PROPERTIES OF EQUALITY

In Geometry deductive reasoning is used to prove conjectures and theorems. In this unit you will begin to examine a logical procedure for verifying geometric relationships. You will start with reviewing the “properties of equality” for real numbers, and then relate them to geometric measures. You will apply the reflexive, symmetric, transitive, and substitution properties to provide reasons for geometric statements and to write informal proofs.

Real Number Properties of Equality – Chart

Reflexive and Symmetric Properties

Transitive and Substitution Properties

Informal Proof

Real Number Properties of Equality

In the table below, each of the properties of equality for real numbers are listed with a general explanation of each property. The properties are used to verify steps in a proof.

Properties of Equality for Real Numbers	
Reflexive Property	For every number a, $a = a$.
Symmetric Property	For all numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For all numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	For all numbers a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	For all numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$, and if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	For all numbers a and b, if $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	For all numbers a, b, and c, $a(b + c) = ab + ac$.

Measures of segments and angles are real numbers; thus, the properties of equality may be used to show many relationships in geometry.

Examples: Each of the statements below is supported by a property of equality. Read each statement closely, and then identify the property of equality that is illustrated by the statement.

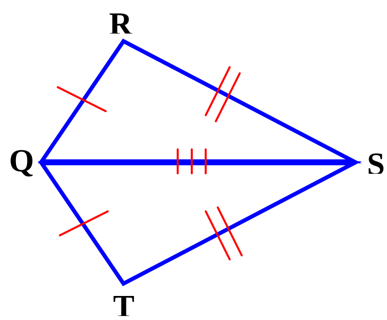
Statement	**Property of Equality**
(1) If $AB = CD$ and $AB = EF$, then $CD = EF$.	Substitution
(2) If $8x = 23$, then $x = \frac{23}{8}$.	Division
(3) $m\angle QTR = m\angle QTR$	Reflexive
(4) If $DE = FG$ and $FG = HI$, then $DE = HI$.	Transitive
(5) If $QR + RS = TU + RS$, then $QR = TU$.	Subtraction
(7) If $m\angle QTR = m\angle WXY$, then $m\angle WXY = m\angle QTR$.	Symmetric
(8) $9(y + 4) = 9y + 36$	Distributive

Reflexive and Symmetric Properties

Postulate 4-A Reflexive Property	**Any segment or angle is congruent to itself.** $\overline{QS} \cong \overline{QS}$

Reflexive Property for Segments

In the figure, $\overline{QS}$ is shared by $\triangle QRS$ and $\triangle QTS$. In proofs which involve geometric figures such as this one, the reflexive property is used to illustrate that a segment is congruent to itself.



The red hash marks mean congruency.

$\overline{QR}$ is congruent to $\overline{QT}$, denoted by one hash mark on each segment.

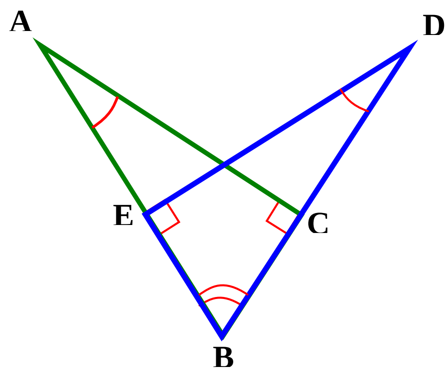
$\overline{RS}$ is congruent to $\overline{TS}$, denoted by two hash marks on each congruent segment.

$\overline{QS}$ is congruent to $\overline{QS}$, itself, denoted by three hash marks on that segment.

*Note: $\overline{QS}$ is shared by $\triangle QRS$ and $\triangle QTS$

Reflexive Property for Angles

In the figure, $\angle ABD$ is shared by $\triangle ABC$ and $\triangle DBE$. In proofs which involve geometric figures such as this one, we will use the reflexive property to illustrate that an angle is congruent to itself.



The curved red hash marks mean congruency. The corner notch (appears to be square in shape) is a mark for denoting right angles.

$\angle ACB$ and $\angle DEB$ are right angles and each are marked with the corner notch for right angles. Thus, they are congruent.

$\angle A \cong \angle D$. These congruent angles are denoted by one curved hash mark on each angle.

$\angle B$ is congruent to $\angle B$, itself, $(\angle B \cong \angle B)$ and marked with two curved hash marks. Note: angle B is shared by both $\triangle ABC$ and $\triangle DBE$.

Property of Symmetry

Postulate 4-B
Symmetric Property

If $\overline{AB} \cong \overline{CD}$, **then** $\overline{CD} \cong \overline{AB}$.
If $\angle CAB \cong \angle DOE$, **then** $\angle DOE \cong \angle CAB$.

Transitive and Substitution Properties

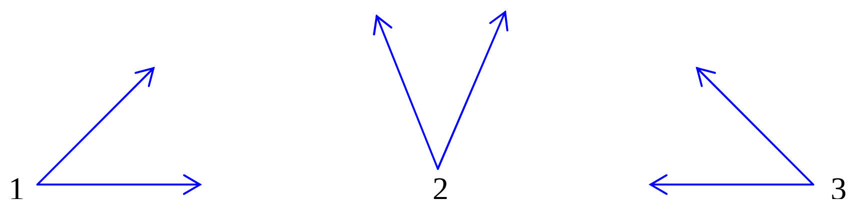
Transitive Property

Theorem 4-A Transitive Property

If any segments or angles are congruent to the same angle, then they are congruent to each other.

If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

Transitive Property for Angles



Given: $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$

$\therefore \quad \angle 1 \cong \angle 3$

*Three dots made in the shape of a triangle are used as a symbol to represent the word "therefore".

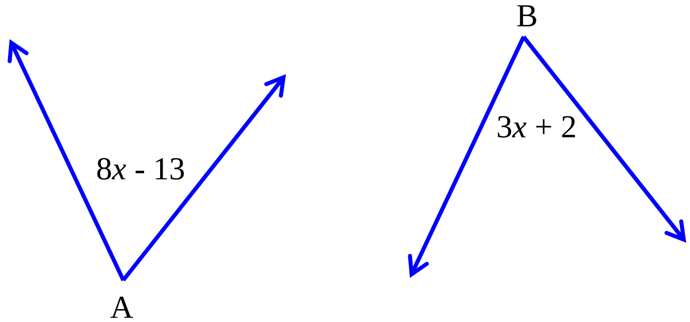
Theorem 4-B Transitive Property

If any segments or angles are congruent to each other, then they are congruent to the same angle. (This statement is the converse of Theorem 4-A.)

Substitution Property

In algebra, equivalent values or expressions may be substituted for each other. The same applies in geometry. Angles or segments that are congruent may be substituted for each other.

Example: Given: $\angle A \cong \angle B$. Find the measurement of each angle.



Congruent angles have equal measures by definition of congruency. Therefore, the expression for $m\angle A$ can be set equal to the expression for the $m\angle B$.

$$
\begin{aligned}
8x - 13 &= 3x + 2 \\
+13 &\quad\ +13 \\
8x &= 3x + 15 \\
-3x &\quad -3x \\
5x &= 15 \\
x &= 3
\end{aligned}
$$

By the substitution property, 3 can be substituted in for x to find the measure of each angle. It is a good idea to substitute the value for x into both expressions even though the measures of the angles are given as equal. Use the expression for $\angle B$ as a check to verify that both measurements are the same.

$$
\begin{aligned}
\angle A &= 8x - 13 & \qquad \angle B &= 3x + 2 \\
\angle A &= 8(3) - 13 & \qquad \angle B &= 3(3) + 2 \\
\angle A &= 11 & \qquad \angle B &= 11
\end{aligned}
$$

The measure of angle B verifies the measure of angle A.

Each angle measures 11°.

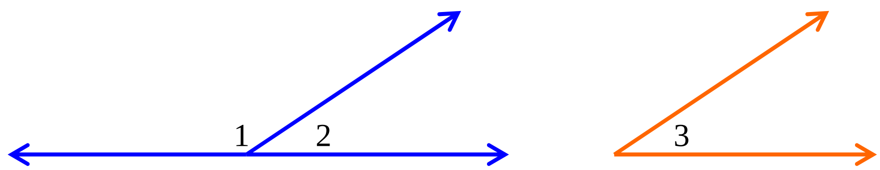
Informal Proof

Informal Proof (paragraph proof) – An informal proof is a paragraph of statements that explain why a conjecture is true.

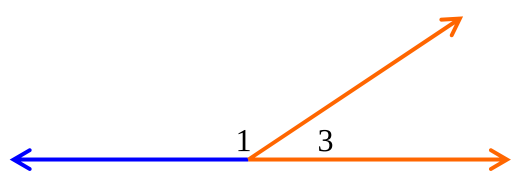
Example 1: Write a paragraph proof to support the conclusion based on the given information.

Given: $\angle 1$ and $\angle 2$ are supplementary; $\angle 2 \cong \angle 3$

Conclusion: $\angle 1$ and $\angle 3$ are supplementary.



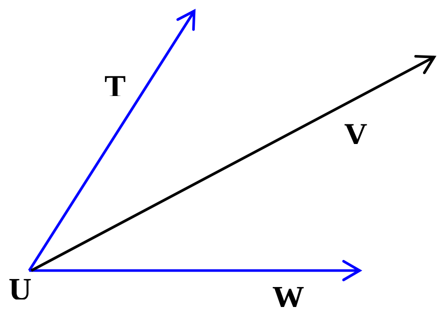
Using the substitution property, $\angle 3$ may be substituted for $\angle 2$ since congruent angles have the same measure. Therefore, $\angle 1$ and $\angle 3$ are supplementary as illustrated below.



Example 2: Write an informal proof for the conjecture.

Given: $\overrightarrow{UV}$ is an angle bisector of $\angle TUW$.

Conjecture: $m\angle TUV = m\angle VUW$



Informal Proof: By definition of angle bisector, $\angle TUV \cong \angle VUW$. Congruent angles, by definition, have equal measures. Therefore, $m\angle TUV = m\angle VUW$.