MULTI-STEP EQUATIONS

This unit is about solving equations that involve more than one operation. First, a review will be provided for one-step equations and two-step equations, and then multi-step equations will be examined. To extend the process of solving equations, there may be times when the distributive property is used to eliminate any parentheses. Then, like terms may be combined and the equation can be solved.

Review of Basic Equations

This section of the unit is a review one-step and two-step equations.

Remember, to solve an equation involving more than one operation, perform the order of operations IN REVERSE to solve for the unknown variable.

Example #1: Solve $y - 6 = -21$ for y.

$$\begin{array}{rcl} y - \cancel{6} & = & -21 \\ \underline{+\cancel{6}} & & \underline{+6} \\ y & = & -15 \end{array}$$

Add 6 to both sides of the equation.

Therefore, $y = -15$.

✓Check the answer by replacing y with –15 in the original equation.

$y - 6 = -21$

$-15 - 6 = -21$

$-21 = -21$ *True*✓

Example #2: Solve $\frac{-m}{3}=5$ for *m*.

*Remember, to solve the equation, isolate *m* on one side.

$$\frac{-m}{3}=5$$

$$\frac{-1m}{3}=5 \qquad -m \text{ can be written as } -1m$$

$$-\frac{1}{3}m=5 \qquad \frac{-1m}{3} \text{ can be written as } -\frac{1}{3}m$$

$$-3\left(-\frac{1}{3}\right)m=-3(5) \qquad \text{Multiply both sides by } -3,$$

the reciprocal of $-\frac{1}{3}$.

$$1m=-15$$

$$m=-15 \qquad 1m \text{ is the same as just } m.$$

Therefore, *m* = –15.

✓Check the answer by replacing *m* with –15 in the original equation.

$$\frac{-m}{3}=5$$

$$\frac{-(-15)}{3}=5$$

$$\frac{15}{3}=5$$

$5=5$ *True*✓

The next few examples will involve combining the two processes from above in one equation.

*Remember that to solve an equation, perform the order of operations in REVERSE ORDER, which means:

- Add or subtract first.
- Multiply or divide second.

Example #3: Solve $5x+6=31$ for *x*.

$$\begin{aligned} 5x \cancel{+6} &= 31 \\ \cancel{-6} \quad & -6 \\ \hline 5x \quad &= 25 \\ x \quad &= 5 \end{aligned}$$

Subtract 6 from both sides of the equation.

Divide both sides by 5.

Therefore, $x = 5$.

✓Check the answer by replacing *x* with 5 in the original equation.

$5x+6=31$

$5(5)+6=31$

$25+6=31$

$31=31$ *True*✓

Example #4: Solve for $-6z-18=-132$ for *z*.

$$\begin{aligned} -6z \cancel{-18} &= -132 \\ \cancel{+18} \quad & +18 \\ \hline -6z \quad &= -114 \\ z \quad &= 19 \end{aligned}$$

Add 18 to both sides of the equation.

$(-132+18=-114)$

Divide both sides by -6.

$(-114 \div -6 = +19)$

Therefore, $z = 19$.

✓Check the answer by replacing *z* with 19 in the original equation.

$-6z-18=-132$

$-6(19)-18=-132$

$-114-18=-132$

$-132=-132$ *true*✓

Solving Multi-Step Equations

Now, let's take a look at solving equations with variables on both sides of the equals sign.

Example 1: Solve 8*x* + 5 = 2*x* – 16 for *x*.

Step #1: Move the variables (with coefficients) to one side and the numbers (with no variable "attached") to the other side. Use algebra to justify the adjustments.

$$\begin{array}{l} 8x \; + 5 = \cancel{2x} - 16 \\ -2x \qquad \cancel{-2x} \\ \hline 6x \cancel{+5} = -16 \end{array}$$

Subtract 2x from both sides of the equation.
$(8x - 2x = 6x \qquad 2x - 2x = 0)$

$$\begin{array}{l} \quad \cancel{-5} \quad -5 \\ \hline 6x = -21 \end{array}$$

Subtract 5 from both sides.
$(-16 - 5 = -16 + -5 = -21)$

Step #2: Divide both sides by 6 to solve for the unknown.

$$\frac{\cancel{6}x}{\cancel{6}} = \frac{-21}{6}$$

$$x = -3.5$$

Divide both sides by 6.

$$\frac{-21}{6} = \frac{-7}{2} = -3\frac{1}{2} = -3.5$$

Therefore, *x* = –3.5.

✓Check the answer by replacing *x* with –3.5 in the original equation.

$$8x + 5 = 2x - 16$$

$$8(-3.5) + 5 = 2(-3.5) - 16$$

$$-28 + 5 = -7 - 16$$

$$-23 = -23 \; \textit{true} \checkmark$$

To extend the process of solving equations, there may be times when the distributive property is used to eliminate any parentheses. Then, like terms may be combined and the equation can be solved.

Example #2: Solve $5(d+4)=7(d-2)$ for d.

Step #1: Eliminate the parentheses by using the distributive property on each of the quantities.

$$5(d+4)=7(d-2)$$
$$5d+20=7d-14$$

$$5(d+4)=5(d)+5(4)=5d+20$$
$$7(d-2)=7(d)-7(2)=7d-14$$

Step #2: Move the variables (with coefficients) to one side and the numbers (with no variables "attached") to the other side.

$$\begin{array}{rcl} \cancel{5d}+20 & = & 7d-14 \\ \cancel{-5d} & & -5d \\ \hline 20 & = & 2d\cancel{-14} \\ +14 & & \cancel{+14} \\ 34 & = & 2d \end{array}$$

Step #3: Divide both sides by 2 to solve for the unknown.

$$\frac{34}{2}=\frac{2d}{2}$$
$$17=d$$
$$d=17$$

Therefore, $d = 17$.

✓Check the answer by replacing *d* with 17 in the original equation.

$$5(d+4)=7(d-2)$$
$$5(17+4)=7(17-2)$$
$$5(21)=7(15)$$
$$105=105 \textit{ true}✓$$

Example #3: Solve $3(x-6)+2=4(x+2)-21$ for x.

Step #1: Eliminate the parentheses by using the distributive property on each of the quantities.

$$3(x-6)+2=4(x+2)-21$$
$$3x-18+2=4x+8-21$$

$$3(x-6)=3(x)-3(6)=3x-18$$
$$4(x+2)=4(x)+4(2)=4x+8$$

Step #2: Combine any like terms on either side of the equals sign. In this case, combine (–18 + 2) on the left and (+8 – 21) on the right.

$$3x-18+2=4x+8-21 \qquad -18+2=-16 \quad +8-21=-13$$
$$3x-16=4x-13$$

Step #3: Move the variables (with coefficients) to one side and the numbers (with no variables “attached”) to the other side.

$$\cancel{3x}-16=4x-13$$
$$\cancel{-3x} \qquad -3x$$
$$-16=1x\cancel{-13}$$
$$+13 \qquad \cancel{+13}$$
$$-3=1x$$
$$x=-3$$

Therefore, $x = -3$.

✓Check the answer by replacing *x* with –3 in the original equation.

$$3(x-6)+2=4(x+2)-21$$
$$3(-3-6)+2=4(-3+2)-21$$
$$3(-9)+2=4(-1)-21$$
$$-27+2=-4-21$$
$$-25=-25 \text{ } \textit{true}✓$$