Theorems and Postulates

| Postulate 1-A Protractor Postulate | Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r. |

| Definition of Right, Acute and Obtuse Angles | $\angle A$ is a right angle if $m\angle A$ is 90. $\angle A$ is an acute angle if $m\angle A$ is less than 90. $\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180. |

| Postulate 1-B Angle Addition | If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$. If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$. |

Vertical angles are congruent.

The sum of the measures of the angles in a linear pair is 180°.

The sum of the measures of complementary angles is 90°.

| Postulate 2-A Ruler | Two points on a line can be paired with real numbers so that, given any two points R and S on the line, R corresponds to zero, and S corresponds to a positive number. |

Point R could be paired with 0, and S could be paired with 10.
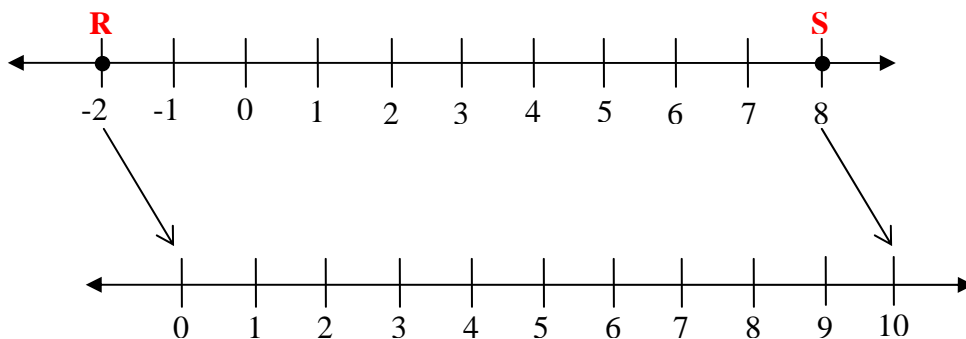
| Postulate 2-B Segment Addition | If N is between M and P, then MN + NP = MP. Conversely, if MN + NP = MP, then N is between M and P. |

Theorem 2-A **Pythagorean** **Theorem**	In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.
Distance Formula	The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Midpoint **Definition**	The midpoint, **M**, of $\overline{AB}$ is the point between **A** and **B** such that **AM** = **MB**.
Midpoint Formula **Number Line**	With endpoints of A and B on a number line, the midpoint of $\overline{AB}$ is $\dfrac{A+B}{2}$.
Midpoint Formula **Coordinate Plane**	In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2})$.
Theorem 2-B **Midpoint Theorem**	If M is the midpoint of $\overline{PQ}$, then $\overline{PM} \cong \overline{MQ}$.
Postulate 3-A **Law of** **Detachment**	If $p \Rightarrow q$ is true, and p is true, then q is true.
Postulate 3-B **Law of Syllogism**	If $p \Rightarrow q$ is true and $q \Rightarrow r$ is true, then $p \Rightarrow r$ is true.
Postulate 4-A **Reflexive** **Property**	Any segment or angle is congruent to itself. $\overline{QS} \cong \overline{QS}$
Postulate 4-B **Symmetric** **Property**	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. If $\angle CAB \cong \angle DOE$, then $\angle DOE \cong \angle CAB$.

Theorem 4-A **Transitive** **Property**	If any segments or angles are congruent to the same angle, then they are congruent to each other.

If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3.$

Theorem 4-B **Transitive** **Property**	If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 4-A.)

Theorem 5-A **Addition** **Property**	If a segment is added to two congruent segments, then the sums are congruent.

Theorem 5-B **Addition** **Property**	If an angle is added to two congruent angles, then the sums are congruent.

Theorem 5-C **Addition** **Property**	If congruent segments are added to congruent segments, then the sums are congruent.

Theorem 5-D **Addition** **Property**	If congruent angles are added to congruent angles, then the sums are congruent.

Theorem 5-E **Subtraction** **Property**	If a segment is subtracted from congruent segments, then the differences are congruent.

Theorem 5-F **Subtraction** **Property**	If an angle is subtracted from congruent angles, then the differences are congruent.

Theorem 5-G **Subtraction** **Property**	If congruent segments are subtracted from congruent segments, then the differences are congruent.

Theorem 5-H Subtraction Property	If congruent angles are subtracted from congruent angles, then the differences are congruent.
Theorem 5-I Multiplication Property	If segments are congruent, then their like multiples are congruent.
Theorem 5-J Multiplication Property	If angles are congruent, then their like multiples are congruent.
Theorem 5-K Division Property	If segments are congruent, then their like divisions are congruent.
Theorem 5-L Division Property	If angles are congruent, then their like divisions are congruent.
Theorem 7-A	Congruence of angles is reflexive, symmetric, and transitive.
Theorem 7-B	If two angles form a linear pair, then they are supplementary angles.
Theorem 7-C	Angles supplementary to the same angle are congruent.
Theorem 7-D	Angles supplementary to congruent angles are congruent.

Theorem 7-E	Angles complementary to the same angle are congruent.
Theorem 7-F	Angles complementary to congruent angles are congruent.
Theorem 7-G	Right angles are congruent.
Theorem 7-H	Vertical angles are congruent.
Theorem 7-I	Perpendicular lines intersect to form right angles.
Postulate 7-A	If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.
Theorem 7-J	If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.
Theorem 7-K	If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.
Theorem 7-L	If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Theorem 7-M	**If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.**

The definition of slope states that, given two points (x_1, y_1) and (x_2, y_2), the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

Postulate 8-A	**Two non-vertical lines have the same slope if and only if they are parallel.**
Postulate 8-B	**Two non-vertical lines are perpendicular if and only if the product of their slopes is –1.**
Postulate 8-C	**If two lines in a plane are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.**
Postulate 8-D	**If there is a line and a point that is not on the line, then there exists exactly one line that passes through the point that is parallel to the given line.**
Theorem 8-A	**If two lines in a plane are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.**
Theorem 8-B	**If two lines in a plane are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.**
Theorem 8-C	**If two lines in a plane are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.**
Theorem 8-D	**If two lines in a plane are perpendicular to the same line, then the lines are parallel.**

The distance from a point, which is not on a line, and a line is the length of a line segment that is perpendicular from the point to the line.

The distance between two parallel lines is the distance between one line and any point on the other line.

Theorem 10-A **Angle Sum** **Theorem**	The sum of the measures of the angles of a triangle is 180.
Theorem 10-B **Third Angle** **Theorem**	If two of the angles in one triangle are congruent to two of the angles in a second triangle, then the third angles of each triangle are congruent.
Theorem 10-C **Exterior Angle** **Theorem**	In a triangle, the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.
Corollary 10-A-1	The acute angles of a right triangle are complementary.
Corollary 10-A-2	There can be at most one right angle in triangle.
Corollary 10-A-3	There can be at most one obtuse angle in triangle.
Corollary 10-A-4	The measure of each angle in an equiangular triangle is 60.

Definition of Congruent Triangles
(CPCTC)

Two triangles are congruent if and only if their corresponding parts are congruent.

Postulate 10-A	Any segment or angle is congruent to itself. (Reflexive Property)

Postulate 11-A **SSS Postulate**	**If the sides of a triangle are congruent to the sides of a second triangle, then the triangles are congruent.**

SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.

Postulate 11-B **SAS Postulate**	**If two sides and the included angle of a triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.**

SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.

Postulate 11-C **ASA Postulate**	**If two angles and the included side of a triangle are congruent to the two angles and included side of a second triangle, then the triangles are congruent.**

ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.

Theorem 11-A **AAS Theorem**	**If two angles and a non-included side of a triangle are congruent to two angles and a non-included side of a second triangle, then the two triangles are congruent.**

AAS	Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and side of the other triangle.

Theorem 11-B **Isosceles Triangle Theorem**	**If two sides of a triangle are congruent, then the angles that are opposite those sides are congruent.**

Theorem 11-C	If two angles of a triangle are congruent, then the sides that are opposite those angles are congruent.
Corollary 11-B-1	A triangle is equilateral if and only if it is equiangular.
Corollary 11-B-2	Each angle of an equilateral triangle measures $60°$.
Postulate 12-A HL Postulate	If there exists a correspondence between the vertices of two right triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the second triangle, then the two right triangles are congruent.

The shortest distance between two points is a straight line.

Postulate 12-B	A line segment is the shortest path between two points.
Theorem 12-A	A point on a perpendicular bisector of a segment is equidistant from the endpoints of the segment.
Theorem 12-B	A point that is equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.
Theorem 12-C	A point on the bisector of an angle is equidistant from the sides of the angle.
Theorem 12-D	A point that is in the interior of an angle and is equidistant from the sides of the angle lies on the bisector of the angle.

Comparison Property	$a < b$, $a = b$, or $a > b$.

Transitive Property	1. If $a < b$ and $b < c$, then $a < c$. 2. If $a > b$ and $b > c$, then $a > c$.

Addition Property	1. If $a > b$, then $a + c > b + c$. 2. If $a < b$, then $a + c < b + c$.

Subtraction Property	1. If $a > b$, then $a - c > b - c$. 2. If $a < b$, then $a - c < b - c$.

Multiplication Properties	1. If $c > 0$ and $a < b$, then $ac < bc$. 2. If $c > 0$ and $a > b$, then $ac > bc$. 3. If $c < 0$ and $a < b$, then $ac > bc$. 4. If $c < 0$ and $a > b$, then $ac < bc$.
Division Properties	1. If $c > 0$ and $a < b$, then $\dfrac{a}{c} < \dfrac{b}{c}$. 2. If $c > 0$ and $a > b$, then $\dfrac{a}{c} > \dfrac{b}{c}$. 3. If $c < 0$ and $a < b$, then $\dfrac{a}{c} > \dfrac{b}{c}$. 4. If $c < 0$ and $a > b$, then $\dfrac{a}{c} < \dfrac{b}{c}$.

Theorem 13-A Exterior Angle Inequality Theorem	If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its remote interior angles.
Theorem 13-B	If a side of a triangle is longer than another side, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.
Theorem 13-C	In a triangle, if the measure of an angle is greater than the measure of a second angle, then the side that is opposite the larger angle is longer than the side opposite the smaller angle.
Theorem 13-D	The shortest segment from a point to a line is a perpendicular line segment between the point and the line.
Theorem 13-E Triangle Inequality Theorem	The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
Theorem 13-F SAS Inequality (Hinge Theorem)	If two sides of a triangle are congruent to two sides of a second triangle, and if the included angle of the first triangle is greater than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

**Theorem 13-G
SSS Inequality**

If two sides of a triangle are congruent to two sides of a second triangle, and if the third side in the first triangle is longer than the third side in the second triangle, then the included angle between the congruent sides in the first triangle is greater than the included angle between the congruent sides in the second triangle.