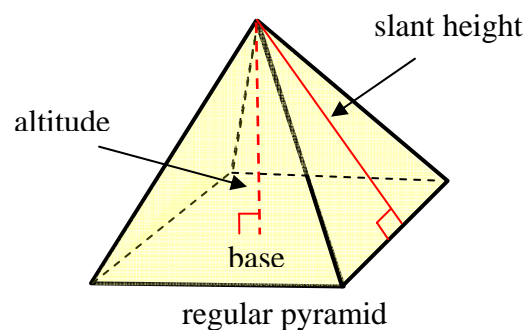
lateral edges – The lateral edges of a pyramid are the edges of the lateral faces that have the vertex as an endpoint.



altitude – The altitude of a pyramid is a perpendicular segment between the vertex and the base.

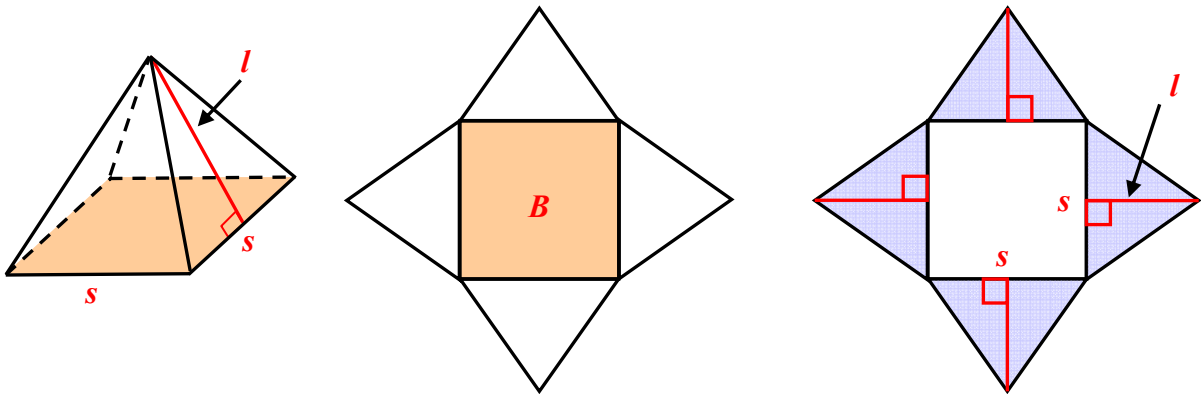
slant height – The slant height of a regular pyramid is the height of one of its lateral faces.

regular pyramid – A regular pyramid is a pyramid that has a regular polygon as its base and the altitude is at the center of the base. In a regular pyramid, all of the lateral faces are congruent isosceles triangles.



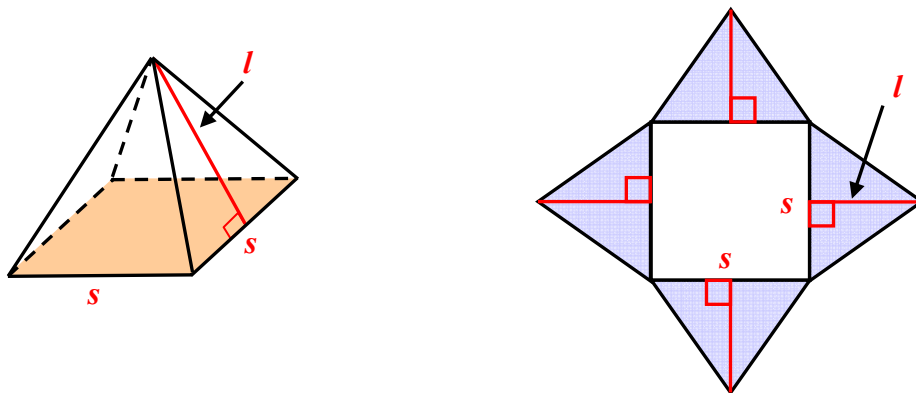
A regular pyramid and its net are drawn below to explore the formulas for the lateral and the surface area of pyramids.

The length of one side of the regular polygonal base (square in this example) is represented by s , the slant height of one of the congruent isosceles triangles is represented by l , and the area of the base is represented by B .



Lateral Area

lateral area – The lateral area of a pyramid is the sum of the areas of the lateral triangular faces.



We will develop a formula similar to finding the lateral area of a prism.

$$L = \frac{1}{2}sl + \frac{1}{2}sl + \frac{1}{2}sl + \frac{1}{2}sl$$

Sum of the Areas of Each Triangular Lateral Face

$$L = \frac{1}{2}l(s + s + s + s)$$

Distributive Property

$$P = s + s + s + s$$

Perimeter of the Regular Square Base

$$L = \frac{1}{2}lP$$

Substitution

$$L = \frac{1}{2}Pl$$

Commutative Property

Lateral Area of a Regular Pyramid

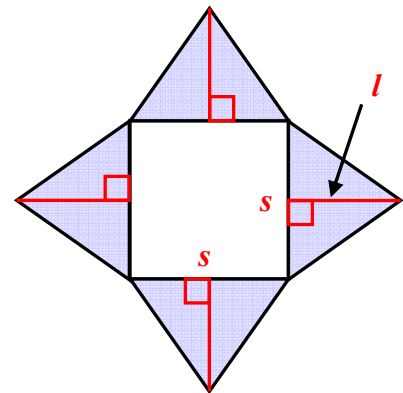
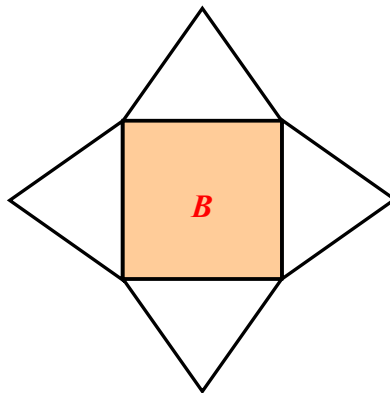
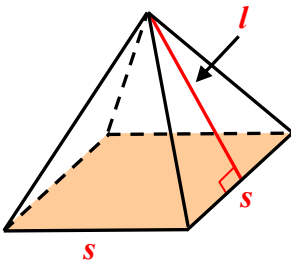
The lateral area L of a regular pyramid is the product of the perimeter P of its base and $(1/2)$ its slant height l .

$$L = \frac{1}{2}Pl$$

Surface Area

surface area – The surface area of a pyramid is the sum of the areas of its base and the lateral area.

$$\begin{array}{rcccc} \text{Surface area} & = & \text{Base Area} & + & \text{Lateral Area} \\ T & = & B & + & L \end{array}$$



The surface area (T) of a pyramid may be determined as follows:

$$T = B + L$$

Surface Area of a Cylinder

$$T = B + \frac{1}{2}Pl$$

Substitution ($L = \frac{1}{2}Pl$)

Surface Area of a Regular Pyramid

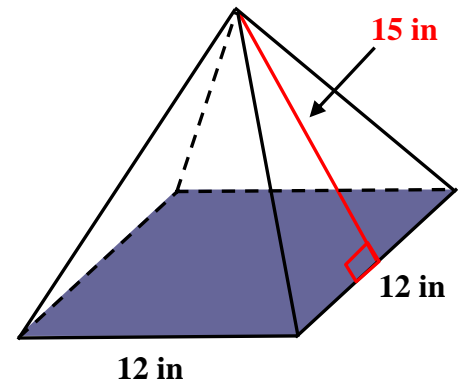
The surface area T of a regular pyramid is the sum of the area of its base B and the product of the perimeter P of its base and $(1/2)$ its slant height l .

$$T = B + \frac{1}{2}Pl$$

Example 1: Find the surface area of a regular pyramid with a square base measuring 12 inches on one side and a slant height of 15 inches.

Plan:

- (1) Find the area of the base.
- (2) Find the perimeter of the base.
- (3) Substitute values into the surface area formula for pyramids.



Step 1: The shape of the base is a square. Find the area of a square.

$$A = s^2$$

$$A = 12^2$$

$$A = 144$$

The area of the base B is 144 square inches.

Step 2: Find the perimeter of the square base.

$$P = 4s$$

$$P = 4(12)$$

$$P = 48$$

The perimeter of the square base is 48 inches.

Step 3: Find the surface area of the pyramid based on the previous calculations.

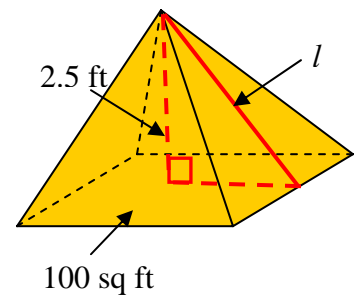
$$T = B + \frac{1}{2}Pl \quad \text{Formula for the Surface Area of a Pyramid}$$

$$T = 144 + \frac{1}{2}(48)(15) \quad \text{Substitution } (B = 144, P = 48, h = 15)$$

$$T = 504 \quad \text{Simplify}$$

The surface area of the pyramid is 504 square inches.

Example 2: A sun port that is shaped like a regular square pyramid shades an area of 100 square feet when the sun is directly overhead. If the height of the sun port from the base of the canvas to the vertex is 2.5 feet, how many square feet of canvas was used to make the sun port?



The canvas area of the sun port is the lateral area of a pyramid.

The shaded area is the area of the base of the pyramid (a square).

Plan: To use the formula, $T = B + \frac{1}{2}Pl$,

(1) Calculate the perimeter.

(2) Find the slant height (l).

(3) Substitute into the lateral area formula for a Pyramid. (The height is given as 2.5 feet.)

Step 1: The area is given as 100 square feet and the shape of the base is a square. To find the perimeter, first find the length of one side of the square, and then calculate the perimeter.

$$\begin{array}{ll} A = s^2 & \text{Formula for the area of a square.} \\ 100 = s^2 & \text{Substitution } (A = 100) \\ s = 10 & \text{Take square root of both sides of the equation.} \end{array}$$

$$\begin{array}{ll} P = 4s & \text{Formula for the perimeter of a square.} \\ P = 4(10) & \text{Substitution } (s = 10) \\ P = 40 & \end{array}$$

The perimeter of the square base is 40 feet.

Step 2: The slant height is part of a right triangle that is drawn from the vertex to the center of the square and out to the slant height. One leg is 2.5 feet (height) and the other leg is half of the length across the square and parallel to the edge of the square ($(10 \div 2 = 5)$). Now, use the Pythagorean Theorem to determine the slant height.

$$\begin{array}{ll} a^2 + b^2 = c^2 & \text{Pythagorean Theorem (The slant height of the} \\ & \text{pyramid is the hypotenuse of the right triangle.} \\ (2.5)^2 + 5^2 = l^2 & \text{Substitution } (a = h = 2.5, b = \text{half the length of} \\ & \text{one side of the square} = 5, c = \text{hypotenuse} = l) \\ 31.25 = l^2 & \text{Simplify} \\ l \approx 5.6 & \text{Take the square root of both sides of the equation.} \end{array}$$

The slant height is approximately 5.6 feet.

Step 3: Now determine the area of the canvas (lateral area of the pyramid).

$$\begin{array}{ll} L = \frac{1}{2}Pl & \text{Formula for the lateral area of a regular pyramid.} \\ L = \frac{1}{2}(40)(5.6) & \text{Substitution } (P = 40, l = 5.6) \\ L = 112 & \text{Simplify} \end{array}$$

It would take a minimum of 112 square feet of canvas to make the sun port.

Surface Area of a Cone

cone - A cone is a three-dimensional figure that has a circular base and one vertex. The lateral face is a circle sector.

base – The base of a cone is a circle.

axis – The axis of a cone is a segment with endpoints at the vertex and center of the circular base.

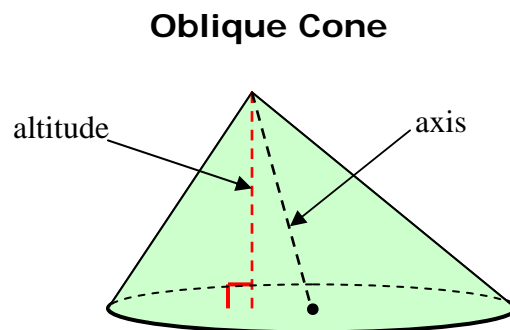
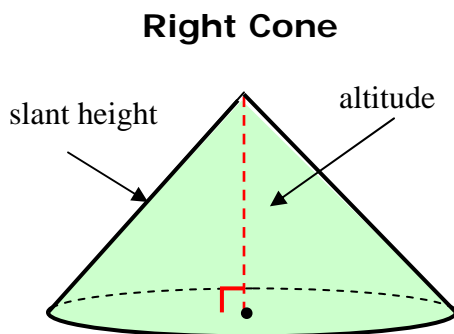
altitude – The altitude of a cone is a segment that has an endpoint at the vertex and is perpendicular to the base.

height - The height of a cone is the length of its altitude.

right cone – A right cone is a cone in which the axis and the altitude are the same segment.

oblique cone– An oblique cone is a cone that is not a right cone. The axis and the altitude are different segments.

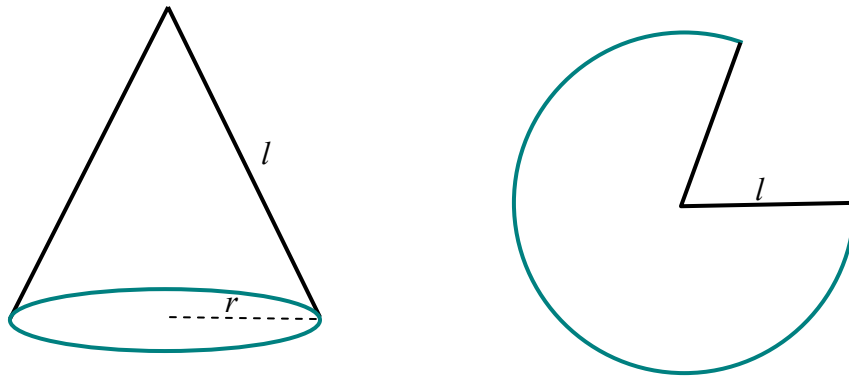
slant height – The slant height of a right cone is the length of any segment that joins the vertex to the edge of the base.



Lateral Area

To find the lateral surface area of a cone, examine the rationale below.

The curved area of a right cone is shown as a flat surface below, a circle sector. The slant height of the cone is represented with l and is the radius of the curved area. The radius of the base (a circle) is represented with r .



In the diagram shown above, the length around the circular base, $2\pi r$, (first figure) is the same length as the length around the sector (second figure). Both lengths are shown as teal-colored curves.

Now let's consider the following ratio about the circle sector (the cone laid flat) shown above:

$$\frac{\text{area of a sector of a circle}}{\text{area of the whole circle}} = \frac{\text{length of the arc of the sector}}{\text{circumference of the whole circle}}$$

(of which the circle is a part) (of which the arc is a part)

$\left. \begin{aligned} &\text{area of a sector of a circle} \\ &\text{area of the whole circle} \\ &\text{(of which the circle is a part)} \\ &\text{length of the arc of the sector} \\ &\text{circumference of the whole circle} \\ &\text{(of which the arc is a part)} \end{aligned} \right\}$	$\left. \begin{aligned} &\text{To Be Determined} \\ &A = \pi r^2, r = l, \therefore A = \pi l^2 \\ &2\pi r \text{ (the circumference of the circular} \\ &\text{base of the cone)} \\ &C = 2\pi r, r = l, \therefore C = 2\pi l \end{aligned} \right\}$
---	---

$$\frac{\text{area of a sector of a circle}}{\pi l^2} = \frac{2\pi r}{2\pi l}$$

Substitute the information from above.

$$\frac{\text{area of a sector of a circle}}{\pi l^2} = \frac{\cancel{2} \pi r}{\cancel{2} \pi l}$$

Cancel

$$\frac{\text{area of a sector of a circle}}{\pi l^2} = \frac{r}{l}$$

Simplify

$$l(\text{area of a sector of a circle}) = r\pi l^2$$

Cross Multiply

$$\text{area of a sector of a circle} = r\pi l$$

Divide both sides by l .

$$\text{lateral area of a cone} = \pi r l$$

Commutative Property

Lateral Area of a Cone

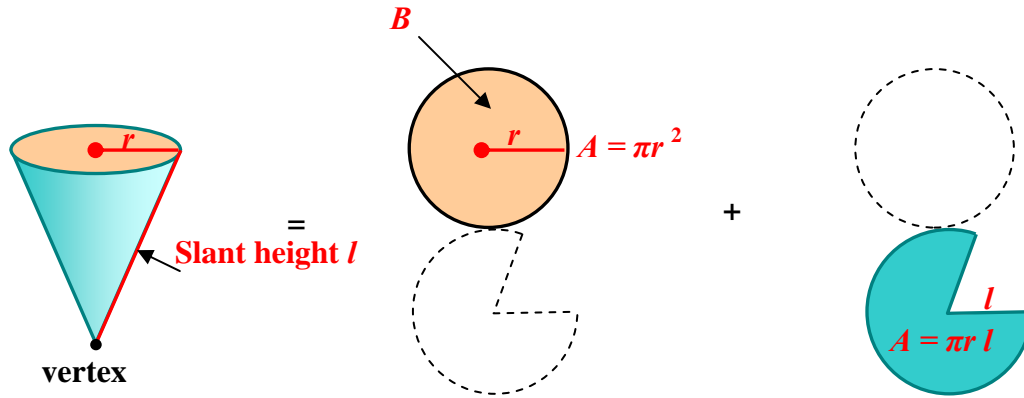
The lateral area of a cone is the product of the slant height of a cone and “pi” times the radius.

$$L = \pi r l$$

Surface Area

surface area – The surface area of a cone is the sum of the lateral area and the base area.

$$\begin{array}{rccccccc} \text{Surface area} & = & \text{Base Area} & + & \text{Lateral Area} \\ T & = & B & + & \pi r l \end{array}$$



The surface area T of a cone may be determined as follows:

$$T = B + L \quad \text{Surface Area of a Cone}$$

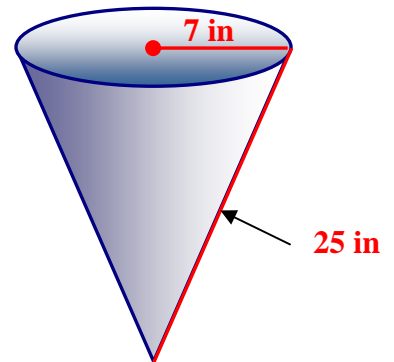
$$T = \pi r^2 + \pi r l \quad \text{Substitution}$$

Surface Area of a Cone

The surface area T of a cone is the sum of the area of its base and its lateral area.

$$T = \pi r^2 + \pi r l$$

Example: Find the surface area of a cone with a slant height of 25 inches and a radius of 7 inches. Round the answer to the nearest square inch.



$$T = \pi r^2 + \pi r l \quad \text{Formula for Surface Area of a Cone}$$

$$T = \pi(7)^2 + \pi(7)(25) \quad \text{Substitution}$$

$$T = 49\pi + 175\pi \quad \text{Simplify}$$

$$T = 224\pi \quad \text{Simplify}$$

$$T = 703.36 \quad \text{Simplify}$$

The surface area of the cone is approximately 703 square inches.

Surface Area of a Sphere

sphere – A sphere is a three-dimensional figure with all points equidistant from a fixed point called its center.

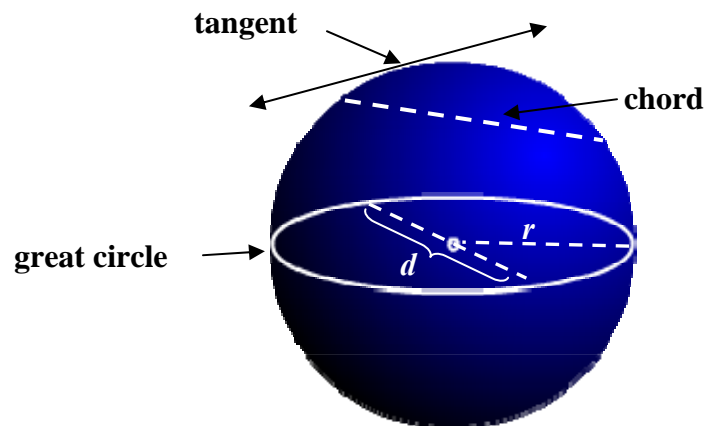
center of a sphere – The center of a sphere is the fixed point from which all points on a sphere are a given distance.

radius of a sphere – The radius of a sphere is a segment that has one endpoint on the sphere and the other at the center of the sphere.

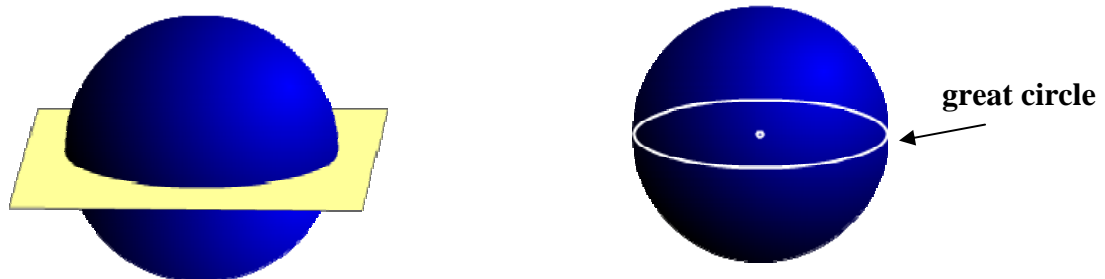
chord of a sphere – A chord of a sphere is a segment that has its endpoints on the sphere.

diameter of a sphere – A diameter of a sphere is a chord that passes through the center of the sphere.

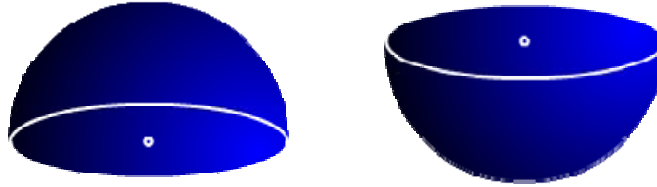
tangent of a sphere – A tangent of a sphere is a line that intersects (touches) the sphere at exactly one point.



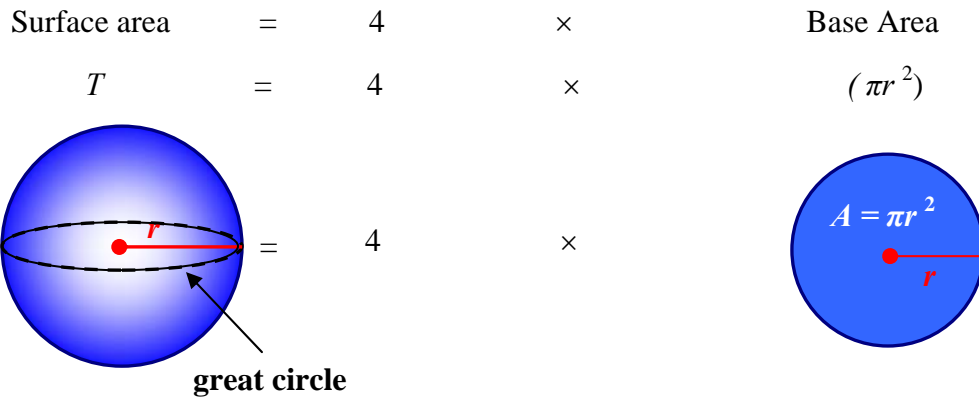
great circle – A great circle is the circle formed when a sphere is sliced such that the slice contains the center of the sphere. The equator is the Earth's great circle.



hemisphere – a hemisphere is half of a sphere. A great circle divides a sphere into two congruent hemispheres.



The surface area of a sphere equals four areas of its great circle.

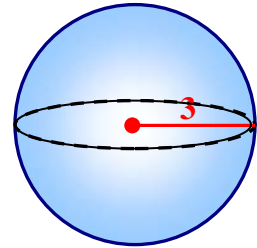


Surface Area of a Sphere

The surface area of a sphere is four times the area of its great circle.

$$T = 4\pi r^2$$

Example: Find the surface area of a sphere with a radius of 3 inches. Round the answer to the nearest square inch.



$$T = 4\pi r^2 \quad \text{Formula for Surface Area of a Sphere}$$

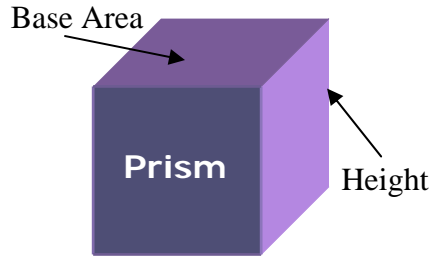
$$T = 4\pi(3)^2 \quad \text{Substitution}$$

$$T = 36\pi \quad \text{Simplify}$$

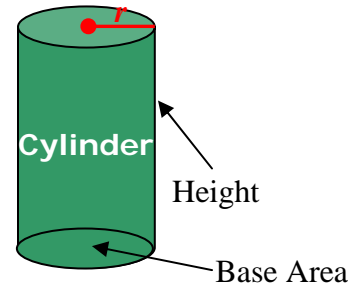
$$T = 113.04 \quad \text{Simplify}$$

The surface area of the sphere is approximately 113 square inches.

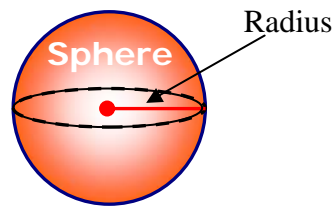
Surface Area Formulas



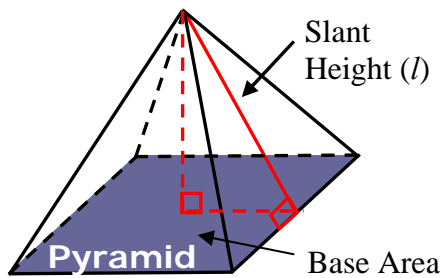
$$S = 2B + Ph$$



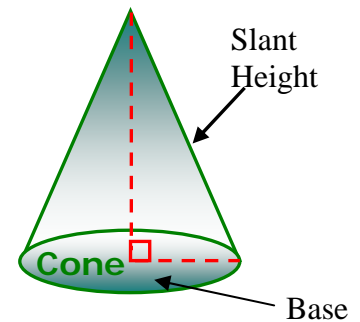
$$S = 2\pi r^2 + 2\pi rh$$



$$S = 4\pi r^2$$



$$S = B + \frac{1}{2}Pl$$



$$S = \pi r^2 + \pi rl$$

Key for Variables:

B - area of base (circle or polygon)

r - radius of circular base

L - lateral area of curved surface

h - height (always perpendicular to base)

l - slant height

P - perimeter of base