

Theorems and Postulates

Postulate 1-A Protractor Postulate

Given \overline{AB} and a number r between 0 and 180, there is exactly one ray with endpoint A , extending on either side of \overline{AB} , such that the measure of the angle formed is r .

Definition of Right, Acute and Obtuse Angles

$\angle A$ is a right angle if $m\angle A$ is 90.

$\angle A$ is an acute angle if $m\angle A$ is less than 90.

$\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180.

Postulate 1-B Angle Addition

If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.

If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.

Vertical angles are congruent.

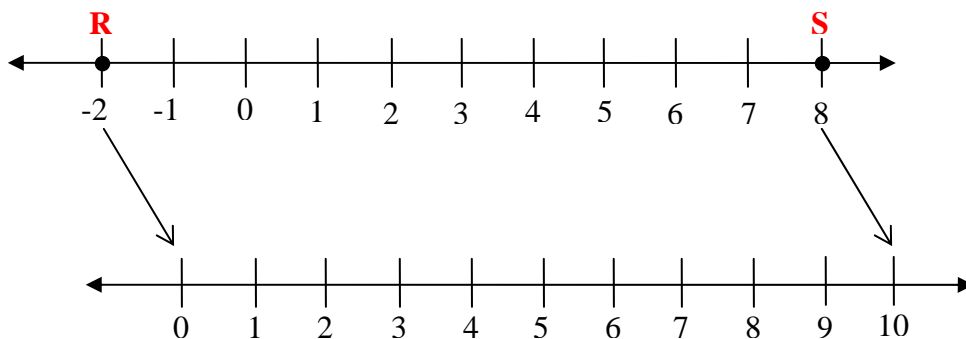
The sum of the measures of the angles in a linear pair is 180° .

The sum of the measures of complementary angles is 90° .

Postulate 2-A Ruler

Two points on a line can be paired with real numbers so that, given any two points R and S on the line, R corresponds to zero, and S corresponds to a positive number.

Point R could be paired with 0, and S could be paired with 10.



Postulate 2-B Segment Addition

If N is between M and P , then $MN + NP = MP$.

Conversely, if $MN + NP = MP$, then N is between M and P .

**Theorem 2-A
Pythagorean
Theorem**

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Distance Formula

The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

**Midpoint
Definition**

The midpoint, M , of \overline{AB} is the point between A and B such that $AM = MB$.

**Midpoint Formula
Number Line**

With endpoints of A and B on a number line, the midpoint of \overline{AB} is $\frac{A+B}{2}$.

**Midpoint Formula
Coordinate Plane**

In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.

**Theorem 2-B
Midpoint Theorem**

If M is the midpoint of \overline{PQ} , then $\overline{PM} \cong \overline{MQ}$.

**Postulate 3-A
Law of
Detachment**

If $p \Rightarrow q$ is true, and p is true, then q is true.

**Postulate 3-B
Law of Syllogism**

If $p \Rightarrow q$ is true and $q \Rightarrow r$ is true, then $p \Rightarrow r$ is true.

**Postulate 4-A
Reflexive
Property**

Any segment or angle is congruent to itself.

$$\overline{QS} \cong \overline{QS}$$

**Postulate 4-B
Symmetric
Property**

If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

If $\angle CAB \cong \angle DOE$, then $\angle DOE \cong \angle CAB$.

**Theorem 4-A
Transitive
Property**

If any segments or angles are congruent to the same angle, then they are congruent to each other.

**If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.**

**Theorem 4-B
Transitive
Property**

If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 4-A.)

**Theorem 5-A
Addition
Property**

If a segment is added to two congruent segments, then the sums are congruent.

**Theorem 5-B
Addition
Property**

If an angle is added to two congruent angles, then the sums are congruent.

**Theorem 5-C
Addition
Property**

If congruent segments are added to congruent segments, then the sums are congruent.

**Theorem 5-D
Addition
Property**

If congruent angles are added to congruent angles, then the sums are congruent.

**Theorem 5-E
Subtraction
Property**

If a segment is subtracted from congruent segments, then the differences are congruent.

**Theorem 5-F
Subtraction
Property**

If an angle is subtracted from congruent angles, then the differences are congruent.

**Theorem 5-G
Subtraction
Property**

If congruent segments are subtracted from congruent segments, then the differences are congruent.

**Theorem 5-H
Subtraction
Property**

If congruent angles are subtracted from congruent angles, then the differences are congruent.

**Theorem 5-I
Multiplication
Property**

If segments are congruent, then their like multiples are congruent.

**Theorem 5-J
Multiplication
Property**

If angles are congruent, then their like multiples are congruent.

**Theorem 5-K
Division
Property**

If segments are congruent, then their like divisions are congruent.

**Theorem 5-L
Division
Property**

If angles are congruent, then their like divisions are congruent.

Theorem 7-A

Congruence of angles is reflexive, symmetric, and transitive.

Theorem 7-B

If two angles form a linear pair, then they are supplementary angles.

Theorem 7-C

Angles supplementary to the same angle are congruent.

Theorem 7-D

Angles supplementary to congruent angles are congruent.

Theorem 7-E

Angles complementary to the same angle are congruent.

Theorem 7-F

Angles complementary to congruent angles are congruent.

Theorem 7-G

Right angles are congruent.

Theorem 7-H

Vertical angles are congruent.

Theorem 7-I

Perpendicular lines intersect to form right angles.

Postulate 7-A

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Theorem 7-J

If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.

Theorem 7-K

If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

Theorem 7-L

If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Theorem 7-M

If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.

The definition of slope states that, given two points (x_1, y_1) and (x_2, y_2) , the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

Postulate 8-A

Two non-vertical lines have the same slope if and only if they are parallel.

Postulate 8-B

Two non-vertical lines are perpendicular if and only if the product of their slopes is -1 .

Postulate 8-C

If two lines in a plane are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.

Postulate 8-D

If there is a line and a point that is not on the line, then there exists exactly one line that passes through the point that is parallel to the given line.

Theorem 8-A

If two lines in a plane are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

Theorem 8-B

If two lines in a plane are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

Theorem 8-C

If two lines in a plane are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.

Theorem 8-D

If two lines in a plane are perpendicular to the same line, then the lines are parallel.

The distance from a point, which is not on a line, and a line is the length of a line segment that is perpendicular from the point to the line.

The distance between two parallel lines is the distance between one line and any point on the other line.

**Theorem 10-A
Angle Sum
Theorem**

The sum of the measures of the angles of a triangle is 180.

**Theorem 10-B
Third Angle
Theorem**

If two of the angles in one triangle are congruent to two of the angles in a second triangle, then the third angles of each triangle are congruent.

**Theorem 10-C
Exterior Angle
Theorem**

In a triangle, the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Corollary 10-A-1

The acute angles of a right triangle are complementary.

Corollary 10-A-2

There can be at most one right angle in triangle.

Corollary 10-A-3

There can be at most one obtuse angle in triangle.

Corollary 10-A-4

The measure of each angle in an equiangular triangle is 60.

**Definition of Congruent Triangles
(CPCTC)**

Two triangles are congruent if and only if their corresponding parts are congruent.

Postulate 10-A

Any segment or angle is congruent to itself.
(Reflexive Property)

**Postulate 11-A
SSS Postulate**

If the sides of a triangle are congruent to the sides of a second triangle, then the triangles are congruent.

SSS

The three sides of one triangle must be congruent to the three sides of the other triangle.

**Postulate 11-B
SAS Postulate**

If two sides and the included angle of a triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

SAS

Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.

**Postulate 11-C
ASA Postulate**

If two angles and the included side of a triangle are congruent to the two angles and included side of a second triangle, then the triangles are congruent.

ASA

Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.

**Theorem 11-A
AAS Theorem**

If two angles and a non-included side of a triangle are congruent to two angles and a non-included side of a second triangle, then the two triangles are congruent.

AAS

Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and side of the other triangle.

**Theorem 11-B
Isosceles Triangle
Theorem**

If two sides of a triangle are congruent, then the angles that are opposite those sides are congruent.

Theorem 11-C

If two angles of a triangle are congruent, then the sides that are opposite those angles are congruent.

Corollary 11-B-1

A triangle is equilateral if and only if it is equiangular.

Corollary 11-B-2

Each angle of an equilateral triangle measures 60° .