Theorems and Postulates

Postulate 1-A
Protractor Postulate

Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.

Definition of Right, Acute and Obtuse Angles

$\angle A$ is a right angle if $m\angle A$ is 90.
$\angle A$ is an acute angle if $m\angle A$ is less than 90.
$\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180.

Postulate 1-B
Angle Addition

If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.
If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.

Vertical angles are congruent.

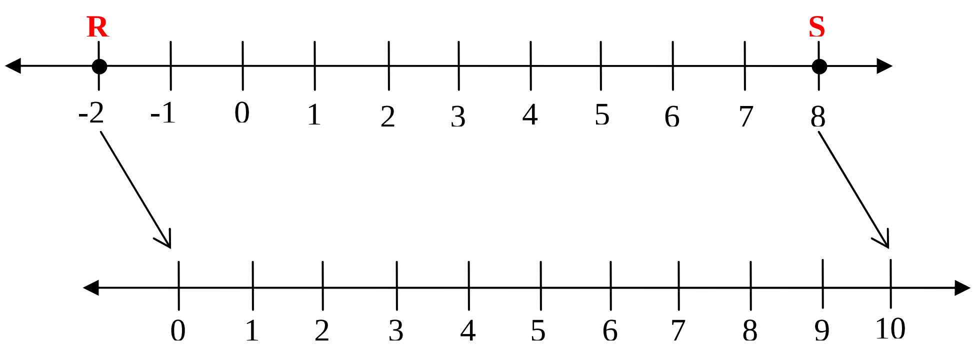
The sum of the measures of the angles in a linear pair is 180°.

The sum of the measures of complementary angles is 90°.

Postulate 2-A
Ruler

Two points on a line can be paired with real numbers so that, given any two points R and S on the line, R corresponds to zero, and S corresponds to a positive number.

Point R could be paired with 0, and S could be paired with 10.



Postulate 2-B
Segment Addition

If N is between M and P, then MN + NP = MP.
Conversely, if MN + NP = MP, then N is between M and P.

Theorem 2-A Pythagorean Theorem	**In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.**
Distance Formula	**The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **.**
Midpoint Definition	**The midpoint, M, of $\overline{AB}$ is the point between A and B such that AM = MB.**
Midpoint Formula Number Line	**With endpoints of A and B on a number line, the midpoint of $\overline{AB}$ is** $\frac{A+B}{2}$**.**
Midpoint Formula Coordinate Plane	**In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are** $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$**.**
Theorem 2-B Midpoint Theorem	If M is the midpoint of $\overline{PQ}$, then $\overline{PM} \cong \overline{MQ}$.
Postulate 3-A Law of Detachment	**If** $p \Rightarrow q$ **is true, and** p **is true, then** q **is true.**
Postulate 3-B Law of Syllogism	**If** $p \Rightarrow q$ **is true and** $q \Rightarrow r$ **is true, then** $p \Rightarrow r$ **is true.**
Postulate 4-A Reflexive Property	**Any segment or angle is congruent to itself.** $\overline{QS} \cong \overline{QS}$
Postulate 4-B Symmetric Property	**If** $\overline{AB} \cong \overline{CD}$, **then** $\overline{CD} \cong \overline{AB}$**.** **If** $\angle CAB \cong \angle DOE$**, then** $\angle DOE \cong \angle CAB$**.**

Theorem 4-A Transitive Property

If any segments or angles are congruent to the same angle, then they are congruent to each other.

If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

Theorem 4-B Transitive Property

If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 4-A.)

Theorem 5-A Addition Property

If a segment is added to two congruent segments, then the sums are congruent.

Theorem 5-B Addition Property

If an angle is added to two congruent angles, then the sums are congruent.

Theorem 5-C Addition Property

If congruent segments are added to congruent segments, then the sums are congruent.

Theorem 5-D Addition Property

If congruent angles are added to congruent angles, then the sums are congruent.

Theorem 5-E Subtraction Property

If a segment is subtracted from congruent segments, then the differences are congruent.

Theorem 5-F Subtraction Property

If an angle is subtracted from congruent angles, then the differences are congruent.

Theorem 5-G Subtraction Property

If congruent segments are subtracted from congruent segments, then the differences are congruent.

Theorem 5-H Subtraction Property	**If congruent angles are subtracted from congruent angles, then the differences are congruent.**
Theorem 5-I Multiplication Property	**If segments are congruent, then their like multiples are congruent.**
Theorem 5-J Multiplication Property	**If angles are congruent, then their like multiples are congruent.**
Theorem 5-K Division Property	**If segments are congruent, then their like divisions are congruent.**
Theorem 5-L Division Property	**If angles are congruent, then their like divisions are congruent.**
Theorem 7-A	**Congruence of angles is reflexive, symmetric, and transitive.**
Theorem 7-B	**If two angles form a linear pair, then they are supplementary angles.**
Theorem 7-C	**Angles supplementary to the same angle are congruent.**
Theorem 7-D	**Angles supplementary to congruent angles are congruent.**

Theorem 7-E	**Angles complementary to the same angle are congruent.**
Theorem 7-F	**Angles complementary to congruent angles are congruent.**
Theorem 7-G	**Right angles are congruent.**
Theorem 7-H	**Vertical angles are congruent.**
Theorem 7-I	**Perpendicular lines intersect to form right angles.**
Postulate 7-A	**If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.**
Theorem 7-J	**If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.**
Theorem 7-K	**If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.**
Theorem 7-L	**If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.**

Theorem 7-M

If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.

The definition of slope states that, given two points (x_1, y_1) and (x_2, y_2), the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

Postulate 8-A

Two non-vertical lines have the same slope if and only if they are parallel.

Postulate 8-B

Two non-vertical lines are perpendicular if and only if the product of their slopes is –1.

Postulate 8-C

If two lines in a plane are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.

Postulate 8-D

If there is a line and a point that is not on the line, then there exists exactly one line that passes through the point that is parallel to the given line.

Theorem 8-A

If two lines in a plane are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

Theorem 8-B

If two lines in a plane are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

Theorem 8-C

If two lines in a plane are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.

Theorem 8-D

If two lines in a plane are perpendicular to the same line, then the lines are parallel.

The distance from a point, which is not on a line, and a line is the length of a line segment that is perpendicular from the point to the line.

The distance between two parallel lines is the distance between one line and any point on the other line.

Theorem 10-A Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180.
Theorem 10-B Third Angle Theorem	If two of the angles in one triangle are congruent to two of the angles in a second triangle, then the third angles of each triangle are congruent.
Theorem 10-C Exterior Angle Theorem	In a triangle, the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.
Corollary 10-A-1	The acute angles of a right triangle are complementary.
Corollary 10-A-2	There can be at most one right angle in triangle.
Corollary 10-A-3	There can be at most one obtuse angle in triangle.
Corollary 10-A-4	The measure of each angle in an equiangular triangle is 60.

Definition of Congruent Triangles (CPCTC)

Two triangles are congruent if and only if their corresponding parts are congruent.

Postulate 10-A	Any segment or angle is congruent to itself. (Reflexive Property)

Postulate 11-A

SSS Postulate

If the sides of a triangle are congruent to the sides of a second triangle, then the triangles are congruent.

SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.

Postulate 11-B

SAS Postulate

If two sides and the included angle of a triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.

Postulate 11-C

ASA Postulate

If two angles and the included side of a triangle are congruent to the two angles and included side of a second triangle, then the triangles are congruent.

ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.

Theorem 11-A

AAS Theorem

If two angles and a non-included side of a triangle are congruent to two angles and a non-included side of a second triangle, then the two triangles are congruent.

AAS	Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and side of the other triangle.

Theorem 11-B

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles that are opposite those sides are congruent.

Theorem 11-C	**If two angles of a triangle are congruent, then the sides that are opposite those angles are congruent.**
Corollary 11-B-1	**A triangle is equilateral if and only if it is equiangular.**
Corollary 11-B-2	**Each angle of an equilateral triangle measures 60°.**
Postulate 12-A **HL Postulate**	**If there exists a correspondence between the vertices of two right triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the second triangle, then the two right triangles are congruent.**

The shortest distance between two points is a straight line.

Postulate 12-B	**A line segment is the shortest path between two points.**
Theorem 12-A	**A point on a perpendicular bisector of a segment is equidistant from the endpoints of the segment.**
Theorem 12-B	**A point that is equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.**
Theorem 12-C	**A point on the bisector of an angle is equidistant from the sides of the angle.**
Theorem 12-D	**A point that is in the interior of an angle and is equidistant from the sides of the angle lies on the bisector of the angle.**

Comparison Property	$a < b$, $a = b$, or $a > b$.
Transitive Property	1. If $a < b$ and $b < c$, then $a < c$. 2. If $a > b$ and $b > c$, then $a > c$.
Addition Property	1. If $a > b$, then $a + c > b + c$. 2. If $a < b$, then $a + c < b + c$.
Subtraction Property	1. If $a > b$, then $a - c > b - c$. 2. If $a < b$, then $a - c < b - c$.

Multiplication Properties	1. If $c > 0$ and $a < b$, then $ac < bc$. 2. If $c > 0$ and $a > b$, then $ac > bc$. 3. If $c < 0$ and $a < b$, then $ac > bc$. 4. If $c < 0$ and $a > b$, then $ac < bc$.

Division Properties	1. If $c > 0$ and $a < b$, then $\frac{a}{c} < \frac{b}{c}$. 2. If $c > 0$ and $a > b$, then $\frac{a}{c} > \frac{b}{c}$. 3. If $c < 0$ and $a < b$, then $\frac{a}{c} > \frac{b}{c}$. 4. If $c < 0$ and $a > b$, then $\frac{a}{c} < \frac{b}{c}$.

Theorem 13-A Exterior Angle Inequality Theorem	**If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its remote interior angles.**
Theorem 13-B	**If a side of a triangle is longer than another side, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.**
Theorem 13-C	**In a triangle, if the measure of an angle is greater than the measure of a second angle, then the side that is opposite the larger angle is longer than the side opposite the smaller angle.**
Theorem 13-D	**The shortest segment from a point to a line is a perpendicular line segment between the point and the line.**
Theorem 13-E Triangle Inequality Theorem	**The sum of the lengths of any two sides of a triangle is greater than the length of the third side.**
Theorem 13-F SAS Inequality (Hinge Theorem)	**If two sides of a triangle are congruent to two sides of a second triangle, and if the included angle of the first triangle is greater than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.**

Theorem 13-G SSS Inequality	**If two sides of a triangle are congruent to two sides of a second triangle, and if the third side in the first triangle is longer than the third side in the second triangle, then the included angle between the congruent sides in the first triangle is greater than the included angle between the congruent sides in the second triangle.**
Theorem 14-A	**The opposite sides of a parallelogram are congruent.**
Theorem 14-B	**The opposite angles of a parallelogram are congruent.**
Theorem 14-C	**The consecutive pairs of angles of a parallelogram are supplementary.**
Theorem 14-D	**The diagonals of a parallelogram bisect each other.**
Theorem 14-E	**Either diagonal of a parallelogram separates the parallelogram into two congruent triangles.**
Theorem 14-F	**In a quadrilateral if both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.**
Theorem 14-G	**In a quadrilateral if both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.**
Theorem 14-H	**In a quadrilateral if its diagonals bisect each other, then the quadrilateral is a parallelogram.**

Theorem 14-I	**In a quadrilateral if one pair of opposite sides is both congruent and parallel, then the quadrilateral is a parallelogram.**
Theorem 14-J	**If a parallelogram is a rectangle, then its diagonals are congruent.**
Theorem 14-K	**If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.**
Theorem 15-A	**The diagonals of a rhombus bisect its four angles.**
Theorem 15-B	**The diagonals of a rhombus are perpendicular.**
Theorem 15-C	**If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.**
Theorem 15-D	**In an isosceles trapezoid, both pairs of base angles are congruent.**
Theorem 15-E	**In an isosceles trapezoid, the diagonals are congruent.**
Theorem 15-F Mid-Segment Theorem	**The median of a trapezoid is parallel to the bases and its length is one-half the sum of the lengths of the bases.**

Theorem 15-G

The diagonals of a kite are perpendicular.

Equality of Cross Products

For any real numbers, *a*, *b*, *c*, and *d*, where *b* and *d* are not equal to zero,

$\frac{a}{b} = \frac{c}{d}$ if and only if, $ad = bc$.

Postulate 16-A

AA Similarity

If two angles of one triangle are congruent to two angles of a second triangle, then the triangles are similar.

Theorem 16-A
SSS Similarity

If the measure of the corresponding sides of two triangles is proportional, then the triangles are similar.

Theorem 16-B
SAS Similarity

If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of a second triangle, and the included angles are congruent, then the triangles are similar.

Theorem 16-C

The similarity of triangles is reflexive, symmetric, and transitive.

Theorem 19-A

If a line is parallel to one side of a triangle and intersects the other two sides, then those sides are separated into segments of proportional lengths.

Theorem 19-B

A line that divides two sides of a triangle proportionally is parallel to the third side of the triangle.

Theorem 19-C
Triangle Mid-segment Theorem

If a segment's endpoints are the midpoints of two sides of a triangle, then it is parallel to the third side of the triangle and one-half its length.

Corollary 19-A-1	**If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.**
Corollary 19-A-2	**If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.**
Theorem 19-D	**If two triangles are similar, then their perimeters are proportional to the measures of the corresponding sides.**
Theorem 19-E	**If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.**
Theorem 19-F	**If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.**
Theorem 19-G	**If two triangles are similar, then the measures of the corresponding angle bisectors of the two triangles are proportional to the measures of the corresponding sides.**
Theorem 19-H Angle Bisector Theorem	**In a triangle an angle bisector separates the opposite side into segments that have the same ratio as the other two sides.**
Theorem 20-A	**In a right triangle, if an altitude is drawn from the vertex of the right angle to the hypotenuse, then the two triangles formed are similar to each other and to the given triangle.**

Theorem 20-B

In a right triangle, the measures of the altitude drawn from the vertex of the right angle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse created by the intersection of the hypotenuse and the altitude.

Theorem 20-C

In a right triangle with the altitude drawn to the hypotenuse, the measure of a leg is the geometric mean between the measure of the hypotenuse and the measure of the segment of the hypotenuse that is adjacent to the leg.

Theorem 20-D Converse of the Pythagorean Theorem

If the sum of the squares of the measures of the two legs of a right triangle equals the square of the hypotenuse, then the triangle is a right triangle.

Suppose that m and n are two positive integers with $m < n$, then $n^2 - m^2$, $2mn$, and $n^2 + m^2$ is a **Pythagorean triple**.

Theorem 20-E

In a 45-45-90 degree right triangle, the length of the hypotenuse can be determined by multiplying $\sqrt{2}$ times the leg.

leg a = leg b	$\rightarrow$	x
hypotenuse	$\rightarrow$	$x\sqrt{2}$

Theorem 20-F

In a 30-60-90 degree right triangle, the length of the hypotenuse is twice as long as the shorter leg, and the longer leg equals the shorter leg multiplied by $\sqrt{3}$.

shorter leg	$\rightarrow$	x
longer leg	$\rightarrow$	$x\sqrt{3}$
hypotenuse	$\rightarrow$	$2x$