LINE SEGMENTS AND COORDINATE GEOMETRY

In this unit you will review plotting points and lines in the coordinate plane. Next you will examine the ruler postulate, measuring line segments, and the segment addition postulate. You will also review the Pythagorean Theorem and apply it to the distance formula. The final topic in this unit is finding the midpoint of two points and finding the distance between them.

Graphing Points in Quadrant I

Graphing Points in Quadrants II, III, and IV

Ruler Postulate and Segment Addition

Pythagorean Theorem and Distance Formula

Midpoint Formula

Graphing in Quadrant I of the Coordinate Plane

Ordered pair - An ordered pair is a pair of numbers that represent the location in a grid. In the coordinate plane, an ordered pair is the *x* and *y*-coordinate of a point represented as (*x*, *y*).

Origin - The origin is the beginning point in the coordinate plane. It is the point where the *x*-axis and the *y*-axis intersect. The coordinates of the origin are (0,0).

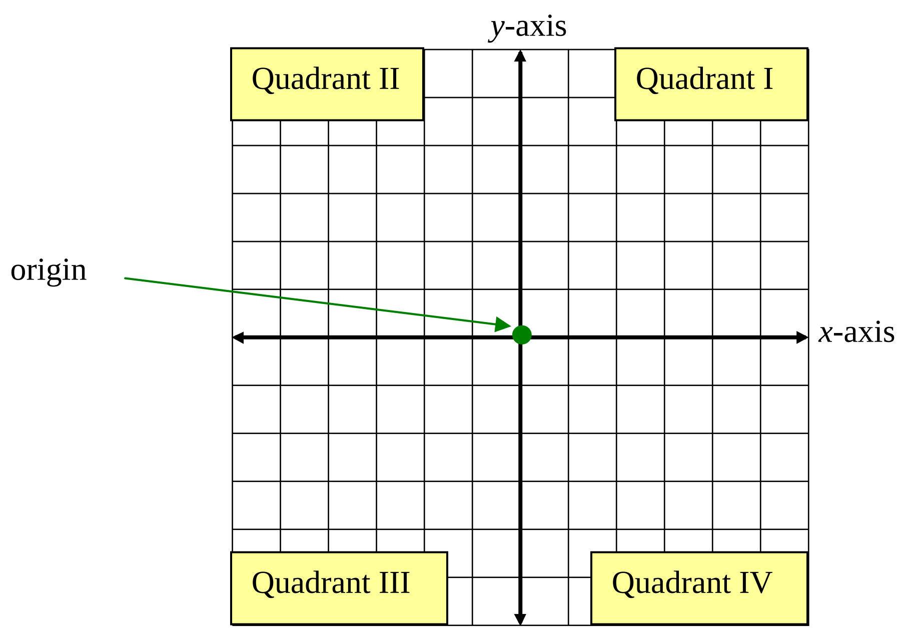
Quadrants - Quadrants are the four regions of the coordinate plane. The *x* and *y*-axis divide the coordinate plane into four quadrants.

Axes - Axes is the plural of axis. There are two axes in a coordinate plane. The *x*-axis is the horizontal axis and is a number line. The *y*-axis is the vertical axis and is a vertical number line.

Coordinates - Coordinates are the components of an ordered pair. In an ordered pair, the first number is called the *x*-coordinate and the second number is called the *y*-coordinate.

Coordinate plane - The coordinate plane is a numbered grid system that has a horizontal and a vertical number line in the center. These lines are perpendicular to each other and meet at the origin, the point considered the starting point of the system. The origin is numbered as (0, 0).

In a **coordinate plane**, points may be located by **plotting** them. The coordinate plane is divided into **four quadrants** by the ***x*–axis** and the ***y*-axis**. The starting point, the **origin**, is the center, or point where the *x* and *y*-axis intersect (cross).



A point is designated by both an ***x*-coordinate** and a ***y*-coordinate.** The origin's coordinates are (0, 0). The *x*-coordinate is the first number and the *y*-coordinate is the second number.

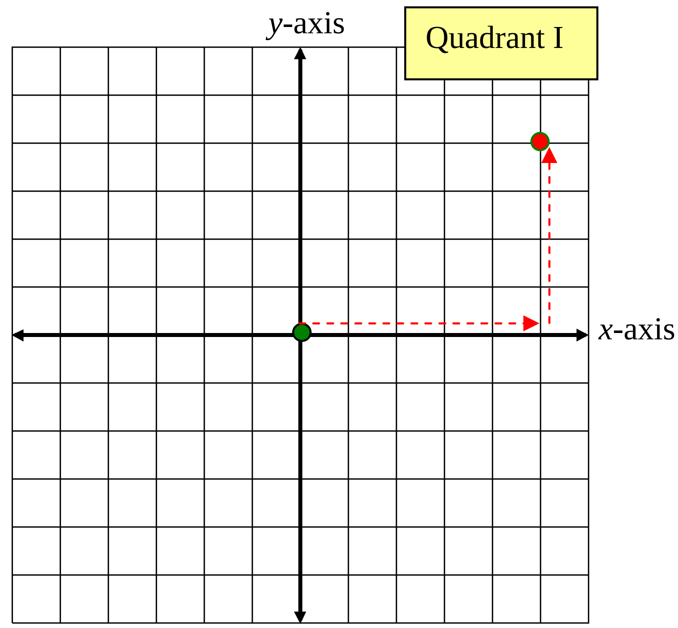
The ***x*-coordinate** is how far you count **right or left** of the origin. The ***y*-coordinate** is how far you then count **up or down**. A point's location is written as an **ordered pair (*x*, *y*)**.

In this grid, each space represents one unit.

Plot (5, 4)

When plotting points, start at the origin. Count right if the *x*-coordinate is positive, left if it is negative. Then count up if the *y*-coordinate is positive, count down if it is negative.

To plot (5,4) start at the origin, count 5 units to the right, and then count 4 units up.



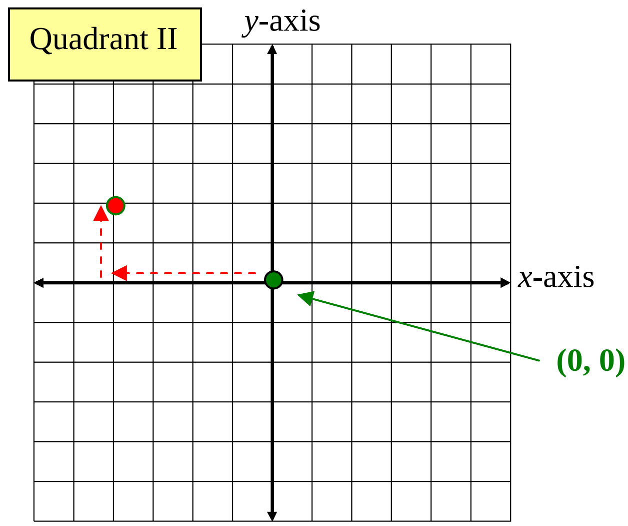
Graphing in Quadrants II, III, and IV

In these grids each space represents one unit. The starting point is the origin (0,0).

Plot (–4, 2)

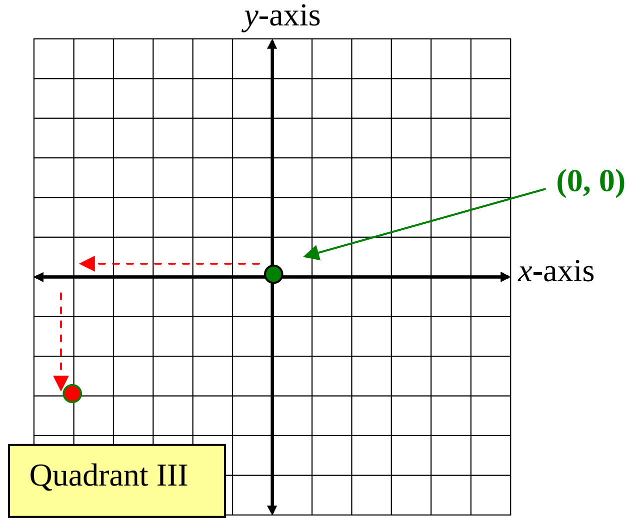
When plotting points, start at the origin. Count right if the *x*-coordinate is positive, left if it is negative. Then count up if the *y*-coordinate is positive, count down if it is negative.

To plot (–4, 2) starting at the origin, count 4 units to the left, and then count 2 units up.



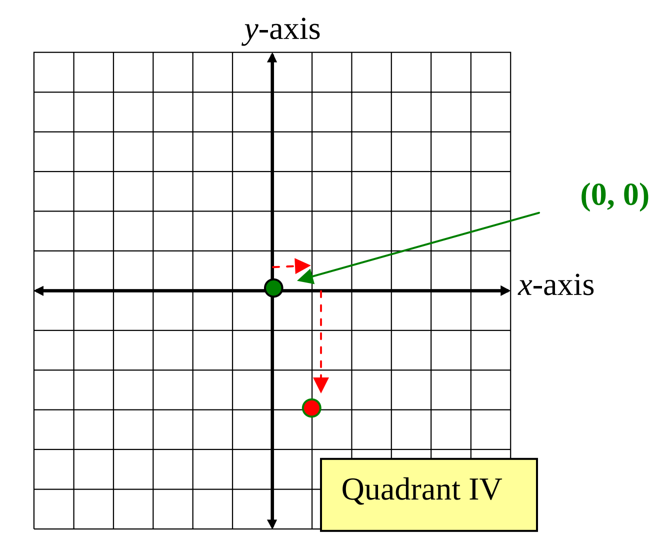
Plot (–5, –3)

To plot (–5, 3), start at the origin, count 5 units to the left, and then count 3 units down.



Plot (1, –3)

To plot (1, –3), start at the origin, count 1 unit to the right, and then count 3 units down.



Ruler Postulate and Segment Addition

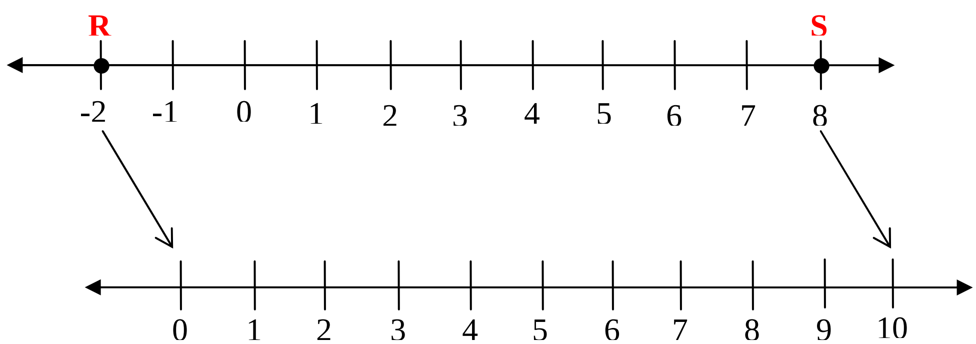
Congruent line segments - Congruent line segments are line segments that measure the same length. The symbol for congruency is $\cong$.

Absolute value - The absolute value of a number is the number's distance from zero on the number line. The symbol for absolute value is | |.

Postulate 2-A Ruler

Two points on a line can be paired with real numbers so that, given any two points R and S on the line, R corresponds to zero, and S corresponds to a positive number.

Point R could be paired with 0, and S could be paired with 10.



Example 1: Find the Distance from **R** to **S**.

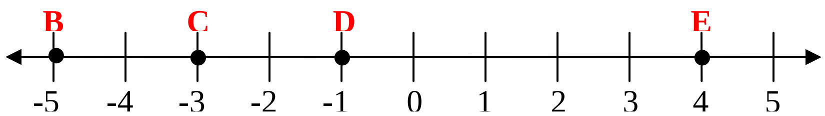
To solve, find the absolute value of the difference between the two points. When finding absolute value, order doesn't matter.

$$|-2-8| = |-10| = 10$$

OR

$$|8-(-2)| = |8+2| = 10$$

Refer to the numberline to answer the following questions.



Example 2: Find the lengths of **CD**, **DE**, and **CE**.

CD = $|-3-(-1)|$

= $|-3+1|$

= $|-2|$ or 2

DE = $|-1-4|$

= $|-5|$ or 5

CE = $|-3-4|$

= $|-7|$ or 7

Example 3: Find two congruent segments.

CD = $|-3-(-1)|$

= $|-3+1|$

= $|-2|$ or 2

BC = $|-5-(-3)|$

= $|-5+3|$

= $|-2|$ or 2

CD $\cong$ **BC**

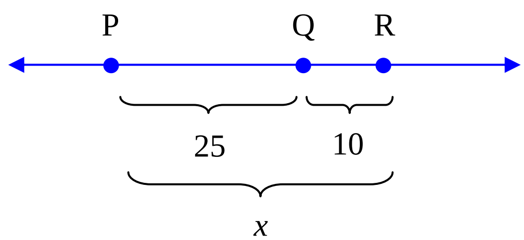
CD is congruent to **BC**.

Postulate 2-B
Segment Addition

If N is between M and P, then MN + NP = MP.
Conversely, if MN + NP = MP, then N is between M and P.

Example 1: P, Q, and R are collinear and Q is between P and R. If segment PQ = 25 inches long and QR = 10 inches long, then how long is PR?

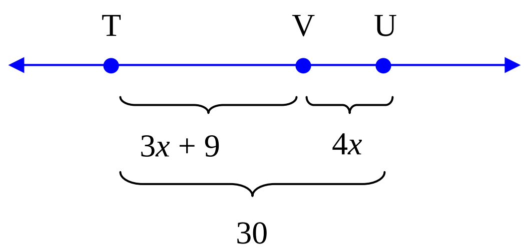
Draw a picture to help visualize the problem.



$$\begin{aligned} PQ + QR &= PR \\ 25 + 10 &= x \\ 35 &= x \\ 35 &= PR \end{aligned}$$

Example 2: T, U, and V are collinear and V is between T and U. The length of TU is 30 inches. If segment TV = 3*x* +9 inches long and VU is 4*x* units long, find the length of VU.

Draw a picture to help visualize the problem.



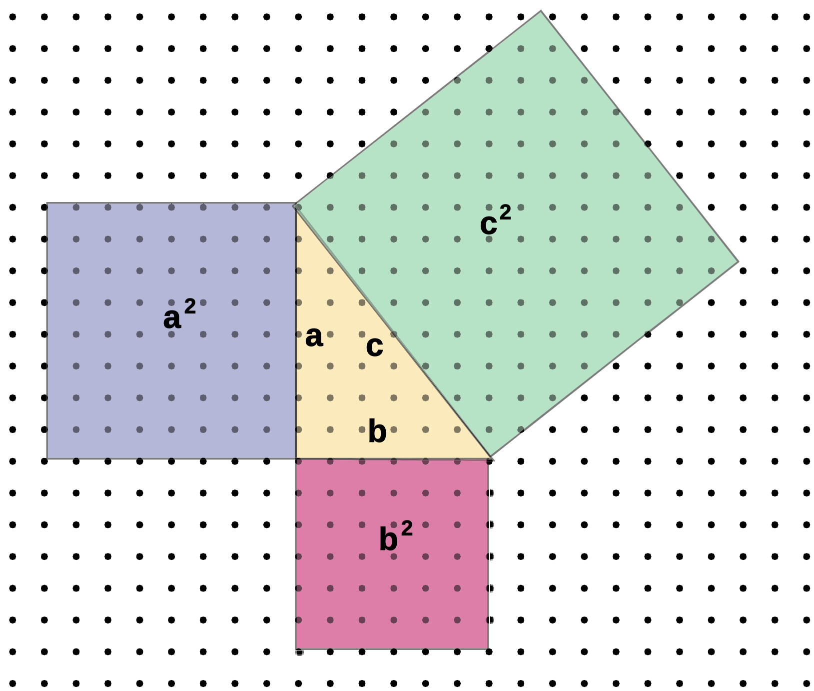
$$\begin{aligned} TV + VU &= TU \\ 3x + 9 + 4x &= 30 \\ 7x + 9 &= 30 \\ 7x &= 21 \\ x &= 3 \end{aligned}$$

Through substitution, VU = 4*x* = 4(3) = 12 inches.

Pythagorean Theorem and Distance Formula

Theorem - A theorem is a mathematical statement that must be proven before it is accepted as being true.

Pythagorean Theorem - The Pythagorean Theorem is a relationship between the three sides of a right triangle. The sum of the squares of the two sides of the right triangle that make up the right angle are equal to the square of the third side, the hypotenuse which is the side opposite the right angle.



Special names are given to the sides of a right triangle. The two sides that make up the right triangle are called “**legs**” and the side opposite the right angle is called the “**hypotenuse**”.

A special relationship exists between the sides of a right triangle. The sum of the squares of the two legs equals the square of the hypotenuse.

$$c^2 = a^2 + b^2$$

In the example above, the legs measure 6 and 8 units. What does the diagonal measure?

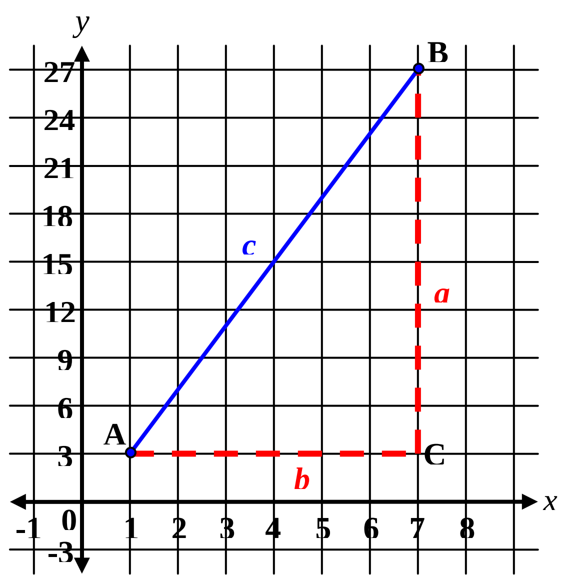
$c^2 = 6^2 + 8^2$
$c^2 = 36 + 64$
$c^2 = 100$
$c = \sqrt{100}$
$c = 10$

Theorem 2-A Pythagorean Theorem

In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Example 1: Use the Pythagorean Theorem to find the distance from A to B.

*Note: Each space on the *x*-axis equals one unit. Each space on the *y*-axis equals 3 units.



Point A (1, 3) Point B (7, 27) Point C (7, 3)

Leg *a*: Find the length of $\overline{BC}$.

Point C (7, 3) to Point B (7, 27)

To find the distance, look at the change in the y-coordinates.

$a = \overline{BC} = |3 - 27| = |-24| = 24$

Leg *b*: Find the length of $\overline{AC}$.

Point A (1, 3) to Point C (7, 3)

To find the distance, look at the change in the *x*-coordinates.

$b = \overline{AC} = |1 - 7| = |-6| = 6$

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
24^2 + 6^2 &= c^2 \\
576 + 36 &= c^2 \\
612 &= c^2 \\
\pm\sqrt{612} &= c \\
24.7 &\approx c
\end{aligned}
$$

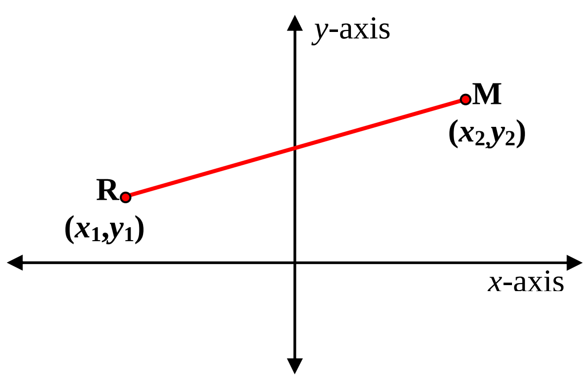
*612 is an irrational number

*612 is an irrational number and its root extends on forever and never develops into a repeating pattern. For this course, round irrational answers as directed. The symbol for approximately equal is $\approx$. In geometry, the negative root is often ignored because the problems are mostly about distance as is true in this problem.

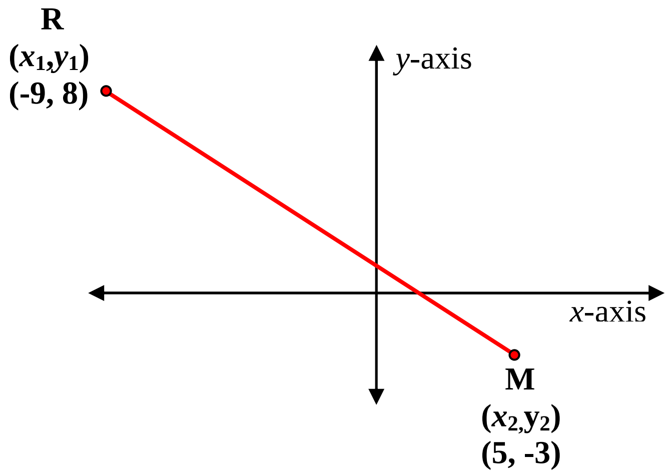
In coordinate geometry, the Pythagorean Theorem can be adapted to the Distance Formula.

Distance Formula

The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **.**



Example 2: Find the length of $\overline{RM}$ for R(–9, 8) and M(5, –3).



Let R be Point 1 and represented by (x_1,y_1). $(x_1,y_1) = (-9, 8)$

Let M be Point 2 and represented by (x_2,y_2). $(x_2,y_2) = (5, -3)$

Thus,

$x_1 = -9$ $y_1 = 8$ $x_2 = 5$ $y_2 = -3$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(5-(-9))^2 + (-3-8)^2}$$
$$d = \sqrt{(14)^2 + (-11)^2}$$
$$d = \sqrt{196+121}$$
$$d = \sqrt{317}$$
$$d \approx 17.8$$

Midpoint Formula

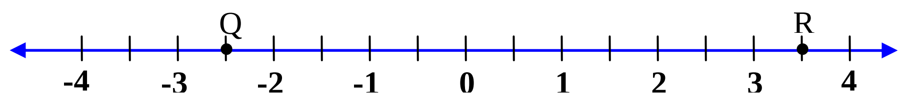
Midpoint – The midpoint of a line segment is the point that is halfway between the two end points.

Midpoint Definition	**The midpoint, M, of $\overline{AB}$ is the point between A and B such that AM = MB.**

To find the midpoint between two points on a number line, find the average distance between the two points.

Midpoint Formula Number Line	**With endpoints of A and B on a number line, the midpoint of $\overline{AB}$ is $\frac{A+B}{2}$.**

Example 1: Find the midpoint of Q and R.



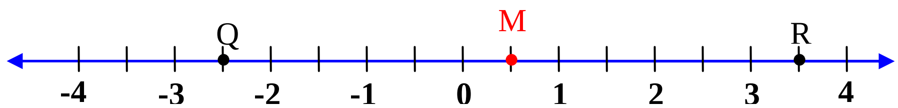
Let A represent point Q at –2.5.

Let B represent point R at 3.5.

A= –2.5

B= 3.5

$$\text{M} = \frac{A+B}{2} = \frac{-2.5+3.5}{2} = \frac{1.0}{2} = 0.5$$

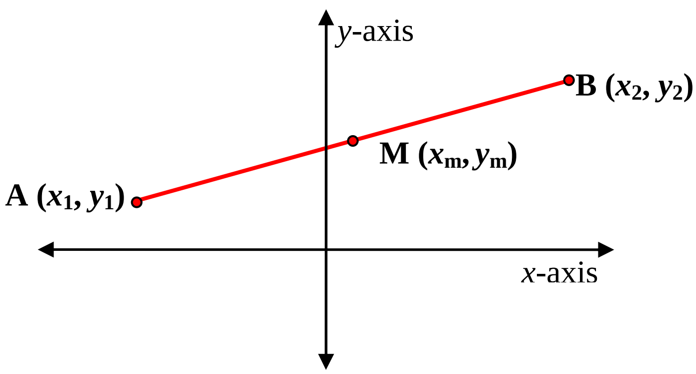


Midpoint Formula Coordinate Plane

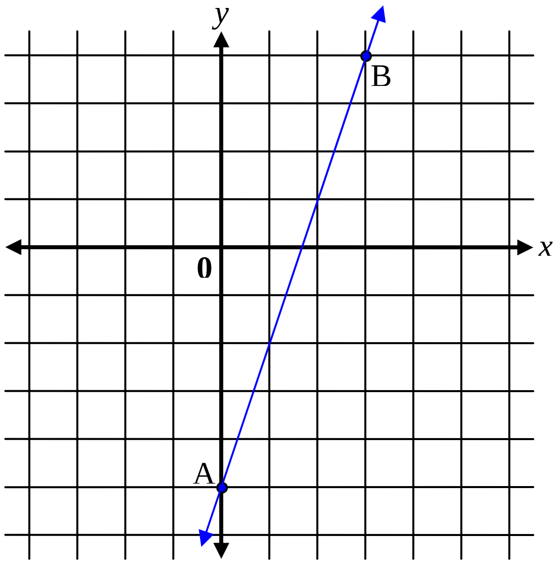
In the coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

To find the midpoint between two points in the coordinate plane, find the average distance between the two points. Study this formula and diagram.

$$\mathbf{M} = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Example 2: Find the midpoint (M) of $\overline{AB}$.



Point A (x_1, y_1) Point B (x_2, y_2)

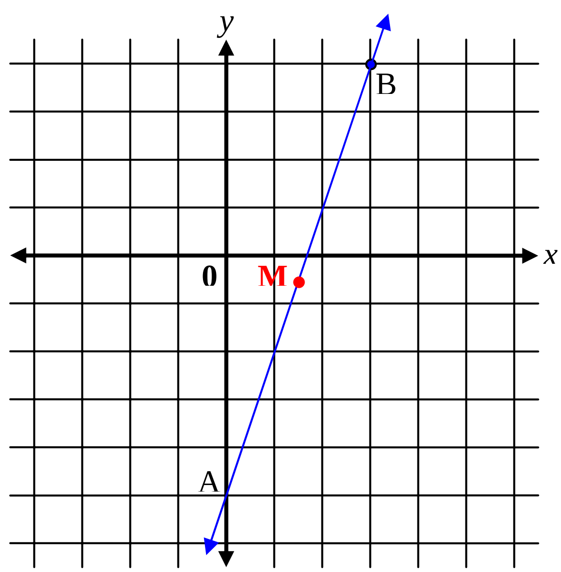
Point A (0, –5) Point B (3, 4)

$$M = (x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = (x_m, y_m) = \left(\frac{0+3}{2}, \frac{-5+4}{2}\right)$$

$$M = (x_m, y_m) = \left(\frac{3}{2}, \frac{-1}{2}\right)$$

$$M = (x_m, y_m) = \left(1\frac{1}{2}, -\frac{1}{2}\right)$$



Theorem 2-B
Midpoint Theorem

If M is the midpoint of $\overline{PQ}$, then $\overline{PM} \cong \overline{MQ}$.