PROPERTIES OF TRIANGLES

In this unit you will begin with a review of triangles and their properties. You will look at classifying triangles by both angles and sides. You will examine the Angle Sum Theorem and other theorems that apply to triangles. In the final part of this unit, you will explore congruency between triangles by examining their corresponding angles and sides and learn a very useful definition of congruent triangles (CPCTC).

Classifying Triangles by Angles

Classifying Triangles by Sides

Angles of a Triangle

Congruent Triangles

Classifying Triangles by Angles

triangle – A triangle is a three-sided polygon.

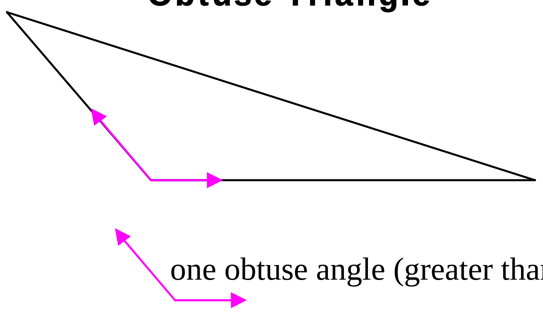
right triangle – A right triangle is a triangle that has one right angle.

obtuse triangle – An obtuse triangle is a triangle that has one obtuse angle.

acute triangle – An acute triangle is a triangle in which all the angles are acute.

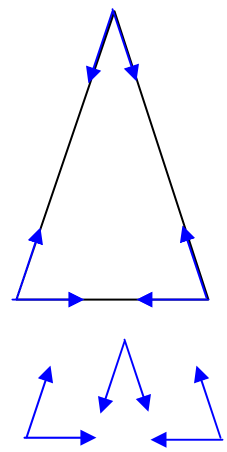
equiangular triangle – An equiangular triangle is a triangle in which all three angles are equal in measure.

Obtuse Triangle



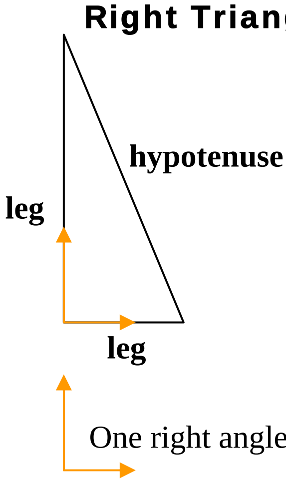
one obtuse angle (greater than 90°)

Acute Triangle



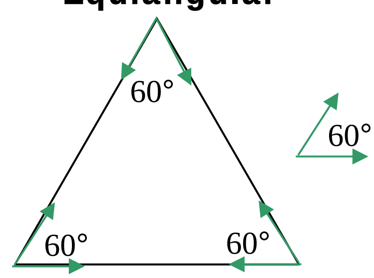
All are acute angles (angles that measure less than 90°).

Right Triangle



One right angle (90°)

Equiangular



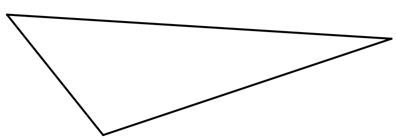
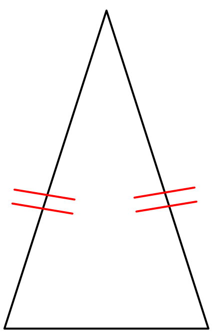
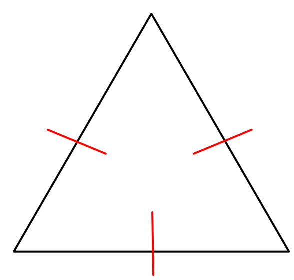
All angles measure the same(60°).

Classifying Triangles by Sides

scalene triangle – A scalene triangle is a triangle that has all sides measuring different lengths.

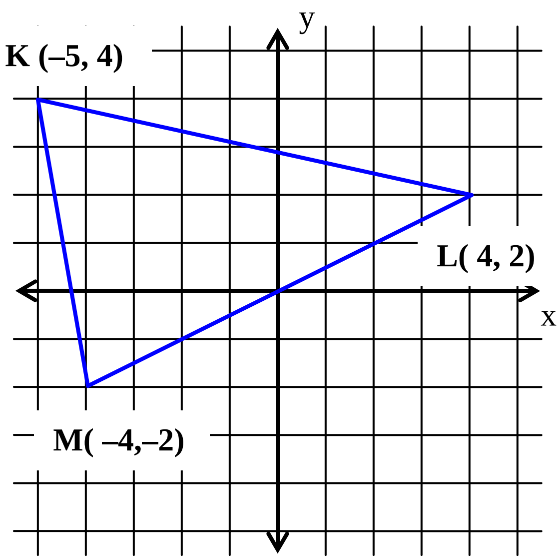
isosceles triangle – An isosceles triangle is a triangle that has two congruent sides.

equilateral triangle – An equilateral triangle is a triangle that has all sides congruent.

Scalene Triangle	Isosceles Triangle	Equilateral Triangle
		
All sides are different lengths.	Two sides are the same lengths.	All three sides are the same length.

Let's examine a triangle in the coordinate plane to determine what type of triangle it is.

Given $\triangle KLM$. Use the distance formula to determine the type of triangle based on the length of it sides.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$KL = \sqrt{(-5-4)^2 + (4-2)^2}$

$d = \sqrt{(-9)^2 + (2)^2}$

$d = \sqrt{85}$

$LM = \sqrt{[4-(-4)]^2 + [2-(-2)]^2}$

$d = \sqrt{(8)^2 + (4)^2}$

$d = \sqrt{80}$

$KM = \sqrt{[-5-(-4)]^2 + [4-(-2)]^2}$

$d = \sqrt{(-1)^2 + (6)^2}$

$d = \sqrt{37}$

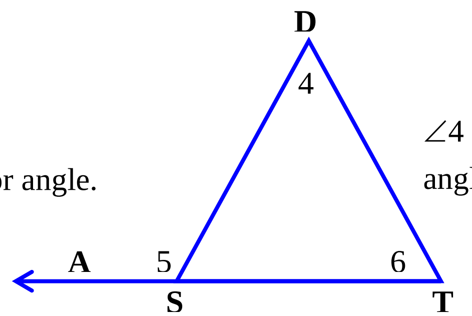
$\triangle KLM$ is a scalene triangle because each side measures a different length.

Angles of a Triangle

exterior angle – An exterior angle is an angle in the exterior of a triangle and is formed with one side of a triangle and an extension of the adjacent side.

remote interior angles – Remote interior angles are the interior angles of a triangle that are not adjacent to an exterior angle.

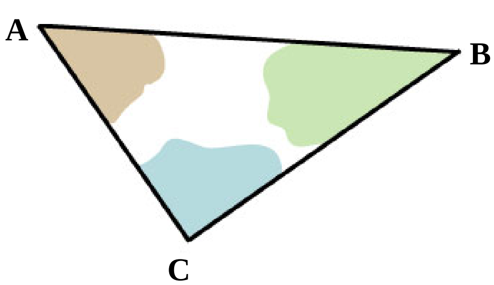
$\angle 5$ is an exterior angle.

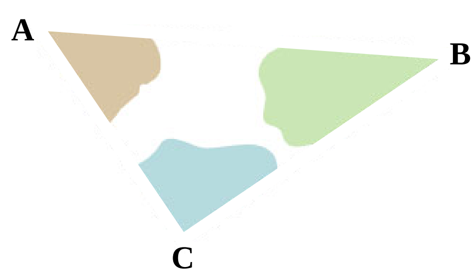


$\angle 4$ and $\angle 6$ are the remote interior angles of exterior $\angle 5$.

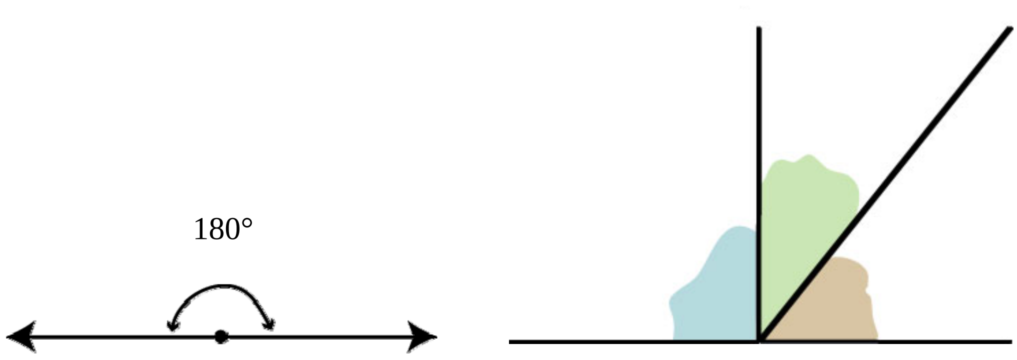
Theorem 10-A Angle Sum Theorem	**The sum of the measures of the angles of a triangle is 180.**

Activity: Look at the angles in the triangle below. Draw a triangle similar to the one shown below. It does not have to be the exact same size. Cut the angles away from the triangle.





Place the angles side by side at one vertex point, so that there are no gaps between the angles as shown below.



The three angles of the triangle total 180°.

Example 1: In $\triangle ABC$, what is the $m\angle A$ if the $m\angle C = 95°$ and the $m\angle B = 40°$?

$m\angle A + m\angle B + m\angle C = 180$	Angle Sum Theorem (Theorem 10-A)
$m\angle A + 40 + 95 = 180$	Substitution
$m\angle A + 135 = 180$	Simplify
$m\angle A = 45$	Subtraction

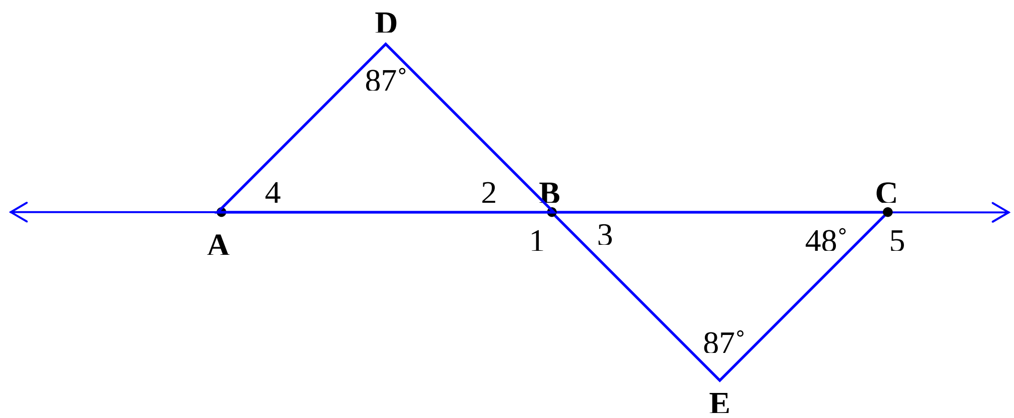
Theorem 10-B
Third Angle Theorem

If two of the angles in one triangle are congruent to two of the angles in a second triangle, then the third angles of each triangle are congruent.

Theorem 10-C
Exterior Angle Theorem

In a triangle, the measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Use the figure below to solve the following examples.



Example 2: Find the $m\angle 1$.

In this problem, we will apply the Exterior Angle Theorem since $\angle 1$ is the exterior angle of remote interior angles, $\angle BCE$ and $\angle E$, and the measures of these angles are given.

$m\angle BCE = 48°;\ m\angle E = 87°$	Given
$m\angle 1 = m\angle BCE + m\angle E$	Exterior Angle Theorem (Theorem 10-C)
$m\angle 1 = 48° + 87°$	Substitution
$m\angle 1 = 135°$	Simplify

Example 3: Find the $m\angle 2$.

$\angle 1$ and $\angle 2$ are a linear pair because they form a straight line.

$m\angle 2 + m\angle 1 = 180°$	Linear pairs are supplementary. (Theorem 7-B)
$m\angle 1 = 135°$	Determined in the previous example.
$m\angle 2 + 135° = 180°$	Substitution
$m\angle 2 = 45°$	Subtraction

Example 4: Find the $m\angle 3$.

$\angle 2$ and $\angle 3$ are vertical angles.

$\angle 2 \cong \angle 3$	Vertical angles are congruent. (Theorem 7-H)
$m\angle 2 = m\angle 3$	Definition of congruency.
$m\angle 2 = 45°$	Determined in the previous example.
$m\angle 3 = 45°$	Substitution ($m\angle 2 = 45°$)

Example 5: Find the $m\angle 4$.

For this problem, we will apply the Third Angle Theorem (10-B) even though we could also use the Triangle Sum Theorem (10-A).

$m\angle D = m\angle E$	Given
$m\angle 2 = m\angle 3$	Vertical angles have same measure.
$m\angle 4 = m\angle C = 48°$	Third Angle Theorem (Theorem 10-B)
	(The measure of the third angle of △BCE = 48°.)

Example 6: Find the $m\angle 5$.

$\angle 5$ and $\angle BCE$ are a linear pair because they form a straight line.

$m\angle 5 + m\angle BCE = 180°$	Linear pairs are supplementary. (Theorem 7-B)
$m\angle BCE = 48°$	Given
$m\angle 5 + 48° = 180°$	Substitution
$m\angle 5 = 132°$	Subtraction

corollary – A corollary is a theorem that can be easily proved by means of a closely related theorem.

We will now look at some theorems that are closely related to the Angle Sum Theorem (10-A); thus, instead of referring to them as theorems, we will name them corollaries.

Corollary 10-A-1

The acute angles of a right triangle are complementary.

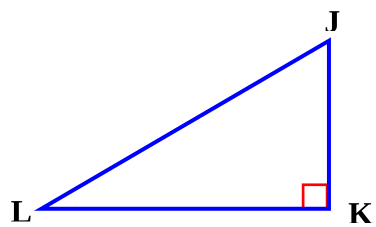
Corollary 10-A-2

There can be at most one right angle in triangle.

Given: $\triangle JKL$

$\angle K$ is a right angle

Prove: $\angle J$ and $\angle L$ are acute angles



Statements	Reasons
$\angle K$ is a right angle.	Given
$m\angle K = 90$	Definition of a right angle.
$m\angle K + m\angle L + m\angle J = 180$	Angle Sum Theorem (Theorem 10-A)
$90 + m\angle L + m\angle J = 180$	Substitution Property
$m\angle L + m\angle J = 90$	Subtraction Property
$m\angle L = 90 - m\angle J$	Subtraction Property
$m\angle J = 90 - m\angle L$	Subtraction Property
$\angle$L and $\angle$J are acute angles.	Definition of acute angles

Corollary 10-A-3

There can be at most one obtuse angle in triangle.

Corollary 10-A-4

The measure of each angle in an equiangular triangle is 60.

Congruent Triangles

We will now examine congruent triangles through transformations. The first transformation is a slide.

Slide

$\triangle ABC \cong \triangle XYZ$

Slide $\triangle ABC$ to the right and down until it fits exactly on top of $\triangle XYZ$. These two triangles are indeed congruent.

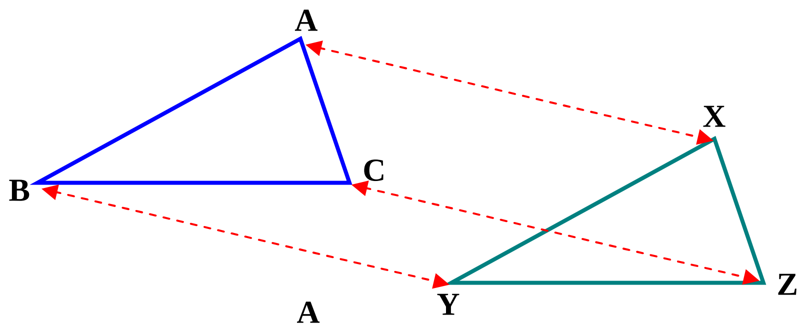
When two geometric figures are congruent, they have congruent corresponding parts.

$\angle$A corresponds to $\angle$X. The symbol $\leftrightarrow$ means "corresponds to".

$\angle A \leftrightarrow \angle X \qquad \therefore \angle A \cong \angle X$

$\angle C \leftrightarrow \angle Z \qquad \therefore \angle C \cong \angle Z$

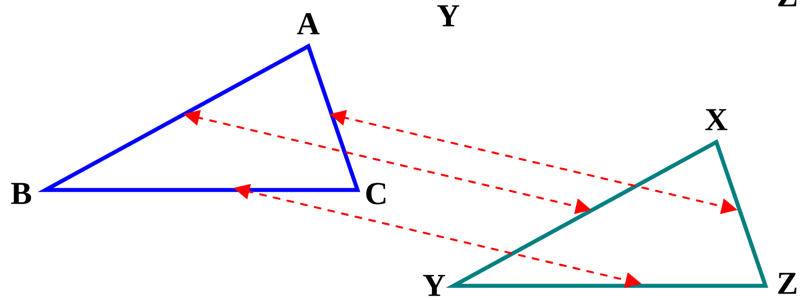
$\angle B \leftrightarrow \angle Y \qquad \therefore \angle B \cong \angle Y$



$\overline{AC} \leftrightarrow \overline{XZ} \qquad \therefore \overline{AC} \cong \overline{XZ}$

$\overline{BC} \leftrightarrow \overline{YZ} \qquad \therefore \overline{BC} \cong \overline{YZ}$

$\overline{AB} \leftrightarrow \overline{XY} \qquad \therefore \overline{AB} \cong \overline{XY}$



Definition of Congruent Triangles (CPCTC)

Two triangles are congruent if and only if their corresponding parts are congruent.

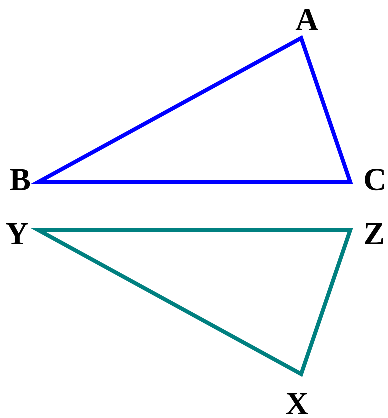
This definition is one that you will use many times in proofs about triangles. Be sure to memorize it!

Corresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent!

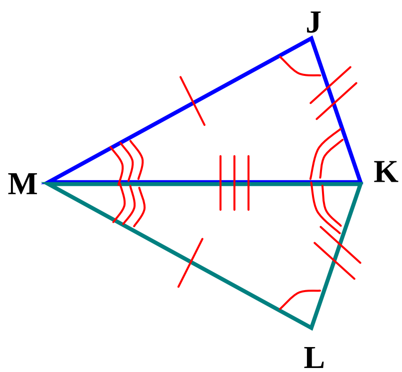
Flip (Reflection)

These figures show congruent triangles that are a reflection of each other.

$\triangle ABC \cong \triangle XYZ$



$\triangle JKM \cong \triangle LKM$



$\angle J \leftrightarrow \angle L$ $\quad \therefore \angle J \cong \angle L$

$\angle JKM \leftrightarrow \angle LKM$ $\quad \therefore \angle JKM \cong \angle LKM$

$\angle JMK \leftrightarrow \angle LMK$ $\quad \therefore \angle JMK \cong \angle LMK$

$\overline{JK} \leftrightarrow \overline{LK}$ $\quad \therefore \overline{JK} \cong \overline{LK}$

$\overline{JM} \leftrightarrow \overline{LM}$ $\quad \therefore \overline{JM} \cong \overline{LM}$

$\overline{MK} \leftrightarrow \overline{MK}$ $\quad \therefore \overline{MK} \cong \overline{MK}$

*Note: In this figure, $\overline{MK}$ corresponds to itself.

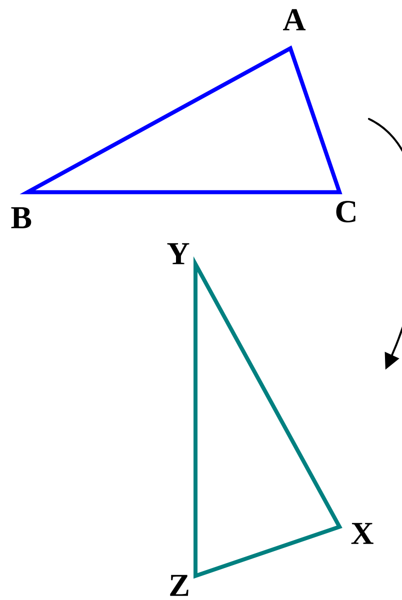
Postulate 10-A

Any segment or angle is congruent to itself. (Reflexive Property)

Rotation

These figures show congruent triangles that are rotated. The second figure is a 90 degree rotation of the first figure.

$\triangle ABC \cong \triangle XYZ$



$\angle A \leftrightarrow \angle X \quad \therefore \angle A \cong \angle X$

$\angle C \leftrightarrow \angle Z \quad \therefore \angle C \cong \angle Z$

$\angle B \leftrightarrow \angle Y \quad \therefore \angle B \cong \angle Y$

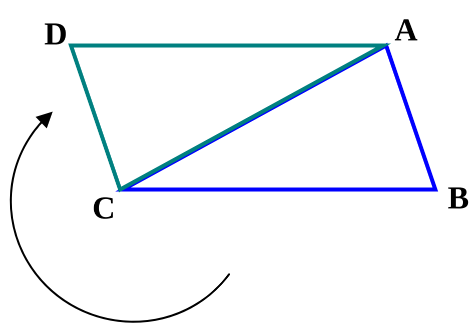
$\overline{AC} \leftrightarrow \overline{XZ} \quad \therefore \overline{AC} \cong \overline{XZ}$

$\overline{BC} \leftrightarrow \overline{YZ} \quad \therefore \overline{BC} \cong \overline{YZ}$

$\overline{AB} \leftrightarrow \overline{XY} \quad \therefore \overline{AB} \cong \overline{XY}$

In this figure, $\triangle ACD$ is a 180 degree rotation of $\triangle ACB$ and vice versa.

$\triangle ABC \cong \triangle ADC$



Example 1: Complete each correspondence and congruence statement.

$\angle BAC \leftrightarrow$ ____	$\angle BAC \cong$ ____
$\angle B \leftrightarrow$ ____	$\angle B \cong$ ____
$\angle BCA \leftrightarrow$ ____	$\angle BCA \cong$ ____
$\overline{AB} \leftrightarrow$ ____	$\overline{AB} \cong$ ____
$\overline{BC} \leftrightarrow$ ____	$\overline{BC} \cong$ ____
$\overline{AC} \leftrightarrow$ ____	$\overline{AC} \cong$ ____

Solution:

$\angle BAC \leftrightarrow \angle ACD$	$\angle BAC \cong \angle ACD$	$\overline{AB} \leftrightarrow \overline{CD}$	$\overline{AB} \cong \overline{CD}$
$\angle B \leftrightarrow \angle D$	$\angle B \cong \angle D$	$\overline{BC} \leftrightarrow \overline{DA}$	$\overline{BC} \cong \overline{DA}$
$\angle BCA \leftrightarrow \angle DAC$	$\angle BCA \cong \angle DAC$	$\overline{AC} \leftrightarrow \overline{AC}$	$\overline{AC} \cong \overline{AC}$

Example 2: Which congruence statement in the previous example is an example of Postulate 10-A? Please explain.

Solution: $\overline{AC} \cong \overline{AC}$

Explanation: In the diagram, $\overline{AC}$ is a part of both triangles; thus, it is congruent to itself. This is an example of the “reflexive” property of congruent segments, Postulate 10-A.