LOGIC

Geometry is an orderly approach to solving problems. In this unit you will examine inductive reasoning, statements of logic, and deductive reasoning. Through these formal approaches to making logical decisions, you will begin to understand that mathematical conjectures are based on observations and mathematical proofs are based on detailed statements of logic.

Inductive Reasoning

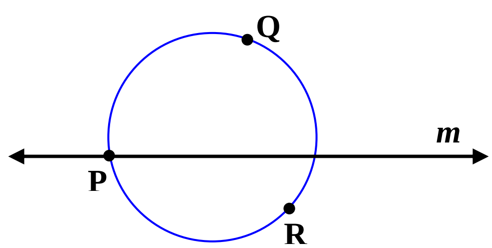
Inductive Reasoning – Inductive reasoning is logic that is used to make generalizations based on observation of specific cases and consideration of patterns. Conclusions are based on specific examples of information gathered in order to make an observation about what must be true. Inductive reasoning is not designed to prove mathematical certainty.

Conjecture – A conjecture is a mathematical statement based on intuitive premise, but not necessarily true until proven.

Counterexample – A counterexample is a false example that proves a conjecture is false. It only takes **one** counterexample to **disprove** a conjecture.

Example 1: By observing the given information and the corresponding figure, make a conjecture.

Given: Point P lies on line *m*, points Q and R are not on line *m*.



Conjecture: Points P, Q and R are non-collinear.

Some conjectures may be incorrect. It only takes one counterexample to disprove a conjecture. Let's take a look at an example.

Example 2: Give a counterexample to disprove the conjecture.

Given: n is an integer
Conjecture: $-n$ is a negative number

Counterexample: If $n = -4$ (an integer), then $-n = -(-4)$ which is 4, a positive number. The conjecture "$-n$ is a negative number" is **false**.

Statements of Logic

Conditional statements – Conditional statements are another name for **If-then** statements of logic. The "if" part is the hypothesis and the "then" part is the conclusion.

Hypothesis – The hypothesis of a conditional statement is the "if" part.

Conclusion – The conclusion of a conditional statement is the "then" part.

We can use *p* to represent the hypothesis and *q* to represent the conclusion.

"If *p*, then *q*" is symbolized as $p \Rightarrow q$ and can also be read as "*p* implies *q*".

Converse – The converse of an if-then statement is formed by interchanging the hypothesis and the conclusion, written as "if *q* then *p*".

The converse is symbolized as $q \Rightarrow p$.

Negation – Negation is a statement of what a thing is not, written as "not *p*".

Negation is symbolized as $\sim p$.

Inverse – The inverse is a denial of a statement, written as "if not *p*, then not *q*".

An inverse statement is symbolized as $\sim p \Rightarrow \sim q$.

Contrapositive – The contrapositive is the negation of the converse, written as "if not *q* then not *p*".

The contrapositive is symbolized as $\sim q \Rightarrow \sim p$.

Now, with all of these new definitions, let's look at examples to help clarify each of the various statements of logic.

Example 1: Write the converse, inverse, and contrapositive of the given statement, and then decide if each new statement is true.

Given: "If it rains, then the open field is wet."

p – It is raining. ~*p* – It is **not** raining.

q – The open field is wet. ~*q* – The open field is **not** wet.

Converse $(q \Rightarrow p)$: If the open field is wet, then it is raining.

Is this true? **Not necessarily!** Could the open field be wet and it not be raining? Yes! There are other ways to make the field wet (dew, flood, etc.)

Inverse $(\sim p \Rightarrow \sim q)$: If it is not raining, then the open field is not wet.

Is this true? **Not necessarily**! If it is not raining, then the open field will not be wet from the absence of rain; but, it could be wet caused by other sources of moisture.

Contrapositive $(\sim q \Rightarrow \sim p)$: If the open field is not wet, then it is not raining.

Is this true? **Most likely!** The open field would not be dry if it was raining.

*Remember: One counterexample that shows the statement is false is all that is needed to determine that the statement is false.

Now, let's look at a more concrete example.

Example 2: Write the converse, inverse, and contrapositive of the given statement, and then decide if each new statement is true.

Given: "If *x* = 3, then *x* > 2."

p: $x = 3$ ~*p*: $x \neq 3$

q: $x > 2$ ~*q*: $x \leq 2$

Converse $(q \Rightarrow p)$: If $x > 2$, then $x = 3$.

Is this true? **No!** Besides 3 being greater than 2, there is an infinite amount of numbers greater than 2.

Inverse $(\sim p \Rightarrow \sim q)$: If $x \neq 3$, then $x \leq 2$.

Is this true? **No**! Even though all numbers less than or equal to 2 are not equal to 3, other numbers are also not equal to 3 like all the numbers greater than 3 and all the numbers between 2 and 3.

Contrapositive $(\sim q \Rightarrow \sim p)$: If $x \leq 2$, then $x \neq 3$.

Is this true? **Yes**! Since x is "numbers less than or equal to 2", then x cannot equal 3.

*Remember: One counterexample is all that is needed to determine that the statement is false.

Deductive Reasoning

Deductive Reasoning – Deductive reasoning is logic that is used to arrive at a conclusion from a given premise. Conclusions are based on a system of particular statements assumed to be true. Deductive reasoning is used to prove mathematical statements in geometry.

In Geometry we will learn to prove mathematical statements. We will approach the proofs with deductive reasoning. In this type of mathematical system, we will have **undefined terms, postulates (assumptions), definitions, and theorems.**

Undefined Term – Undefined terms are terms such as point and line. In an earlier unit, we described a point as a location on a line with no dimensions but represented by a dot. We described a line as a straight length that extends indefinitely into space, but has no width or thickness. The descriptions are not definitions of a point and a line. The terms point and line are considered undefined terms because we have a clear idea of what they are but no concrete definition.

Postulate (assumption) A postulate is a mathematical statement that is accepted as true but remains unproven. Postulates are based on the fundamentals of mathematical belief. Example: In a plane, the shortest distance between two points is a straight line.

Definition – A definition states the meaning of a term or idea. Example: "Right angles are angles that measure 90˚." Definitions are reversible. The reverse of this definition is "Angles that measure 90˚ are right angles." The reverse is still true.

Theorem - A theorem is a mathematical statement that can be proven to be true. Not all theorems are reversible. Example: If two angles measure 45˚, they are congruent. The reverse is not necessarily true. If two angles are congruent, they measure 45˚. This statement is not always true, in fact, it is false more often than true.

Postulate 3-A **Law of Detachment**	**If** $p \Rightarrow q$ **is true, and** p **is true, then** q **is true.**
Postulate 3-B **Law of Syllogism**	**If** $p \Rightarrow q$ **is true and** $q \Rightarrow r$ **is true, then** $p \Rightarrow r$ **is true.**

Example 1: Use the law of syllogism to write a true conclusion.

Given these true statements:

a) If Samantha has a job, then she will earn income. $(p \Rightarrow q)$

b) If Samantha earns income, then she will pay income tax. $(q \Rightarrow r)$

p – Samantha has a job.

q – Samantha will earn income.

r – Samantha will pay income tax.

Therefore, $p \Rightarrow r$, the conclusion is:

"If Samantha has a job, then she will pay income tax."

Example 2: Use the law of syllogism to write a true conclusion.

Given these true statements:

a) If the slope of a line is positive, then $m \neq 0$. $(p \Rightarrow q)$

b) If $m \neq 0$, then the line is not a horizontal line. $(q \Rightarrow r)$

p – The slope of a line is positive.

q – $m \neq 0$

r – The line is not a horizontal line.

Therefore, $p \Rightarrow r$, the conclusion is:

"If the slope of a line is positive, then the line is not a horizontal line."