APPLICATIONS OF POLYGONS AND CIRCLES

In this unit you will investigate some interesting applications of polygons and circles. You will first determine the area of any given regular polygon and explore what happens to the shape of a polygon as the number of sides increase. You will also examine the area formula and how it is derived from an infinite number of sides in a regular polygon. You will then look at geometric probability based on length and area. The final part of this unit is investigating graph theory which is a study of networks that connect nodes with straight or curved paths.

Area of Polygons and Circles

apothem – An apothem is a perpendicular line segment that is drawn from the center of a regular polygon to its side.

If the length of the apothem and perimeter of a regular polygon is known, then the area can be calculated.

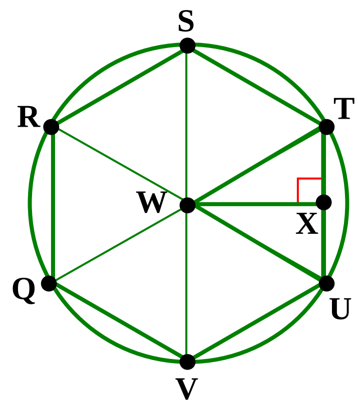
Area of a Regular Polygon

For a regular polygon with an area of *A* square units, a perimeter of *P* units, and an apothem of *a* units, the area equals 1/2 the perimeter times the apothem.

$$A = \frac{1}{2}Pa$$

Let's examine how this formula is derived.

Regular Hexagon QRSTUV is inscribed in Circle W.
Segments RW, SW, TW, UW, VW, and QW are radii.
Segment WX is an apothem.



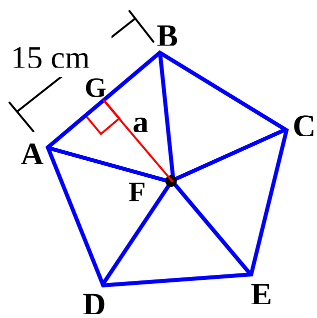
$\triangle TWU$ is isosceles.	$\overline{WT}$ and $\overline{WU}$ are congruent radii.
$\triangle TWU$, $\triangle UWV$, $\triangle VWQ$, $\triangle QWR$, $\triangle RWS$, and $\triangle SWT$ are congruent	SSS (Sides are congruent radii and bases are congruent segments of the regular hexagon.)
The area of the hexagon equals the sum of the area of the six triangles above.	The area of a polygon equals the sum of the areas of its non-overlapping regions.
Area of $\triangle$TWU = $\frac{1}{2}$(TU)(WX)	Formula for the area of a triangle ($A = \frac{1}{2}bh$).

Let *s* represent $\overline{TU}$, the base of triangle TWU (a side of polygon QRSTUV) and *a* represent $\overline{WX}$, the apothem.

$A = \frac{1}{2}sa$	Substitution
$A = 6(\frac{1}{2})sa$	Area of all six congruent triangles in polygon QRSTUV.
$A = 6s(\frac{1}{2})a$	Commutative Property
$P = 6s$	The perimeter (P) of a regular polygon is found by multiplying the number of sides times the length of one side (s).
$A = P(\frac{1}{2})a$	Substitution
$A = \frac{1}{2}Pa$	Commutative Property

Thus, the area of a polygon equals 1/2 the perimeter times the apothem.

Example 1: Find the area of a pentagon that has a side length of 15 centimeters.



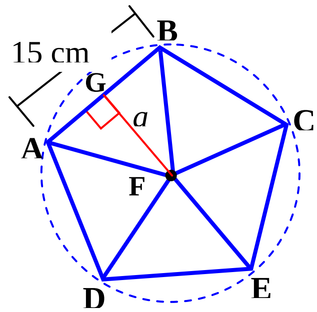
Step 1: Find P(perimeter of the pentagon).

$P = s(5)$	Perimeter of a Regular Pentagon
$P = 15(5)$	Substitution
$P = 75$	Simplify

The perimeter of pentagon ABCDE is 75 centimeters.

Step 2: Find a (apothem). By definition of an apothem, a is perpendicular to $\overline{AB}$.

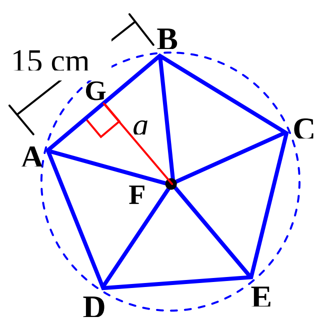
To find the length of the apothem, inscribe the polygon within a circle, and then use trigonometry.



First, find the measure of central angle *AFB*.

$m\overset{\frown}{AB} = 360° / 5$	The five sides of the regular inscribed pentagon are five congruent chords; thus, their arcs are congruent. To find the measure of one arc, divide 360 by 5.
$m\overset{\frown}{AB} = 72°$	Simplify
$m\angle AFB \;=\; 72°$	Definition of Central Angle

Next, determine the measure of $\angle BFG$.



$\triangle BFG \cong \triangle AFG$	Hy-Leg Postutate (Hypotenuses are congruent radii.) ($\overline{\text{MN}}$ is congruent to itself.)
$m\angle BFG = m\angle GFA$	CPCTC
$m\angle BFA = 72°$	The central angle of a regular pentagon equals 72 degrees. (determined above)
$m\angle BFG + m\angle GFA = m\angle BFA$	Angle Addition
$m\angle BFG + m\angle GFA = 72°$	Substitution
$m\angle BFG + m\angle BFG = 72°$	Substitution
$2(m\angle BFG) = 72°$	Collect Like Terms
$m\angle BFG = 36°$	Divide by 2.

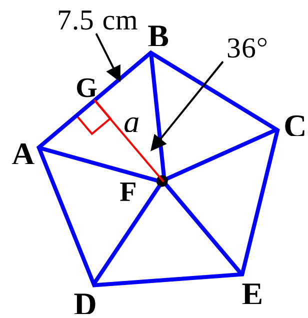
Next, determine the length of $\overline{BG}$.

$BG = GA$	CPCTC
$BA = 15$	Given
$BG + GA = BA$	Segment Addition
$BG + GA = 15$	Substitution
$BG + BG = 15$	Substitution
$2(BG) = 15$	Collect Like Terms
$BG = 7.5$	Divide by 2.

The length of segment BG is 7.5 centimeters.

Now that one side and an angle are known in a right triangle, trigonometry can be applied to determine the length of another side. In this case, the side that we want to determine is the apothem so that we can then calculate the area of the pentagon.

To determine the length of the apothem, apply the tangent function.



$BG = 7.5$ cm $\qquad m\angle BFG = 36°$

$\tan 36° = \dfrac{7.5}{a}$ Definition of Tangent

$a\ (\tan\ 36°) = 7.5$ Cross Products

$a = \dfrac{7.5}{\tan\ 36°}$ Division

$a \approx 10.3\ cm$ Simplify

The length of *a* (apothem) is approximately 10.3 centimeters.

Finally, all the lengths have been determined to find the area of pentagon ABCDE.

$P = 75$ cm $\qquad a \approx 10.3$ cm

$A = \dfrac{1}{2}Pa$ Formula for area of a polygon.

$A = \dfrac{1}{2}(75)(10.3)$ Substitution

$A \approx 386.25\ \text{cm}^2$ Simplify

The area of a regular pentagon with a side that is a length f 15 cm is 386.25 sq cm.

Example 2: Find the area of a regular hexagon that has a side length of 16 centimeters.

Formula: $A = \frac{1}{2}Pa$

Step 1: Find the perimeter of the regular hexagon (6 sides).

$$P = \text{number of sides} \times \text{length of one side}$$
$$P = 6 \times 16$$
$$P = 96 \text{ cm}$$

Step 2: Find the apothem.

(a) Calculate half of the central angle.

$$\frac{1}{2} \times \frac{360}{\text{number of sides in hexagon(6)}} = \frac{1}{2}(60) = 30°$$

(b) Calculate the apothem using trigonometry.

$$\tan = \frac{\text{opp}}{\text{adj}} \qquad \tan 30° = \frac{TX}{WX\,(\text{apothem})}$$

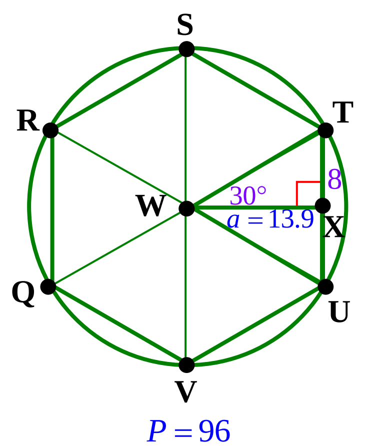
$$\tan 30° = \frac{8}{a} \qquad \text{*Let } a = \text{ apothem, } TX = \frac{TU}{2} = \frac{16}{2} = 8$$

*Make sure the calculator is set in "degree" mode.

$$a \approx 13.9 \text{ cm} \qquad \frac{\tan 30°}{1} = \frac{8}{a} \to (\tan 30°)a = 8 \to a = \frac{8}{\tan 30°}$$

Step 3: Calculate the area.

$$A = \frac{1}{2}Pa$$
$$A = \frac{1}{2}(96)(13.9)$$
$$A \approx 667.2 \text{ square centimeters}$$



The area of a regular hexagon that has a side length of 16 centimeters is approximately 667.2 square centimeters.

Example 3: Find the area of a regular pentagon that has an apothem that measures nine (9) centimeters.

Formula: $A = \frac{1}{2}Pa$

Step 1: Note that the apothem is given as 9 centimeters.

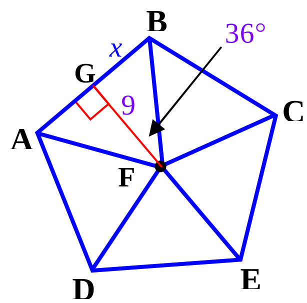
Step 2: Find the perimeter of the regular pentagon (5 sides).

*First use trigonometry to find the length of half of one side of the regular pentagon, double that length, and then calculate the perimeter.

(a) Calculate half of the central angle.

$$\frac{1}{2} \times \frac{360}{\text{number of sides in hexagon(5)}} = \frac{1}{2}(72) = 36°$$

(b) Calculate the length of half of a side of the regular pentagon using trigonometry.



$\tan = \frac{\text{opp}}{\text{adj}}$ $\qquad$ $\tan 36° = \frac{BG}{9\ \text{(apothem)}}$

$\tan 36° = \frac{x}{9}$ $\qquad$ * $x = BG,\ a = 9$

*Make sure the calculator is set in "degree" mode.

$x \approx 6.54$ cm $\qquad$ $\frac{\tan 36°}{1} = \frac{x}{9} \rightarrow x = 9(\tan 36°)$

(c) Calculate the perimeter.

$P =$ number of sides $\times$ length of one side

$P = 5 \times [2(6.54)]$ $\qquad$ *Double x (Segment BG) for the length on one side (Segment AB)

$P \approx 65.4$ cm

Step 3: Calculate the area.

$$A = \frac{1}{2}Pa$$

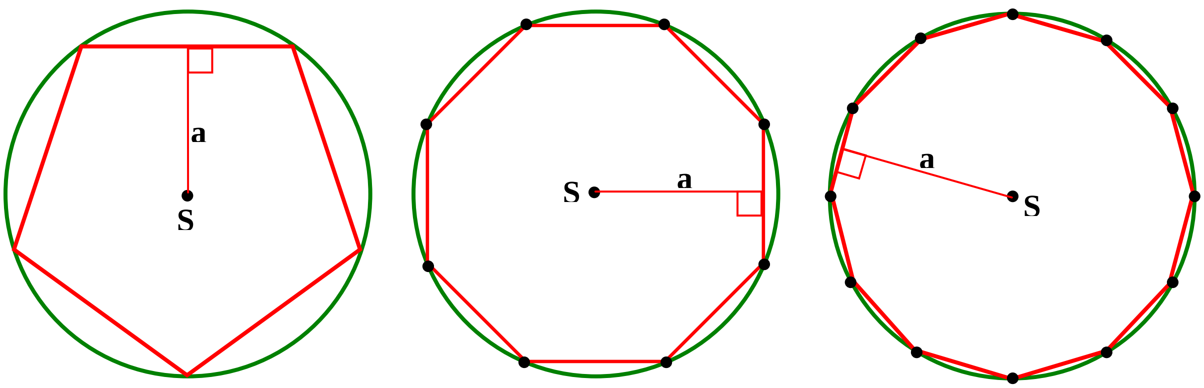
$$A = \frac{1}{2}(65.4)(9) \qquad\qquad P \approx 65.4,\ a = 9$$

$$A \approx 294.3 \text{ square centimeters}$$

The area of a regular pentagon that has an apothem that measures nine (9) centimeters is approximately 294.3 square centimeters.

Area of a Circle (Derivation)

The formula for determining the area of a circle can be derived by using the formula for finding the area of polygons. Notice that as the number of sides in a regular polygon increases, the shape of the polygon approaches the appearance of a circle.



n = 5 **n = 8** **n = 12**

Study the regular pentagon, regular octagon, and regular dodecagon above.

* Note as the number of sides of the regular polygons increase:

1) The areas of the regular polygons approach the area of the circle in which they are inscribed.

2) The length of the apothem (*a*) approaches the length of the radius of the circle,

3) The perimeter of the regular polygon approaches the circumference of the circle.

To summarize:

As *n* increases:

$a \to r$ and $P \to C$

As the number of sides increases "infinitely" in a regular polygon, the area of the inscribed regular polygon and the area of the circle become "indistinguishable".

Thus, apply this theory to derive the formula for the area of a circle.

$A = \frac{1}{2} aP$	Area of a regular polygon
$A = \frac{1}{2}(r)(C)$	Substitution (for a regular polygon with an infinite number of sides) $a \to r$ and $P \to C$
$A = \frac{1}{2} r(2\pi r)$	Substiution ($C = 2\pi r$)
$A = r\pi r$	$\frac{1}{2} \times 2 = 1$
$A = \pi r^2$	Simplify

Area of a Circle

The area of a circle (A) with a radius of r units equals "pi" times the square of the radius.

$$A = \pi r^2$$

Example 1: If the area of a circular garden is 2826 sq ft, what is its circumference?

First, find the radius.

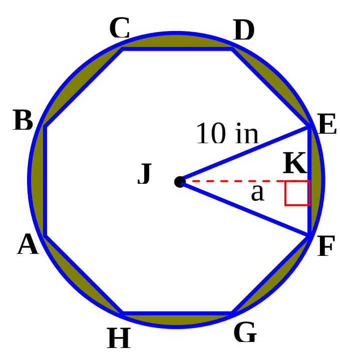
$A = \pi r^2$	Formula for area of a circle.
$2826 = 3.14 r^2$	Substitution
$\frac{2826}{3.14} = r^2$	Division
$900 = r^2$	Simplify
$30 = r$	Take the square root of both sides of the equation.

Now, find the circumference.

$C = 2\pi r$	Circumference
$C = 2(3.14)(30)$	Substitution
$C = 188.4$	Simplify

The circumference of the garden with an area of 2826 square feet is 188.4 feet.

Example 2: A regular octagon is inscribed in a circle with a radius of 10 inches. Find the area of the shaded region (olive green area).



Plan: Find the area of the circle and find the area of the regular octagon.

area of the shaded region = area of the circle – area of the regular octagon

Area of the circle:

$A = \pi r^2$	Formula for area of a circle
$A = 3.14(10^2)$	Substitution
$A = 3.14(100)$	Substitution
$A = 314$	Area of the circle

Area of the regular octagon:

Since a regular octagon divides a circumscribed circle about it into eight congruent arcs, each arc measures 45 degrees (360 / 8 = 45). Thus, central angle *EJF* measures 45 degrees and angle *EJK* measures 22.5 degrees.

To determine the area of the regular octagon,
(a) find the perimeter (*P*) and
(b) find the length of the apothem (*a*)

To determine *P*, first determine the length of *EK*, and then *EF*.

$\sin\ 22.5° = \dfrac{EK}{10}$	Definition of Sine
$EK = 10(\sin\ 22.5°)$	Cross Products
$EK \approx 3.8$	Simplify

Therefore,

$EF \approx 2(3.8) \approx 7.6$	JK is perpendicular to chord EF, and thus, bisects it.

$P = s(8)$	Perimeter of a Regular Octagon
$P = 7.6(8)$	Substitution
$P \approx 60.8$	Simplify

The perimeter of regular octagon ABCDEFGH is 60.8 inches.

To determine *a*, apply the cosine function.

$\cos\ 22.5° = \dfrac{a}{10}$	Definition of Cosine
$a = 10(\cos\ 22.5°)$	Cross products
$a \approx 9.2$	Simplify

Area of the regular octagon:

All parts of the formula for the area of a polygon are determined; thus, the area of the octagon can now be figured.

$P = 60.8$ $\qquad$ $a \approx 9.2$

$A = \dfrac{1}{2} P\ a$	Formula for Area of a Polygon
$A = \dfrac{1}{2}(60.8)(9.2)$	Substitution
$A \approx 279.68\ \text{in}^2$	Simplify

Now we're ready to find the area of the shaded region. Go back to the original plan and use substitution.

Area of the Circle = 314 $\qquad$ Area of the Regular Octagon = 279.68

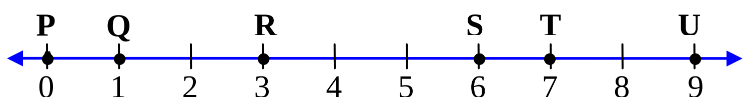
Area of the Shaded Region = Area of the Circle – Area of the Regular Octagon
Area of the Shaded Region = 314 – 279.68
Area of the Shaded Region ≈ 34.32 sq in

Geometric Probability

Postulate 26-A
Length Probability Postulate

If a point on segment AB is chosen at random and point C is between points A and B, then the probability that the point is on segment AC is $\dfrac{\textbf{Length of AC}}{\textbf{Length of AB}}$.

Example 1: What is the probability that a point chosen at random on segment PU is also a point on segment RT?



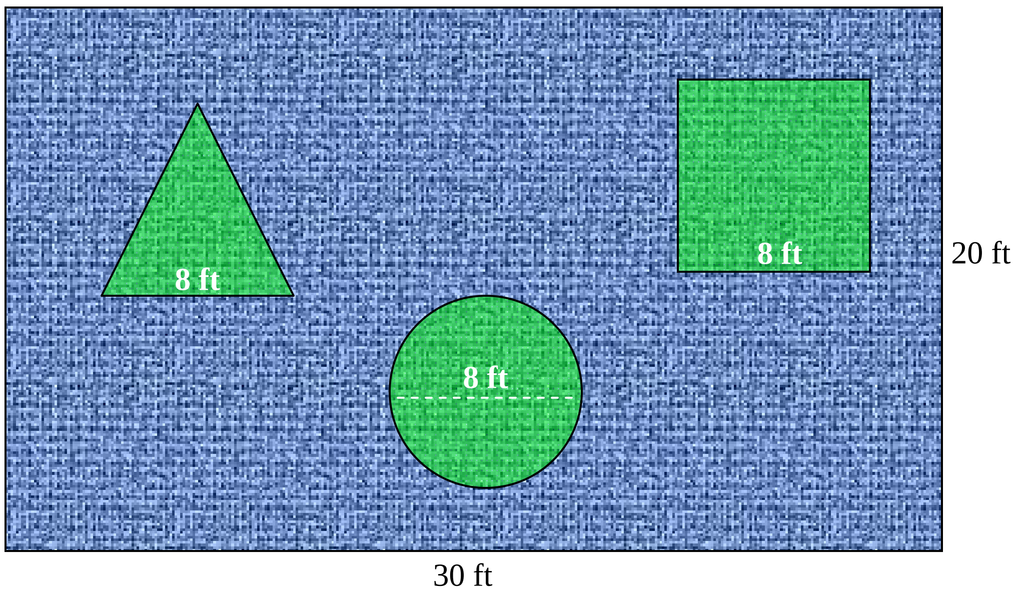
$\dfrac{\text{Length of } RT}{\text{Length of } PU}$	Length Probability Postulate
$\dfrac{4}{9}$	Substitution ($RT = 4,\ PU = 9$)

The probability that a point chosen at random on segment PU is also a point on segment RT is 4/9.

Postulate 26-B
Area Probability Postulate

If a point is chosen at random from region A, the probability that the point is in region B, a region within the interior of region A, is $\dfrac{\textbf{Area of region B}}{\textbf{Area of region A}}$.

Example 2: A rectangular rug measures 30 feet by 20 feet. An equilateral triangle with a side of 8 feet, a circle with a diameter of 8 feet, and a square with a side of 8 feet are painted on the rug. The painted areas are only visible when a special light is shown over the rug. If a person is milling around on the rug, what is the probability that, when the light is turned on, the person will be standing on one of the three shapes. Round the areas to the nearest whole number. Write the probability as a fraction and as a percent.



Plan: Find the areas of the triangle, the circle, the square, and the entire rug. Then, apply the Area Probability Postulate.

$$P(\text{standing on a "hot spot"}) = \frac{\text{area of the triangle} + \text{area of the circle} + \text{area of the square}}{\text{area of the entire rug}}$$

Area of the rug:

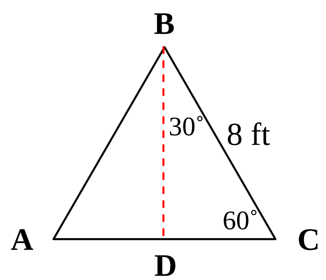
$A = b\ h$	Area of a rectangle
$A = 30(20)$	Substitution
$A = 600$	Simplify

The area of the rug is 600 square feet.

Area of the triangle:

Plan: To determine the area of the triangle, first find the height and then use the area formula for triangles.

Since the triangle is equilateral, the height of the triangle will bisect one of its equal 60-degree angles.



$DC = 4$	The shorter leg of a 30-60-90 degree right triangle equals half of the hypotenuse.
$BD = 4\sqrt{3}$	The longer leg of a 30-60-90 degree right triangle equals the product of half of the hypotenuse times $\sqrt{3}$.

The height of the triangle equals $4\sqrt{3}$ feet.

$A = \frac{1}{2}bh$	Area of a Triangle
$A = \frac{1}{2}(8)(4\sqrt{3})$	Substitution
$A = 16\sqrt{3}$	Simplify
$A \approx 28$	

The area of the triangle is approximately 28 square feet.

Area of the circle:

$A = \pi r^2$	Area of a Circle
$A = 3.14(4^2)$	Substitution ($d = 8,\ r = 4$)
$A = 50.24$	Simplify
$A \approx 50$	

The area of the circle is approximately 50 square feet.

Area of the square:

$$A = s^2 \qquad \text{Area of a Square}$$
$$A = 8^2 \qquad \text{Substitution}$$
$$A = 64 \qquad \text{Simplify}$$

The area of the square is 64 square feet.

Now, all the parts are solved and we can finish the problem by applying the Area Probability Postulate.

$$\mathbf{P(\text{standing on a "hot spot"}) = \frac{\text{area of the triangle + area of the circle + area of the square}}{\text{area of the entire rug}}}$$

$$P(\text{standing on a "hot spot"}) = \frac{28+50+64}{600} \qquad \text{Substitution}$$

$$P(\text{standing on a "hot spot"}) = \frac{142}{600} \qquad \text{Simplify}$$

$$P(\text{standing on a "hot spot"}) \approx 24\%$$

The probability that the person will be standing on a shape is 24%.

Example 3: Find the probability that the person in the previous problem will NOT be standing on one of the shapes.

To solve this problem, find the probability of standing on any area outside of the shapes. Since all the points on the rug would be represented by 100%, then the probability of NOT standing on one of the shapes is the DIFFERENCE between standing on all the points and standing on the shapes.

100% – 24% = 76%

The probability that the person will NOT be standing on a shape is 76%.

sector of a circle – A sector of a circle is a region of a circle that is bounded by a central angle and its intercepted area.

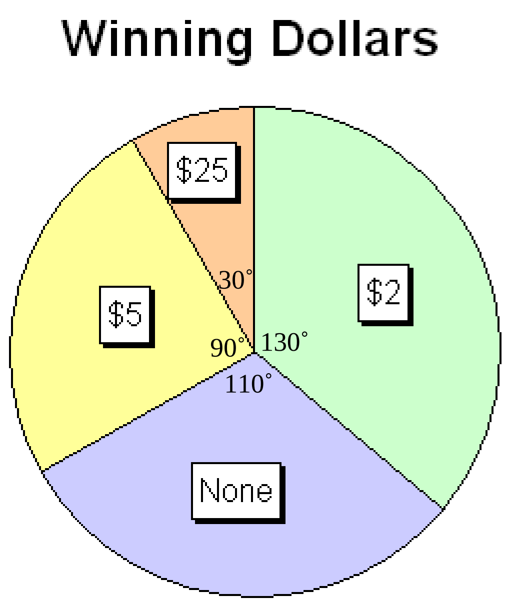
Area of a Sector of a Circle

The area of a sector of a circle (A) is determined by the central angle (n) and radius (r) of the circle such that the area equals n divided by 360 times the area of the circle.

$$A = \frac{n}{360}\pi r^2$$

Example 4: At the local charity fundraiser, Anne can spin the "Winning Dollars" wheel if she donates \$3. What are her chances of winning either \$5 or \$25? The wheel has a radius of 3 feet and the angle sizes for each sector are displayed on the wheel.

*Note: Normally, the angle sizes of a lottery wheel are not shown; but, for this problem, to calculate Anne's chances to win, the angles are displayed and the radius of the wheel is given.



Plan: Find the area of the two sectors that represent the win, find the total area of the circle, and then apply the Area Probability Postulate.

$$P(\text{spinning \$5 or \$25}) = \frac{\text{area of \$5 sector + area of \$25 sector}}{\text{area of the entire Winning Dollars Wheel}}$$

Area of the Winning Dollars Wheel:

$A = \pi r^2$	Formula for Area of a Circle
$A = \pi(3)^2$	Substitution $(r = 3)$
$A = 9\pi$	Simplify

The area of the Winning Dollars Wheel is 9π.

*Note: All of the parts in this problem will be left in terms of "pi" to avoid rounding so that the result will be more accurate.

Area of the $5 Circle Sector:

$A = \frac{n}{360}\pi r^2$	Formula for Area of a Circle Sector
$A = \frac{90}{360}\pi (3)^2$	Substitution $(n = 90, r = 3)$
$A = \frac{1}{4}\pi\ 9$	Simplify
$A = \frac{9}{4}\pi$	Simplify

Area of the $25 Circle Sector:

$A = \frac{n}{360}\pi\ r^2$ Formula for Area of a Circle Sector

$A = \frac{30}{360}\pi\ (3)^2$ Substitution $(n = 30, r = 3)$

$A = \frac{1}{12}\pi\ 9$ Simplify

$A = \frac{3}{4}\pi$ Simplify

Now, all the parts are solved and we can finish the problem by applying the Area Probability Postulate.

$$\mathbf{P(\text{spinning \$5 or \$25}) = \frac{\text{area of \$5 sector + area of \$25 sector}}{\text{area of the entire Winning Dollars Wheel}}}$$

$P(\text{spinning \$5 or \$25}) = \frac{\frac{9}{4}\pi + \frac{3}{4}\pi}{9\pi}$ Substitution

$P(\text{spinning \$5 or \$25}) = \frac{\frac{12}{4}\pi}{9\pi}$ Simplify the numerator.

$P(\text{spinning \$5 or \$25}) = \frac{3\pi}{9\pi}$ Simplify

$P(\text{spinning \$5 or \$25}) = \frac{3\cancel{\pi}}{9\cancel{\pi}}$ Cancel the π's

$P(\text{spinning \$5 or \$25}) = \frac{1}{3}$ Simplify

Anne's chances of winning either \$5 or \$25 are 1/3 or 33 1/3%.

When all outcomes are equally likely, the ratio of the number of favorable outcomes to the number of unfavorable outcomes is called the **odds in favor** of an event.

Odds in Favor of an Event

$$\text{Odds in favor} = \frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}} = a:b$$

a = number of favorable events
b = number of unfavorable events

$a + b$ = total number of outcomes

Let's revisit Anne's chances of winning and examine her odds of winning. We determined that her chance of spinning either \$5 or \$25 was $\frac{1}{3}$.

Thus, Anne has **1** chance out of 3 of winning. This means she has **2** chances out of 3 of losing. (3 –1)

However, the **odds in favor** of her winning are as follows:

Favorable chances: 1 $a = 1$
Unfavorable chances: 2 $b = 2$

$$a:b = 1:2$$

The odds in favor of Anne winning are **1:2**.

When all outcomes are equally likely, the ratio of the number of unfavorable outcomes to the number of favorable outcomes is called the **odds against** an event.

Odds Against of an Event

$$\text{Odds Against} = \frac{\text{number of unfavorable outcomes}}{\text{number of favorable outcomes}} = b : a$$

b = number of unfavorable events
a = number of favorable events

$a + b$ = total number of outcomes

Let's revisit Anne's chances of winning once more, and this time, we'll examine her odds against winning.

Since Anne has **2** chances out of 3 of losing, the against her winning are as follows:

Unfavorable chances: 2	$b = 2$
Favorable chances: 1	$a = 1$

$$b : a = 2 : 1$$

The odds **against** Anne winning are **2:1**.

Example 5: If the game in the previous problem (paying \$3 for a chance to win \$5 or \$25) is played by a lot of people, will the charity be likely to raise money or lose money on this game?

Since the odds are against the players 2 to 1 in winning the game; theoretically, the charity will make money.

*Casinos make lots of money based on this principal. The owners realize that over time and the accumulation of many players trying their luck, if they create games in which the odds are in the casino's favor, the casino will be a very profitable enterprise. A few people will win, but the odds are that most people will lose.

Graph Theory

graph theory – Graph theory is a branch of mathematics that is the study networks and nodes.

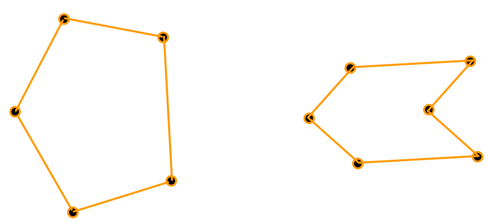
Graph theory can be used when researching the backbone of the Internet, playing games, and solving problems dealing with routing such as transportation routes.

network - A network is a figure consisting of nodes (points) and edges (paths) that join various nodes to one another.

node – A node (in graph theory) is a point of a network.

edge – An edge (in graph theory) is a path connecting two nodes. Edges may be straight or curved.

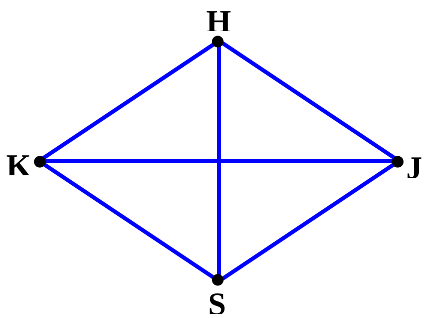
closed graphs – Closed graphs are graphs in which all nodes are connected with at least two edges that lead to other nodes in the network. Below are two examples of closed graphs.

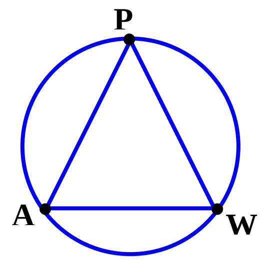


open graphs – Open graphs are graphs that will have at least one edge that ends at a node and the node is not connected to another node in the graph. Below are two examples of open graphs.

complete network – A complete network is a network that has at least one path between all nodes.

Study the two networks shown below. They are complete because between every two nodes there is at least one edge.

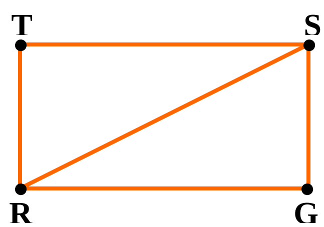




*Note: The paths of a network may be curved.

incomplete network – An incomplete network is a network where at least one pair of nodes is not connected with an edge.

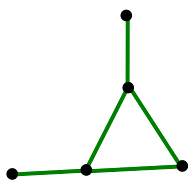
Example 1: Explain why the figure shown below is an incomplete network.



Look closely. There is no edge between nodes T and G. The network is incomplete.

traceable network – A traceable network is a network in which all nodes are connected and each of its edges can be covered exactly once. *In other words, a network is traceable when you can start at a node and trace along the edges without lifting the pencil or crossing over an edge more than once.*

Example 2: Is the network shown below a traceable network?



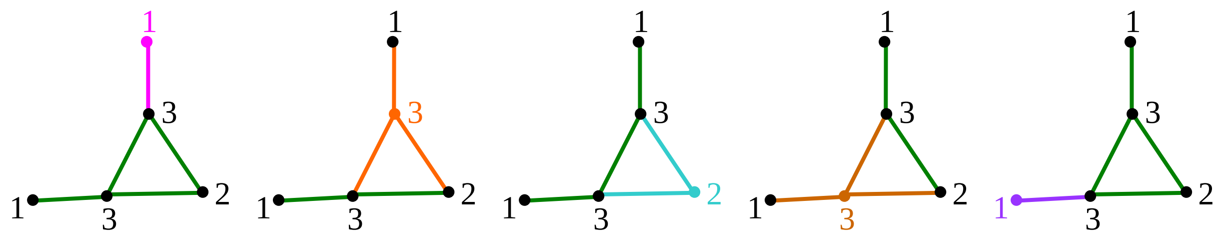
Experiment by starting at one node and tracing through each of the notes. Can you trace a complete path from the first node (your choice as to the starting node) to the last node without lifting your pencil or pointer?

No, you cannot trace along all the edges from node to node without crossing over an edge more than once. This network is not a traceable network.

degree of a node – A degree of a node is determined by counting the number of edges that connect to a node.

Example 3: Determine the degree of each node in the network this is shown in the previous problem.

The degree for each node is shown below. The network is replicated five times and color added so that the edges that connect the node determining the degree can be visualized more easily.



Traceable networks underlie many of the everyday conveniences that most people take for granted – the Internet and the Power Grid are two examples.

Let’s examine a test for determining if a network is traceable.

Network Traceability Test

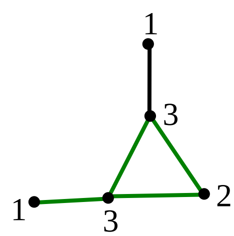
A network is traceable, if and only if, **ONE** of the following is true.

1. Every node in the network has an even degree.

OR

2. Exactly two nodes in the network have an odd degree.

Example 4: Apply the Network Traceability Test to the figure shown below (used in *Example 3* above) and verify that the network is not traceable.

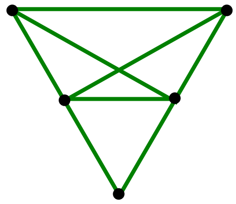


Test 1: Is every node in the network an even degree? **N**o, there is only one node that has an even degree (2).

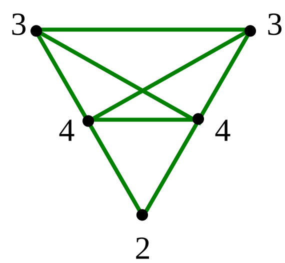
Test 2: Are there exactly two nodes in the network that have an odd degree? **No**, there are four nodes (1, 3, 3, 1) in the network that have an odd degree.

The network is **not traceable** because it **fails both tests**.

Example 5: Determine the degrees for each of the nodes in the network shown below, and then state if the network is traceable or not.



First, determine the degree of each node.



Then, check the Network Traceability Tests

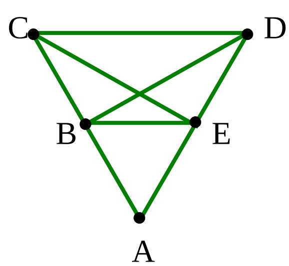
Test 1: Is every node in the network an even degree? **N**o, some nodes have an odd degree (3 and 3).

Test 2: Are there exactly two nodes in the network that have an odd degree? **Yes**, there are exactly two nodes (3 and 3) in the network that have an odd degree.

The network is **traceable** because it **passes only the second test**.

Can you find a traceable path?

.



Try: $C \to D \to A \to C \to E \to B \to D$

*Note: No path was traced more than once; but, paths may be crossed.

When a network is traceable, there are some occasions where any node can be the starting and ending point. On other occasions, only certain nodes can be the starting point.

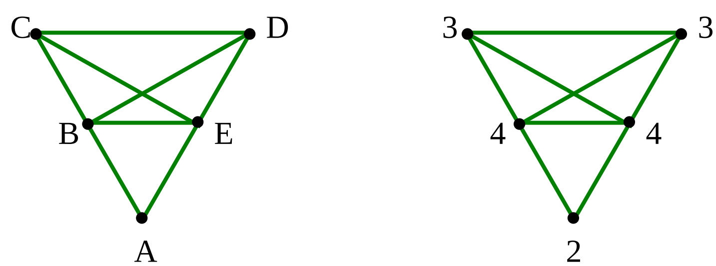
Starting Points for Network Traceability

1. If a node has an odd degree, then the tracing must start or end on that node.

2. If a network has nodes where all have even degrees, then the tracing can start on any node and will end on the starting point.

Example 6: In the previous problem, the network proved to be traceable. One traceable path is shown below. Find another traceable path.

$$C \rightarrow D \rightarrow E \rightarrow A \rightarrow B \rightarrow C \rightarrow E \rightarrow B \rightarrow D$$



Notice that node C has an odd degree; thus, Step 1 (in the chart above) applies to this traceable network.

Also, note that Step 2 (in the chart above) does not apply because all of the nodes in the network do not have an even degree.

To find a second traceable path, we'll start at node D since this node has an odd degree.

$$D \rightarrow B \rightarrow E \rightarrow C \rightarrow B \rightarrow A \rightarrow E \rightarrow D \rightarrow C$$

*Note: The path starts on one odd-numbered node (D) and ends on the other odd-numbered node (C).

There are more! Try to find another traceable path!