

Theorems and Postulates

Postulate 1-A Protractor Postulate

Given \overline{AB} and a number r between 0 and 180, there is exactly one ray with endpoint A , extending on either side of \overline{AB} , such that the measure of the angle formed is r .

Definition of Right, Acute and Obtuse Angles

$\angle A$ is a right angle if $m\angle A$ is 90.

$\angle A$ is an acute angle if $m\angle A$ is less than 90.

$\angle A$ is an obtuse angle if $m\angle A$ is greater than 90 and less than 180.

Postulate 1-B Angle Addition

If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.

If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$.

Vertical angles are congruent.

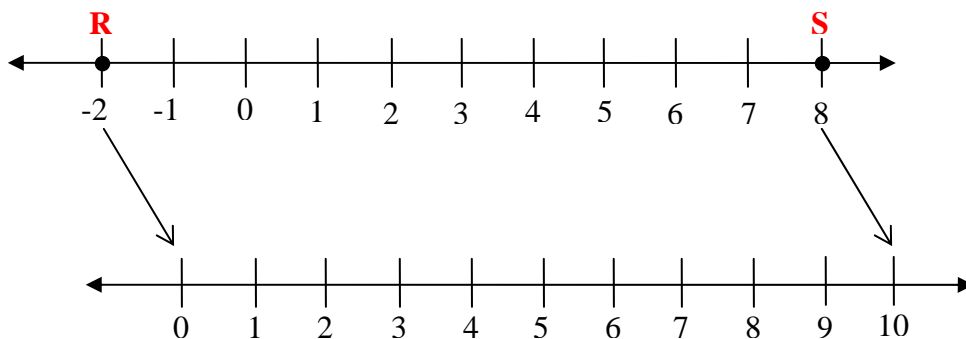
The sum of the measures of the angles in a linear pair is 180° .

The sum of the measures of complementary angles is 90° .

Postulate 2-A Ruler

Two points on a line can be paired with real numbers so that, given any two points R and S on the line, R corresponds to zero, and S corresponds to a positive number.

Point R could be paired with 0, and S could be paired with 10.



Postulate 2-B Segment Addition

If N is between M and P , then $MN + NP = MP$.

Conversely, if $MN + NP = MP$, then N is between M and P .

**Theorem 2-A
Pythagorean
Theorem**

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Distance Formula

The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

**Midpoint
Definition**

The midpoint, M , of \overline{AB} is the point between A and B such that $AM = MB$.

**Midpoint Formula
Number Line**

With endpoints of A and B on a number line, the midpoint of \overline{AB} is $\frac{A+B}{2}$.

**Midpoint Formula
Coordinate Plane**

In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.

**Theorem 2-B
Midpoint Theorem**

If M is the midpoint of \overline{PQ} , then $\overline{PM} \cong \overline{MQ}$.

**Postulate 3-A
Law of
Detachment**

If $p \Rightarrow q$ is true, and p is true, then q is true.

**Postulate 3-B
Law of Syllogism**

If $p \Rightarrow q$ is true and $q \Rightarrow r$ is true, then $p \Rightarrow r$ is true.

**Postulate 4-A
Reflexive
Property**

Any segment or angle is congruent to itself.

$$\overline{QS} \cong \overline{QS}$$

**Postulate 4-B
Symmetric
Property**

If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

If $\angle CAB \cong \angle DOE$, then $\angle DOE \cong \angle CAB$.

**Theorem 4-A
Transitive
Property**

If any segments or angles are congruent to the same angle, then they are congruent to each other.

**If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.**

**Theorem 4-B
Transitive
Property**

If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 4-A.)

**Theorem 5-A
Addition
Property**

If a segment is added to two congruent segments, then the sums are congruent.

**Theorem 5-B
Addition
Property**

If an angle is added to two congruent angles, then the sums are congruent.

**Theorem 5-C
Addition
Property**

If congruent segments are added to congruent segments, then the sums are congruent.

**Theorem 5-D
Addition
Property**

If congruent angles are added to congruent angles, then the sums are congruent.

**Theorem 5-E
Subtraction
Property**

If a segment is subtracted from congruent segments, then the differences are congruent.

**Theorem 5-F
Subtraction
Property**

If an angle is subtracted from congruent angles, then the differences are congruent.

**Theorem 5-G
Subtraction
Property**

If congruent segments are subtracted from congruent segments, then the differences are congruent.

**Theorem 5-H
Subtraction
Property**

If congruent angles are subtracted from congruent angles, then the differences are congruent.

**Theorem 5-I
Multiplication
Property**

If segments are congruent, then their like multiples are congruent.

**Theorem 5-J
Multiplication
Property**

If angles are congruent, then their like multiples are congruent.

**Theorem 5-K
Division
Property**

If segments are congruent, then their like divisions are congruent.

**Theorem 5-L
Division
Property**

If angles are congruent, then their like divisions are congruent.

Theorem 7-A

Congruence of angles is reflexive, symmetric, and transitive.

Theorem 7-B

If two angles form a linear pair, then they are supplementary angles.

Theorem 7-C

Angles supplementary to the same angle are congruent.

Theorem 7-D

Angles supplementary to congruent angles are congruent.

Theorem 7-E

Angles complementary to the same angle are congruent.

Theorem 7-F

Angles complementary to congruent angles are congruent.

Theorem 7-G

Right angles are congruent.

Theorem 7-H

Vertical angles are congruent.

Theorem 7-I

Perpendicular lines intersect to form right angles.

Postulate 7-A

If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Theorem 7-J

If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.

Theorem 7-K

If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

Theorem 7-L

If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

Theorem 7-M

If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.

The definition of slope states that, given two points (x_1, y_1) and (x_2, y_2) , the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

Postulate 8-A

Two non-vertical lines have the same slope if and only if they are parallel.

Postulate 8-B

Two non-vertical lines are perpendicular if and only if the product of their slopes is -1 .

Postulate 8-C

If two lines in a plane are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.

Postulate 8-D

If there is a line and a point that is not on the line, then there exists exactly one line that passes through the point that is parallel to the given line.

Theorem 8-A

If two lines in a plane are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

Theorem 8-B

If two lines in a plane are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

Theorem 8-C

If two lines in a plane are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.

Theorem 8-D

If two lines in a plane are perpendicular to the same line, then the lines are parallel.

The distance from a point, which is not on a line, and a line is the length of a line segment that is perpendicular from the point to the line.

The distance between two parallel lines is the distance between one line and any point on the other line.

**Theorem 10-A
Angle Sum
Theorem**

The sum of the measures of the angles of a triangle is 180.

**Theorem 10-B
Third Angle
Theorem**

If two of the angles in one triangle are congruent to two of the angles in a second triangle, then the third angles of each triangle are congruent.

**Theorem 10-C
Exterior Angle
Theorem**

In a triangle, the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Corollary 10-A-1

The acute angles of a right triangle are complementary.

Corollary 10-A-2

There can be at most one right angle in triangle.

Corollary 10-A-3

There can be at most one obtuse angle in triangle.

Corollary 10-A-4

The measure of each angle in an equiangular triangle is 60.

**Definition of Congruent Triangles
(CPCTC)**

Two triangles are congruent if and only if their corresponding parts are congruent.

Postulate 10-A

Any segment or angle is congruent to itself.
(Reflexive Property)