

## Theorems and Postulates

### Postulate 1-A Protractor Postulate

Given  $\overline{AB}$  and a number  $r$  between 0 and 180, there is exactly one ray with endpoint  $A$ , extending on either side of  $\overline{AB}$ , such that the measure of the angle formed is  $r$ .

### Definition of Right, Acute and Obtuse Angles

$\angle A$  is a right angle if  $m\angle A$  is 90.

$\angle A$  is an acute angle if  $m\angle A$  is less than 90.

$\angle A$  is an obtuse angle if  $m\angle A$  is greater than 90 and less than 180.

### Postulate 1-B Angle Addition

If  $R$  is in the interior of  $\angle PQS$ , then  $m\angle PQR + m\angle RQS = m\angle PQS$ .

If  $m\angle PQR + m\angle RQS = m\angle PQS$ , then  $R$  is in the interior of  $\angle PQS$ .

Vertical angles are congruent.

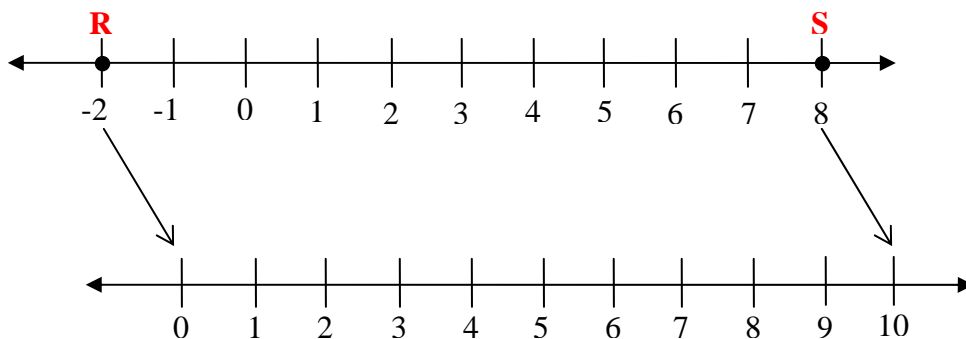
The sum of the measures of the angles in a linear pair is  $180^\circ$ .

The sum of the measures of complementary angles is  $90^\circ$ .

### Postulate 2-A Ruler

Two points on a line can be paired with real numbers so that, given any two points  $R$  and  $S$  on the line,  $R$  corresponds to zero, and  $S$  corresponds to a positive number.

Point  $R$  could be paired with 0, and  $S$  could be paired with 10.



### Postulate 2-B Segment Addition

If  $N$  is between  $M$  and  $P$ , then  $MN + NP = MP$ .

Conversely, if  $MN + NP = MP$ , then  $N$  is between  $M$  and  $P$ .

**Theorem 2-A  
Pythagorean  
Theorem**

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

**Distance Formula**

The distance  $d$  between any two points with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} .$$

**Midpoint  
Definition**

The midpoint,  $M$ , of  $\overline{AB}$  is the point between  $A$  and  $B$  such that  $AM = MB$ .

**Midpoint Formula  
Number Line**

With endpoints of  $A$  and  $B$  on a number line, the midpoint of  $\overline{AB}$  is  $\frac{A+B}{2}$ .

**Midpoint Formula  
Coordinate Plane**

In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$  are  $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ .

**Theorem 2-B  
Midpoint Theorem**

If  $M$  is the midpoint of  $\overline{PQ}$ , then  $\overline{PM} \cong \overline{MQ}$ .

**Postulate 3-A  
Law of  
Detachment**

If  $p \Rightarrow q$  is true, and  $p$  is true, then  $q$  is true.

**Postulate 3-B  
Law of Syllogism**

If  $p \Rightarrow q$  is true and  $q \Rightarrow r$  is true, then  $p \Rightarrow r$  is true.

**Postulate 4-A  
Reflexive  
Property**

Any segment or angle is congruent to itself.

$$\overline{QS} \cong \overline{QS}$$

**Postulate 4-B  
Symmetric  
Property**

If  $\overline{AB} \cong \overline{CD}$ , then  $\overline{CD} \cong \overline{AB}$ .

If  $\angle CAB \cong \angle DOE$ , then  $\angle DOE \cong \angle CAB$ .

**Theorem 4-A  
Transitive  
Property**

**If any segments or angles are congruent to the same angle, then they are congruent to each other.**

**If  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , then  $\overline{AB} \cong \overline{EF}$ .  
If  $\angle 1 \cong \angle 2$  and  $\angle 2 \cong \angle 3$ , then  $\angle 1 \cong \angle 3$ .**

**Theorem 4-B  
Transitive  
Property**

**If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 4-A.)**

**Theorem 5-A  
Addition  
Property**

**If a segment is added to two congruent segments, then the sums are congruent.**

**Theorem 5-B  
Addition  
Property**

**If an angle is added to two congruent angles, then the sums are congruent.**

**Theorem 5-C  
Addition  
Property**

**If congruent segments are added to congruent segments, then the sums are congruent.**

**Theorem 5-D  
Addition  
Property**

**If congruent angles are added to congruent angles, then the sums are congruent.**

**Theorem 5-E  
Subtraction  
Property**

**If a segment is subtracted from congruent segments, then the differences are congruent.**

**Theorem 5-F  
Subtraction  
Property**

**If an angle is subtracted from congruent angles, then the differences are congruent.**

**Theorem 5-G  
Subtraction  
Property**

**If congruent segments are subtracted from congruent segments, then the differences are congruent.**

**Theorem 5-H  
Subtraction  
Property**

**If congruent angles are subtracted from congruent angles, then the differences are congruent.**

**Theorem 5-I  
Multiplication  
Property**

**If segments are congruent, then their like multiples are congruent.**

**Theorem 5-J  
Multiplication  
Property**

**If angles are congruent, then their like multiples are congruent.**

**Theorem 5-K  
Division  
Property**

**If segments are congruent, then their like divisions are congruent.**

**Theorem 5-L  
Division  
Property**

**If angles are congruent, then their like divisions are congruent.**

**Theorem 7-A**

**Congruence of angles is reflexive, symmetric, and transitive.**

**Theorem 7-B**

**If two angles form a linear pair, then they are supplementary angles.**

**Theorem 7-C**

**Angles supplementary to the same angle are congruent.**

**Theorem 7-D**

**Angles supplementary to congruent angles are congruent.**

**Theorem 7-E**

**Angles complementary to the same angle are congruent.**

**Theorem 7-F**

**Angles complementary to congruent angles are congruent.**

**Theorem 7-G**

**Right angles are congruent.**

**Theorem 7-H**

**Vertical angles are congruent.**

**Theorem 7-I**

**Perpendicular lines intersect to form right angles.**

**Postulate 7-A**

**If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.**

**Theorem 7-J**

**If two parallel lines are cut by a transversal, then each of the pair of alternate interior angles is congruent.**

**Theorem 7-K**

**If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.**

**Theorem 7-L**

**If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.**

### **Theorem 7-M**

**If two parallel lines are cut by a transversal that is perpendicular to one of the parallel lines, then the transversal is perpendicular to the other parallel line.**

The definition of slope states that, given two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , the slope of a line containing the points can be determined using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ when } x_2 - x_1 \neq 0$$

### **Postulate 8-A**

**Two non-vertical lines have the same slope if and only if they are parallel.**

### **Postulate 8-B**

**Two non-vertical lines are perpendicular if and only if the product of their slopes is  $-1$ .**

### **Postulate 8-C**

**If two lines in a plane are cut by a transversal and the corresponding angles are congruent, then the lines are parallel.**

### **Postulate 8-D**

**If there is a line and a point that is not on the line, then there exists exactly one line that passes through the point that is parallel to the given line.**

### **Theorem 8-A**

**If two lines in a plane are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.**

### **Theorem 8-B**

**If two lines in a plane are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.**

### **Theorem 8-C**

**If two lines in a plane are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.**

### **Theorem 8-D**

**If two lines in a plane are perpendicular to the same line, then the lines are parallel.**

**The distance from a point, which is not on a line, and a line is the length of a line segment that is perpendicular from the point to the line.**

**The distance between two parallel lines is the distance between one line and any point on the other line.**