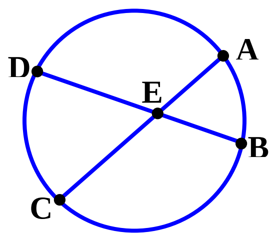
SPECIAL SEGMENTS AND EQUATIONS OF CIRCLES

In this unit you will investigate lengths of segments formed by intersections of chords, secants, and tangents. You will review and apply the quadratic formula to solve equations dealing with circle segments. You will develop the standard equation for circles and graph their solutions. You will apply arcs and the Pythagorean Theorem to graph the values of irrational numbers on a number line.

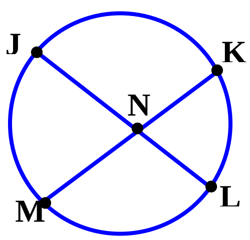
Chord, Secant, and Tangent Relationships

Theorem 24-A	**If two chords intersect in a circle, then the products of the measures of the segments are equal.**



$$AE \bullet EC = DE \bullet EB$$

Example 1: Find the value of *KN* if *JN* = 25, *NL* = 14, and *MN* = 20.



$JN(NL) = MN(NK)$	Theorem 24-A
Let x represent the measure of NK.	
$25(14) = 20x$	Substitution
$350 = 20x$	Simplify
$x = 17.5$	Divide

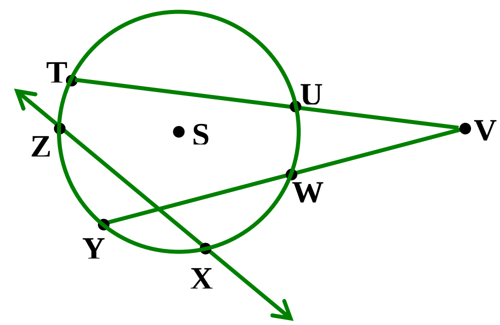
secant – A secant is a line that intersects a circle in exactly two points.

secant segment – A secant segment is a section of a secant line.

external secant segment – An external secant segment is the section of a secant segment that lies in the exterior of a circle.

Example 2: Identify the following parts of $\odot S$.

a) a secant line
b) a secant segment
c) an external secant segment
d) a chord



a) $\overleftrightarrow{XZ}$ is a secant line. It intersects $\odot S$ in exactly two points.

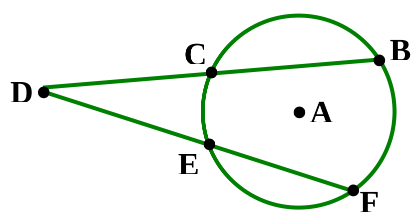
b) $\overline{VT}$ and $\overline{VY}$ are secant segments. They are each a section of a secant line.

c) $\overline{VU}$ and $\overline{VW}$ are external secant segments. They are the section of the secant segment that lies in the exterior of $\odot S$.

d) $\overline{TU}$, $\overline{ZX}$, and $\overline{YW}$ are chords. They are line segments in the interior of $\odot S$ and their endpoints are on $\odot S$.

Theorem 24-B

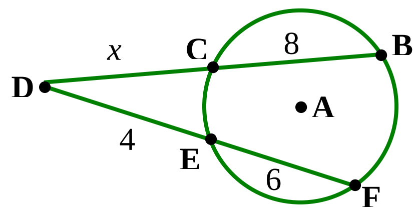
If two secant segments of a circle are drawn from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external segment.



$DB \bullet DC = DF \bullet DE$

Example 3: Find the measure of segment *CD*.

$DB \bullet DC = DF \bullet DE$	Theorem 24-B
$(x+8)(x) = (4+6)(4)$	Substitution
$x^2 + 8x = 40$	Simplify
$x^2 + 8x - 40 = 0$	Subtraction Property



Use the quadratic formula to solve for *x*.

$Ax^2 + Bx + C + 0$	Standard Form
$A = 1,\ B = 8,\ C = -40$	Prepare to substitute.
$x = \dfrac{-b \pm \sqrt{b^2\ \ 4ac}}{2a}$	Quadratic Formula
$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-40)}}{2(1)}$	Substitution
$x = \dfrac{-8 \pm \sqrt{224}}{2}$	Simplify
$x = \dfrac{-8 + \sqrt{224}}{2};\ x = \dfrac{-8 - \sqrt{224}}{2}$	Definition of $\pm$

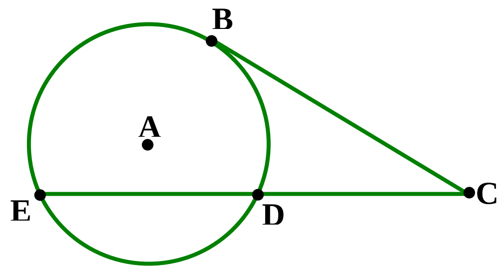
$x \approx 3.48,\qquad x \approx -11.48$

Since this problem is about length, ignore the negative solution.

The length of *CD* is approximately 3.48.

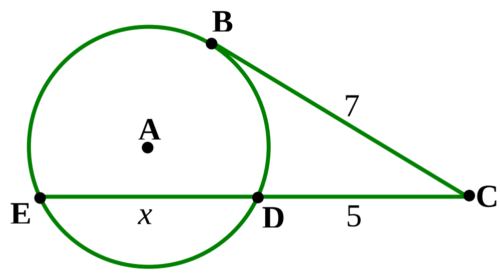
Theorem 24-C

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external segment.



$(BC)^2 = EC \bullet DC$

Example 4: Find the length of segment *JK* in $\odot A$.



$(BC)^2 = EC \bullet DC$

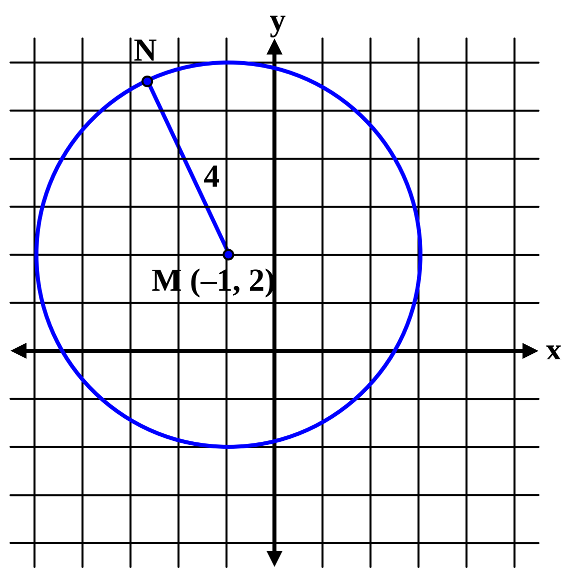
$(BC)^2 = EC \bullet DC$	Theorem 24-C
$(7)^2 = (x+5)(5)$	Substitution
$49 = 5x + 25$	Simplify
$24 = 5x$	Subtraction Property
$x = 4.8$	Division Property

Segment *ED* has a length of 4.8.

Equations and Graphs of Circles

When circles are drawn in the coordinate plane, the distance formula may be used to determine the circle's equation.

> *Example 1*: Use the distance formula to determine the equation for $\odot M$ with radius $MN = 4$ and the center point located at M(–1,2).



The standard equation of a circle with a radius of r and center at (h,k) is:

$$(x-h)^2+(y-k)^2=r^2$$

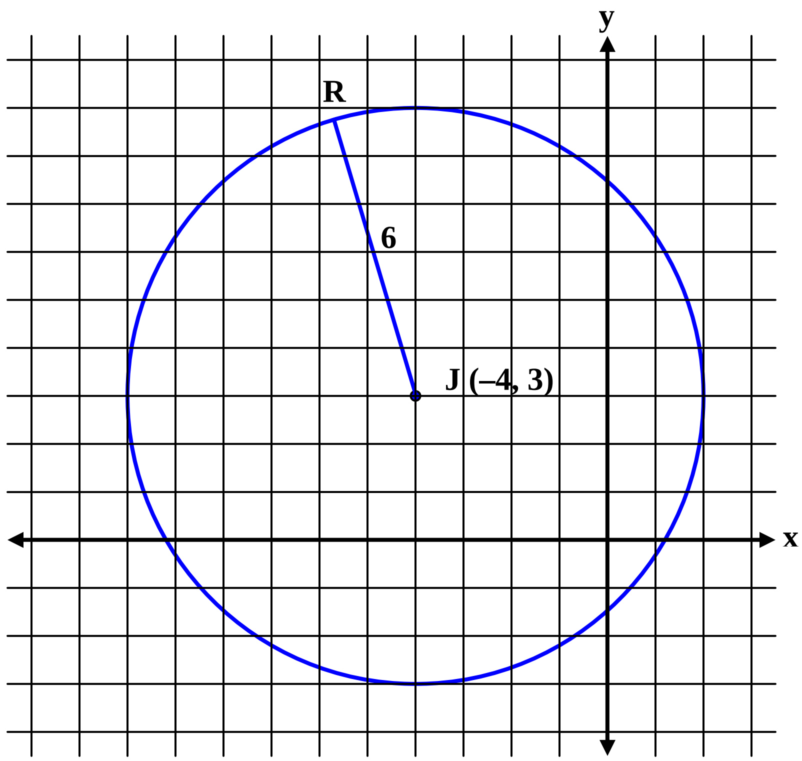
Let $(x,\ y)$ represent the ordered pair for point N.

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$	Distance Formula
$4=\sqrt{[x-(-1)]^2+(y-2)^2}$	Substitution
$4=\sqrt{(x+1)^2+(y-2)^2}$	Simplify
$16=(x+1)^2+(y-2)^2$	Square each side of the equation.
$(x+1)^2+(y-2)^2=16$	Commutative Property

The standard form for the equation of $\odot$M is $(x+1)^2+(y-2)^2=16$.

*This equation may also be written as $(x-(-1))^2+(y-2)^2=4^2$.
Notice that when the equation is written this way, the center point $(-1,2)$ and the radius (4) are easily recognized.

Example 2: Write an equation for $\odot J$ with center J(–4, 3) and a radius of 6 units.



$(x-h)^2+(y-k)^2=r^2$	Standard Circle Equation
$r=6,\ h=-4,\ k=3$	Prepare to substitute.
$[x-(-4)]^2+(y-3)^2=6^2$	Substitution
$(x+4)^2+(y-3)^2=36$	Simplify

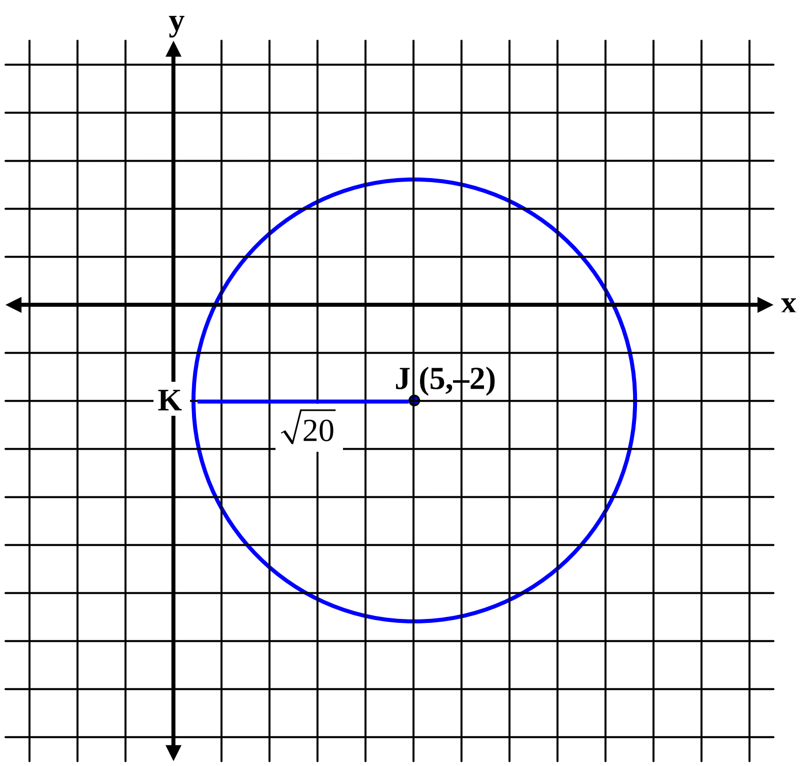
The equation for $\odot J$ is $(x+4)^2+(y-3)^2=36$.

Example 3: Graph the circle with an equation of $(x-5)^2+(y+2)^2=20.$

$(x-h)^2+(y-k)^2=r^2$	Standard Circle Equation
$(x-5)^2+(y+2)^2=20$	Given equation to graph
$(x-5)^2+[y-(-2)]^2=20$	$+2=-(-2)$
$(x-5)^2+[y-(-2)]^2=\left(\sqrt{20}\right)^2$	$20=\left(\sqrt{20}\right)^2$

Therefore, $h=5,\ k=-2,$ and $r=\sqrt{20}.$

Now graph: Center (5, –2), Radius ≈ 4.5



The graph of a circle with an equation of $(x-5)^2+(y+2)^2=20$ has a center at (5,–2) and a radius that is approximately 4.5 units in length.

Graphing Irrational Numbers

Irrational numbers are numbers that have decimals that go on forever, but never develop a repeating pattern. They are numbers that **cannot** be written as fractions where both the numerator and the denominator are integers.

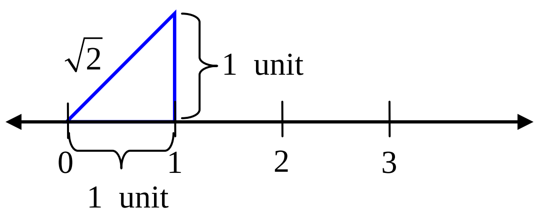
Irrational numbers do not repeat or terminate.

For example:

First 12 digits of $\pi\,(pi)$ = 3.14159265359 and so on...

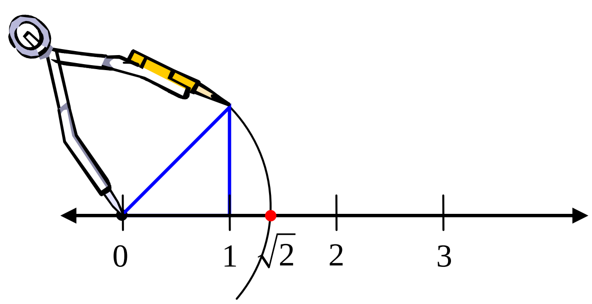
First 7 digits of $\sqrt{2} = 1.414213$ and so on...

Rational numbers can be graphed on a number line. To graph, $\sqrt{2}$, draw a right triangle with one leg on the number line and each leg with a length of 1 unit. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{1^2+1^2} = \sqrt{2}$.



Using a compass, you can transfer the length of the hypotenuse to the number line.

Place the metal point of the compass at zero (0) and the pencil point at the top of the right triangle. This length is the length of the square root of 2. Without changing the setting of the compass, draw an arc that passes through the number line. The point of intersection is the location of the $\sqrt{2}$ on the number line.



How would you locate an irrational number like $\sqrt{20}$ on the number line?

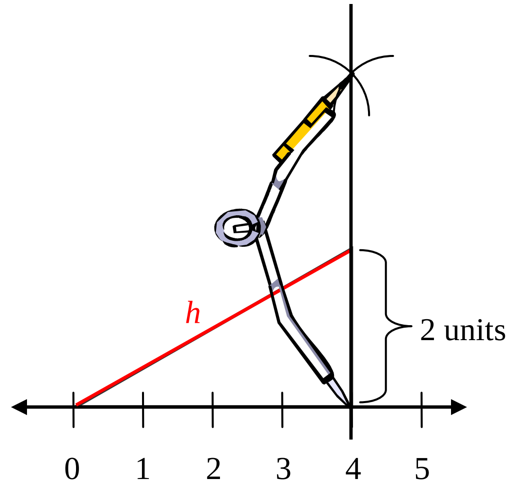
Determine two integers that when squared equal 20.

$2^2 = 4,\ 4^2 = 16,\ 4 + 16 = 20$

Therefore, graph 4 and 2 as the legs of the right triangle.

Step 1: Draw a number line. At 4, construct a perpendicular line segment 2 units in length.

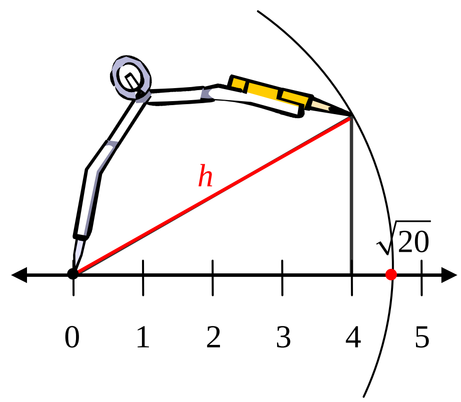
Step 2: Draw the line segment from zero to the end of the line segment that is 2 units in length and label it *h* for hypotenuse.



Step 3: Use the Pythagorean Theorem to show that the hypotenuse is $\sqrt{20}$ units long.

$$a^2 + b^2 = c^2$$
$$2^2 + 4^2 = c^2$$
$$20 = c^2$$
$$\sqrt{20} = c$$

Step 4: Open the compass to the length of *h*. With the tip of the compass at zero and the pencil point at the top of the right triangle, draw an arc that intersects the number line. The distance from 0 to the point on the number line is $\sqrt{20}$ units.



*Note: $\sqrt{20} \approx 4.5$.

The Quadratic Formula

The quadratic formula is used to solve any quadratic equation in standard form, $ax^2 + bx + c = 0$. The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use the quadratic formula

1.) make sure the equation is in standard form
2.) label the values of *a*, *b*, and *c*
3.) replace the values into the equation and solve

Example 1: Use the quadratic formula to solve the given quadratic for "*x*".

$x^2 - 16x - 36 = 0$ $\qquad a = 1,\ b = -16,\ c = -36$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(-36)}}{2(1)}$$

$$x = \frac{16 \pm \sqrt{256 + 144}}{2}$$

$$x = \frac{16 \pm \sqrt{400}}{2}$$

$$x = \frac{16 \pm 20}{2}$$

$x = \dfrac{16 + 20}{2}$ and $x = \dfrac{16 - 20}{2}$

$x = \dfrac{36}{2}$ and $x = \dfrac{-4}{2}$

$x = 18$ and $x = -2$

Example 2: Use the quadratic formula to solve the given quadratic for "*x*".

$$x^2 + 4x - 18 = 0 \qquad a = 1,\ b = 4,\ c = -18$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-18)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 72}}{2}$$

$$x = \frac{-4 \pm \sqrt{88}}{2}$$

$$x = \frac{-4 + \sqrt{88}}{2},\ \ x = \frac{-4 - \sqrt{88}}{2}$$

*These expressions can be simplified and this will be addressed in a later unit.