Theorems and Postulates

Postulate 1-A
Protractor Postulate

Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.

Definition of Right, Acute and Obtuse Angles

$\angle A$ is a right angle if $m\angle A$ is 90.
$\angle A$ is an acute angle if $m\angle A$ is less than 90.
$\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180.

Postulate 1-B
Angle Addition

If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.
If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.

Vertical angles are congruent.

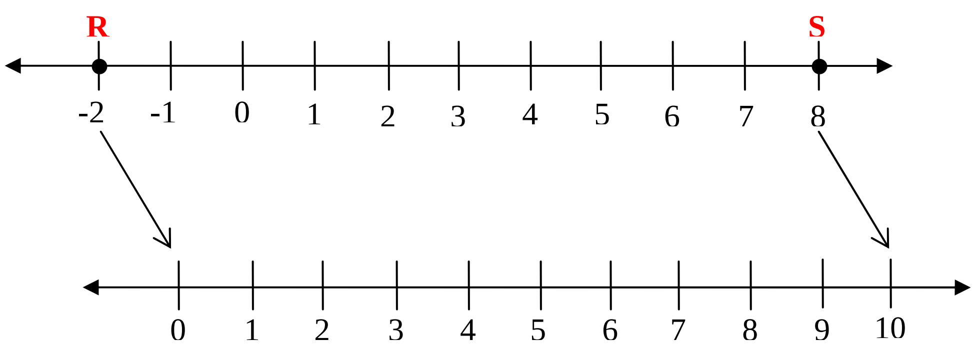
The sum of the measures of the angles in a linear pair is 180°.

The sum of the measures of complementary angles is 90°.

Postulate 2-A
Ruler

Two points on a line can be paired with real numbers so that, given any two points R and S on the line, R corresponds to zero, and S corresponds to a positive number.

Point R could be paired with 0, and S could be paired with 10.



Postulate 2-B
Segment Addition

If N is between M and P, then MN + NP = MP.
Conversely, if MN + NP = MP, then N is between M and P.

Theorem 2-A Pythagorean Theorem	**In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.**
Distance Formula	**The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Midpoint Definition	**The midpoint, M, of $\overline{AB}$ is the point between A and B such that AM = MB.**
Midpoint Formula Number Line	**With endpoints of A and B on a number line, the midpoint of $\overline{AB}$ is** $\frac{A+B}{2}$.
Midpoint Formula Coordinate Plane	**In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
Theorem 2-B Midpoint Theorem	If M is the midpoint of $\overline{PQ}$, then $\overline{PM} \cong \overline{MQ}$.
Postulate 3-A Law of Detachment	**If** $p \Rightarrow q$ **is true, and** p **is true, then** q **is true.**
Postulate 3-B Law of Syllogism	**If** $p \Rightarrow q$ **is true and** $q \Rightarrow r$ **is true, then** $p \Rightarrow r$ **is true.**
Postulate 4-A Reflexive Property	**Any segment or angle is congruent to itself.** $\overline{QS} \cong \overline{QS}$
Postulate 4-B Symmetric Property	**If** $\overline{AB} \cong \overline{CD}$, **then** $\overline{CD} \cong \overline{AB}$. **If** $\angle CAB \cong \angle DOE$, **then** $\angle DOE \cong \angle CAB$.

Theorem 4-A Transitive Property	**If any segments or angles are congruent to the same angle, then they are congruent to each other.** **If** $\overline{AB} \cong \overline{CD}$ **and** $\overline{CD} \cong \overline{EF}$**, then** $\overline{AB} \cong \overline{EF}$**.** **If** $\angle 1 \cong \angle 2$ **and** $\angle 2 \cong \angle 3$**, then** $\angle 1 \cong \angle 3$.
Theorem 4-B Transitive Property	**If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 4-A.)**
Theorem 5-A Addition Property	**If a segment is added to two congruent segments, then the sums are congruent.**
Theorem 5-B Addition Property	**If an angle is added to two congruent angles, then the sums are congruent.**
Theorem 5-C Addition Property	**If congruent segments are added to congruent segments, then the sums are congruent.**
Theorem 5-D Addition Property	**If congruent angles are added to congruent angles, then the sums are congruent.**
Theorem 5-E Subtraction Property	**If a segment is subtracted from congruent segments, then the differences are congruent.**
Theorem 5-F Subtraction Property	**If an angle is subtracted from congruent angles, then the differences are congruent.**
Theorem 5-G Subtraction Property	**If congruent segments are subtracted from congruent segments, then the differences are congruent.**

Theorem 5-H Subtraction Property	**If congruent angles are subtracted from congruent angles, then the differences are congruent.**
Theorem 5-I Multiplication Property	**If segments are congruent, then their like multiples are congruent.**
Theorem 5-J Multiplication Property	**If angles are congruent, then their like multiples are congruent.**
Theorem 5-K Division Property	**If segments are congruent, then their like divisions are congruent.**
Theorem 5-L Division Property	**If angles are congruent, then their like divisions are congruent.**
Theorem 7-A	**Congruence of angles is reflexive, symmetric, and transitive.**
Theorem 7-B	**If two angles form a linear pair, then they are supplementary angles.**
Theorem 7-C	**Angles supplementary to the same angle are congruent.**
Theorem 7-D	**Angles supplementary to congruent angles are congruent.**

Theorem 7-E	**Angles complementary to the same angle are congruent.**
Theorem 7-F	**Angles complementary to congruent angles are congruent.**
Theorem 7-G	**Right angles are congruent.**
Theorem 7-H	**Vertical angles are congruent.**
Theorem 7-I	**Perpendicular lines intersect to form right angles.**
Postulate 7-A	**If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.**
Theorem 7-J	**If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.**
Theorem 7-K	**If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.**
Theorem 7-L	**If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.**

Theorem 7-M

If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.

The definition of slope states that, given two points (x_1, y_1) and (x_2, y_2), the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

Postulate 8-A

Two non-vertical lines have the same slope if and only if they are parallel.

Postulate 8-B

Two non-vertical lines are perpendicular if and only if the product of their slopes is –1.

Postulate 8-C

If two lines in a plane are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.

Postulate 8-D

If there is a line and a point that is not on the line, then there exists exactly one line that passes through the point that is parallel to the given line.

Theorem 8-A

If two lines in a plane are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

Theorem 8-B

If two lines in a plane are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

Theorem 8-C

If two lines in a plane are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.

Theorem 8-D

If two lines in a plane are perpendicular to the same line, then the lines are parallel.

The distance from a point, which is not on a line, and a line is the length of a line segment that is perpendicular from the point to the line.

The distance between two parallel lines is the distance between one line and any point on the other line.

Theorem 10-A Angle Sum Theorem	The sum of the measures of the angles of a triangle is 180.
Theorem 10-B Third Angle Theorem	If two of the angles in one triangle are congruent to two of the angles in a second triangle, then the third angles of each triangle are congruent.
Theorem 10-C Exterior Angle Theorem	In a triangle, the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.
Corollary 10-A-1	The acute angles of a right triangle are complementary.
Corollary 10-A-2	There can be at most one right angle in triangle.
Corollary 10-A-3	There can be at most one obtuse angle in triangle.
Corollary 10-A-4	The measure of each angle in an equiangular triangle is 60.

Definition of Congruent Triangles (CPCTC)

Two triangles are congruent if and only if their corresponding parts are congruent.

Postulate 10-A	Any segment or angle is congruent to itself. (Reflexive Property)

Postulate 11-A

SSS Postulate

If the sides of a triangle are congruent to the sides of a second triangle, then the triangles are congruent.

SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.

Postulate 11-B

SAS Postulate

If two sides and the included angle of a triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.

Postulate 11-C

ASA Postulate

If two angles and the included side of a triangle are congruent to the two angles and included side of a second triangle, then the triangles are congruent.

ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.

Theorem 11-A

AAS Theorem

If two angles and a non-included side of a triangle are congruent to two angles and a non-included side of a second triangle, then the two triangles are congruent.

AAS	Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and side of the other triangle.

Theorem 11-B

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles that are opposite those sides are congruent.

Theorem 11-C	**If two angles of a triangle are congruent, then the sides that are opposite those angles are congruent.**
Corollary 11-B-1	**A triangle is equilateral if and only if it is equiangular.**
Corollary 11-B-2	**Each angle of an equilateral triangle measures 60°.**
Postulate 12-A **HL Postulate**	**If there exists a correspondence between the vertices of two right triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the second triangle, then the two right triangles are congruent.**

The shortest distance between two points is a straight line.

Postulate 12-B	**A line segment is the shortest path between two points.**
Theorem 12-A	**A point on a perpendicular bisector of a segment is equidistant from the endpoints of the segment.**
Theorem 12-B	**A point that is equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.**
Theorem 12-C	**A point on the bisector of an angle is equidistant from the sides of the angle.**
Theorem 12-D	**A point that is in the interior of an angle and is equidistant from the sides of the angle lies on the bisector of the angle.**