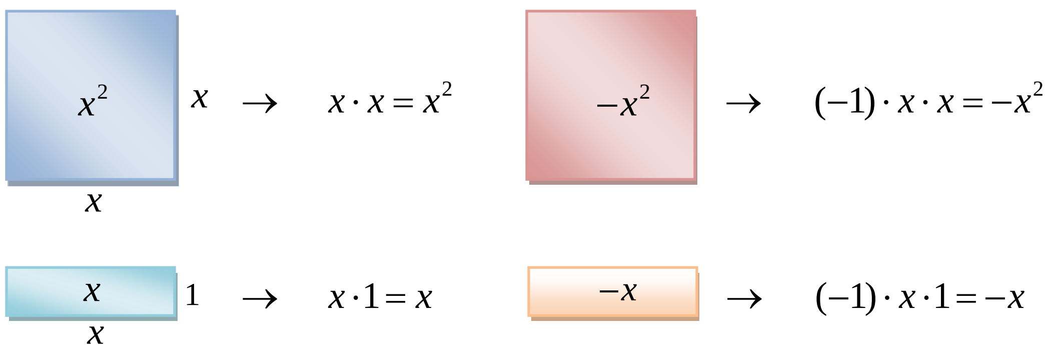
SIMPLIFY ALGEBRAIC EXPRESSIONS

This unit is about polynomials and the algebraic methods related to polynomials. The technique of combining like terms is demonstrated through models that represent the algebraic processes. Further exploration with polynomials will include multiplying a polynomial by a monomial.

Combining Like Terms Using Models

To combine like terms, it is helpful to see models of the problems to interpret the algebraic processes that take place.

First examine the meaning of the algebra tiles shown below based on the area of a rectangle.

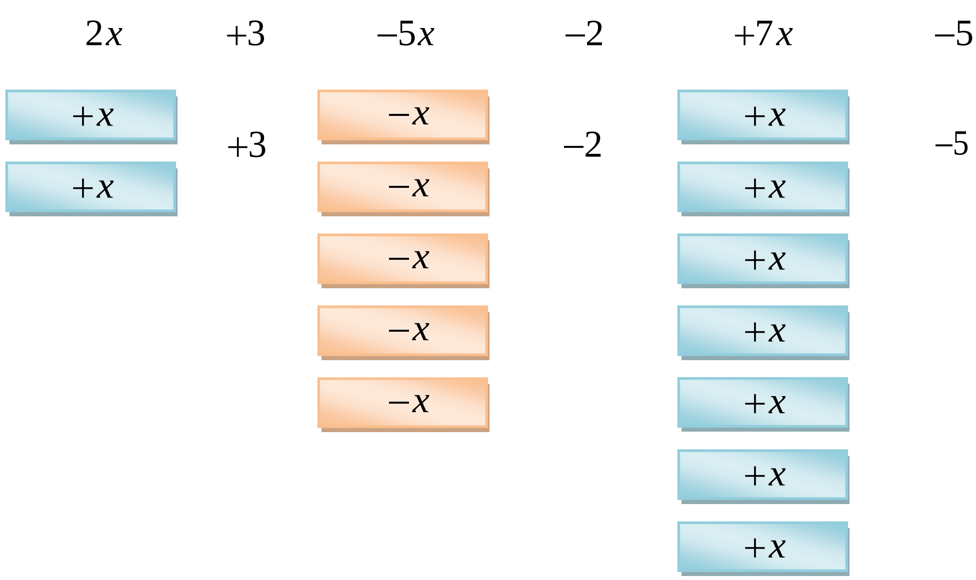


Example 1: Consider the following algebraic expression.

$$2x + 3 - 5x - 2 + 7x - 5$$

The expression may be simplified if we *collect* "like terms". The model below will help to understand "collecting like terms".

First model the expression using algebra tiles.

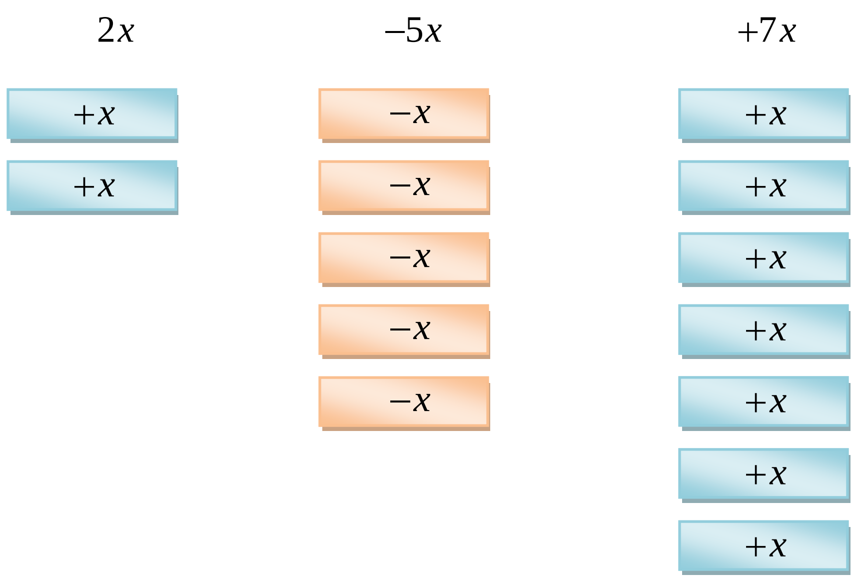


Look to find all of the terms that have x as a variable and combine their integer values. These terms are called “like” terms.

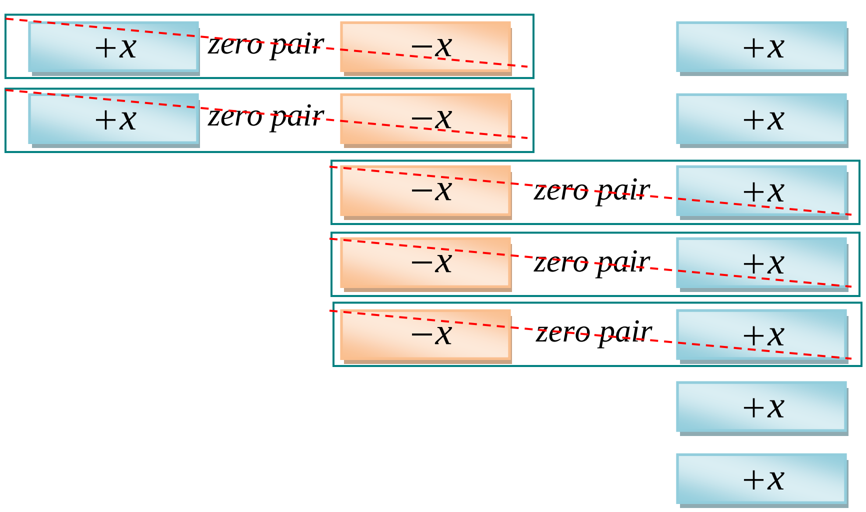
$$2x + 3 - 5x - 2 + 7x - 5$$

*Notice that the sign in front of the term determines whether the rectangles are positive or negative.

First, let’s just focus on the x-terms.



Next, remove pairs of terms that make zero. A $+x$ and a $-x$ make zero.



Next, examine the terms that are left. Combine the remaining terms.

$$\left.\begin{array}{c} +x \\ +x \\ +x \\ +x \end{array}\right\} 4x$$

The x's simplify to $4x$. This is called "collecting like terms".

Finally, examine all of the terms that are numeric and combine them also.

$$2x + 3 - 5x - 2 + 7x - 5$$

*Notice that the sign in front of the numeric term is included with the term.

$$\begin{array}{l} +3 - 2 - 5 = \\ \quad +1 \ - 5 = \\ \qquad -4 \end{array}$$

The numeric values combine to make -4.

Therefore, the expression simplifies to $4x - 4$.

$$2x + 3 - 5x - 2 + 7x - 5 \quad = \quad 4x - 4$$

Seek to find the simplest form by combining all terms of the expression that have the same variables and/or have the same exponents.

Example 2: Simplify the given expression.

$$4n^2 + 3n + 2 - 2n^2 + n - 4$$

There are three types of like terms in this expression:

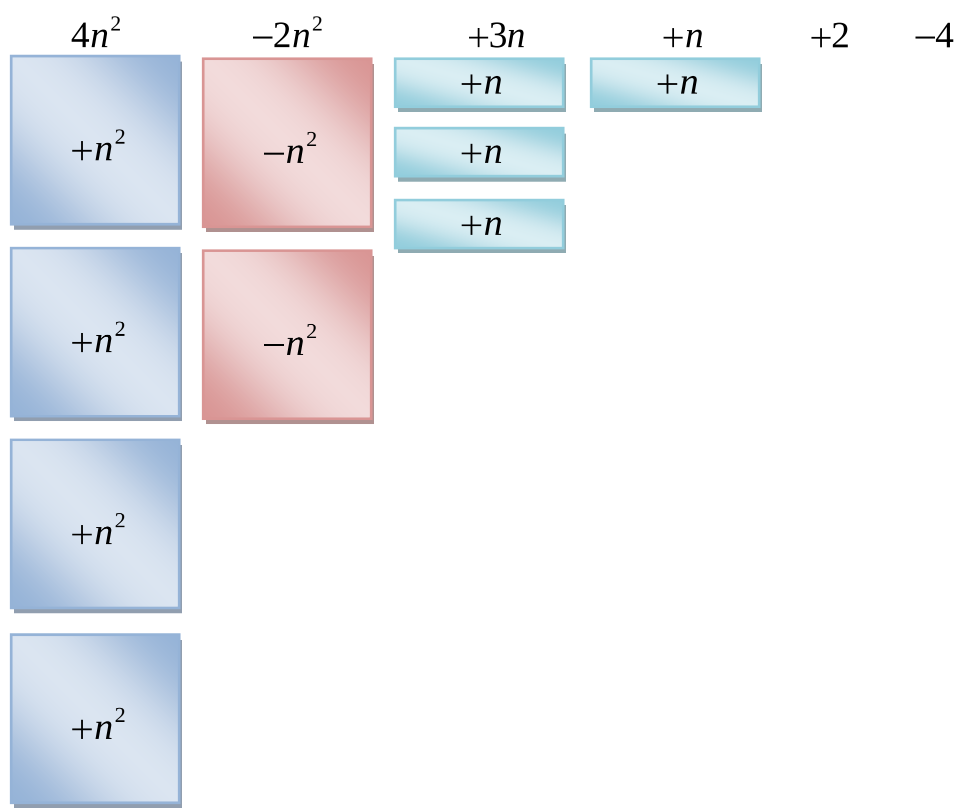
the n^2's, the n's, and the numeric values.

Rewrite the expression to prepare to collect like terms. Put all of the n^2's first, then all of the n's, and then the numeric values.

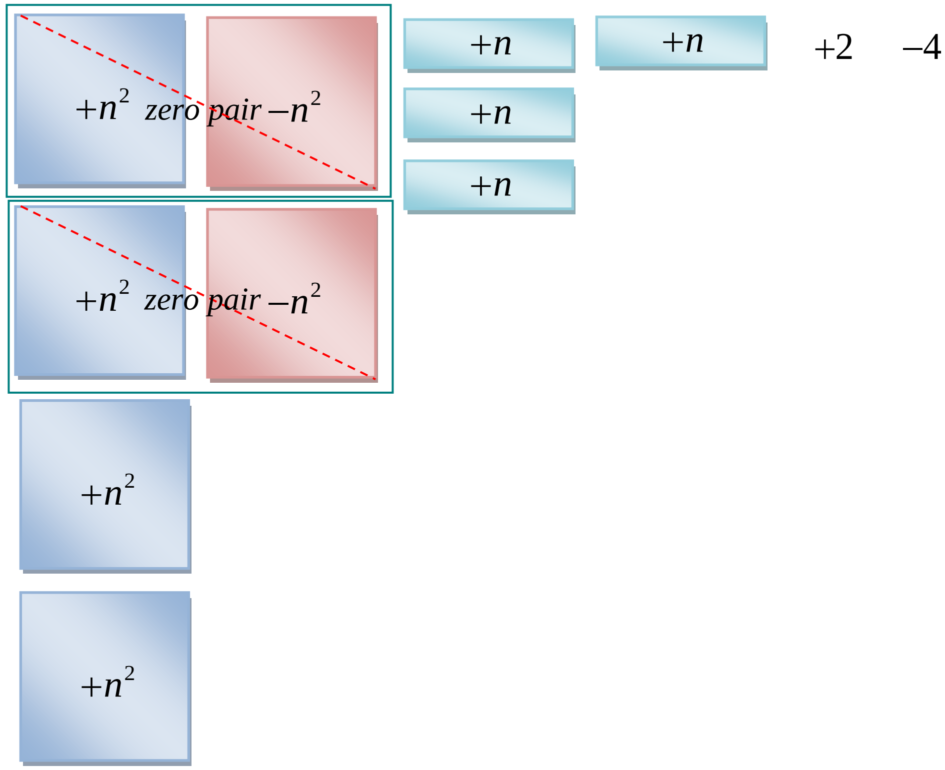
*Remember, the sign in front of the term goes with the term.

$$4n^2 + 3n + 2 - 2n^2 + n - 4 \quad \rightarrow \quad 4n^2 - 2n^2 + 3n + n + 2 - 4$$

Model the expression.



Remove the pairs of terms that make zero.



Collect the like terms.

$$\underbrace{4n^2-2n^2}_{2n^2}\underbrace{+3n+n}_{+4n}\underbrace{+2-4}_{-2}$$

Therefore, the expression simplifies to $2n^2+4n-2$.

**Note: Use great care when negative values are involved.

Combining Like Terms

Let's take a look at "collecting like terms" without using models; but, first, take time to become familiar with terms associated with polynomials.

term – A term is a part of an algebraic expression that consists of a constant multiplier and one or more variables raised to a power.

polynomial – A polynomial is an algebraic expression that is a combination of two or more terms.

constant – A constant is a numeric term only.

coefficient – The coefficient of a variable term is the number that is in front of the variable. The sign in front of the number is included as part of the coefficient.

Example 1: Simplify the given expression.

$$-2x^2+3x-5-4x^2-7x+10$$

*Remember the sign of the term is the sign that is *in front* of it.

Rewrite the expression to prepare to collect like terms.

$$-2x^2+3x-5-4x^2-7x+10$$

$$-2x^2-4x^2+3x-7x-5+10$$

Collect the like terms.

The coefficients of the x^2-terms are -2 and -4.

$$-2-4=-2+(-4)=-6$$

The coefficients of the x-terms are $+3$ and -7.

$$+3-7=+3+(-7)=-4$$

$$\underbrace{-2x^2-4x^2}_{-6x^2}\underbrace{+3x-7x}_{-4x}\underbrace{-5+10}_{+5}$$

The expression simplifies to $-6x^2-4x+5$.

Example 2: Simplify the following polynomial expression.

$$5x^3-8+2x^2-9x+10x^3-3-6x^2$$

Rewrite the expression to prepare to collect like terms.

$$5x^3-8+2x^2-9x+10x^3-3-6x^2$$

$$5x^3+10x^3+2x^2-6x^2-9x-8-3$$

Collect the like terms.

The coefficients of the x^3-terms are 5 and $+10$.

$$5+10=15$$

The coefficients of the x^2-terms are $+2$ and -6.

$$+2-6=+2+(-6)=-4$$

The coefficient of the x-term is -9.

$$\underbrace{5x^3+10x^3}_{15x^3}\underbrace{+2x^2-6x^2}_{-4x^2}\underbrace{-9x}_{-9x}\underbrace{-8-3}_{-11}$$

The expression simplifies to $15x^3-4x^2-9x-11$.

Multiplying a Polynomial by a Monomial

term – A term is a part of an algebraic expression that consists of a constant multiplier and one or more variables raised to a power.

- There are three terms in the polynomial $4x^2 + 3x + 1$.

polynomial – A polynomial is an algebraic expression that is a combination of two or more terms.

- Example: $4x^2 + 3x + 1$

monomial – A monomial is an algebraic expression consisting of only one term.

- Example: $4x^2$

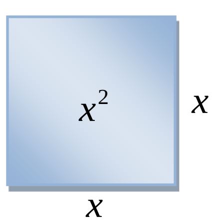
constant – A constant is a numeric term only.

- Example: 1

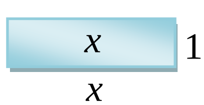
coefficient – The coefficient of a variable term is the number that is in front of the variable. The sign in front of the number is included as part of the coefficient.

- Example: In the polynomial $4x^2 + 3x + 1$, the coefficient of x^2 is 4 and the coefficient of x is 3.

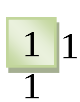
Examine the models and their meanings shown below.



$\rightarrow \quad x \bullet x = x^2$



$\rightarrow \quad x \bullet 1 = x$

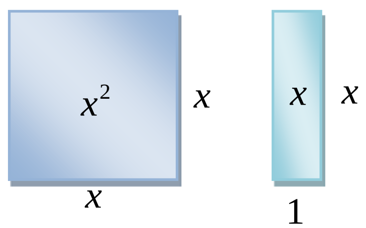


$\rightarrow \quad 1 \bullet 1 = 1$

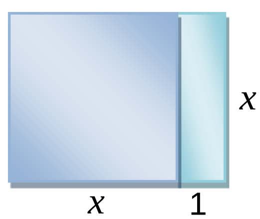
To multiply a polynomial by a monomial, each term of the polynomial is multiplied by the monomial by applying the distributive property.

First we'll use models to show that $x(x+1) = x^2 + x$.

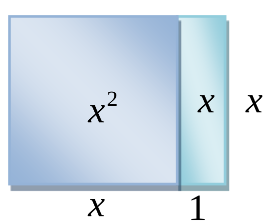
We will work backwards. We use an x^2-tile and an x-tile to represent $x^2 + x$.



Slide the tiles together to form a rectangle.

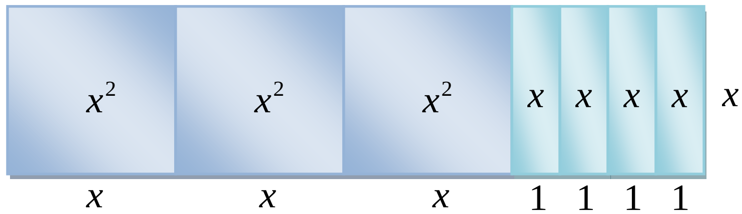


In a rectangle, *area* = *base* × *height*, thus $(x+1) \bullet x = x^2 + x$

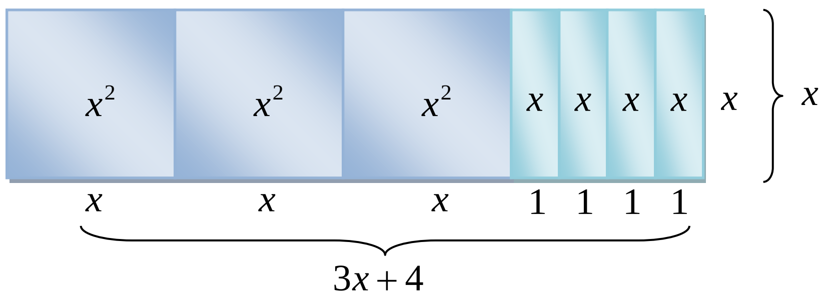


This expression can be written as $x(x+1) = x^2 + x$

Example 1: Write an algebraic statement that is represented by the model. Base the statement on the idea of *area* of a rectangle.



Write an expression to represent the length and width of the complete rectangle.



The base (b) of the rectangle is $3x + 4$.

The height (h) of the rectangle is x.

The area (A) of the rectangle is

$$1x^2 + 1x^2 + 1x^2 + 1x + 1x + 1x + 1x$$

which simplifies to $3x^2 + 4x$.

Thus, $(3x + 4) \bullet x = 3x^2 + 4x.$

This can also be written as follows: $x(3x + 4) = 3x^2 + 4x$.

Example 2: Without using models, multiply $5x(2x+3)$. Apply the distributive property.

$$\begin{aligned} 5x(2x+3) &= 5x(2x+3) \\ &= 5x(2x)+5x(3) \qquad *5\cdot 2\cdot x\cdot x+5\cdot 3\cdot x \\ &= 10x^2+15x \end{aligned}$$

Thus, $5x(2x+3)=10x^2+15x$

Example 3: Multiply $2z(3z^2-4)$.

Apply the distributive property.

$$\begin{aligned} 2z(3z^2-4) &= 2z(3z^2)-2z(4) \\ &= 2z(3z^2)-2z(4) \qquad *2\cdot 3\cdot z\cdot z\cdot z-2\cdot 4\cdot z \\ &= 6z^3-8z \end{aligned}$$

Thus, $2z(3z^2-4)=6z^3-8z$