ONE-STEP EQUATIONS

In this unit, you learn about one-step equations and how to solve the equations algebraically using inverse operations. Learning and using this method with simple equations will help you to solve more complicated equations in later units of this course.

Inverse Statements

Simple Equations: Addition and Subtraction

Simple Equations: Multiplication and Division

Inverse Statements

Inverse: An inverse operation in math is the opposite operation that undoes another operation that may have taken place.

Subtraction works as the inverse of addition and vice versa.

Example 1: If we add the integers 5 + 3 and get 8, what are the related inverse statements?

If we use the inverse operation for addition, which is subtraction, we can write the related sentences, 8 – 3 = 5 or 8 – 5 = 3.

Similarly, division works as the inverse for multiplication and vice versa.

Example 2: If we multiply the integers (–5)(–3) and get 15, what are the related inverse statements?

If we use the inverse operation for multiplication, which is division, then we can write related sentences, 15 / –5 = –3 or 15 / –3 = –5.

To summarize:

Statement		Related Inverse Statements
$5+3=8$	$\rightarrow$	$8-5=3$ or $8-3=5$
$(-5)(-3)=15$	$\rightarrow$	$15\div(-5)=-3$ or $15\div(-3)=-5$

Example 3: Write a related inverse statement for $(-24) \div (-3) = 8$.

Multiplication is the inverse of division. Write a multiplication statement.

Related Inverse Statement $\rightarrow$ $(-3)(8) = -24$

Example 4: Write a related inverse statement for $9 - (-2) = 11$.

Addition is the inverse of subtraction. Write an addition statement.

Related Inverse Statement $\rightarrow$ $11 + (-2) = 9$

Simple Equations: Addition and Subtraction

Keep Equations in Balance

Equations are balanced on the equals sign, so to keep the sides in balance, whatever math operation is completed on one side of the equation must also be completed on the other side of the equation.

Example 1: Solve $x+3.6=8.5$ for *x*.

We need to **isolate** *x* on the left side of the equals sign; that is, we must undo the " + 3.6" by subtracting 3.6. To keep the equation in balance, we must also subtract 3.6 from the right side.

$$\begin{array}{rcl} x + \cancel{3.6} & = & 8.5 \\ \cancel{-3.6} & & -3.6 \\ \hline x & = & 4.9 \end{array}$$

Subtract 3.6 from BOTH sides.

*Subtracting 3.6 on the left side of the equation cancels out the " + 3.6" which leaves just *x* on the left side. This is called "isolating *x*".

For this problem $x=4.9$.

Check: Substitute 4.9 in for *x* in the original equation and simplify.

$$\begin{aligned} x + 3.6 &= 8.5 \\ 4.9 + 3.6 &= 8.5 \\ 8.5 &= 8.5 \checkmark \end{aligned}$$

Example 2: Solve $m-16=-10$ for *m*.

We need to **isolate** *m* on the left side of the equals sign; that is, we must undo the " –16" by adding 16. To keep the equation in balance, we must also add 16 to the right side.

$$\begin{array}{lcr} m \cancel{-16} & = & -10 \\ \cancel{+16} & + & 16 \\ \hline m & = & 6 \end{array}$$

Add 16 to BOTH sides.

*Adding 16 to the left side of the equation cancels out the " –16" which leaves just *m* on the left side.

For this problem $m=6$.

Check: Substitute 6 in for *m* in the original equation and simplify.

$$\begin{aligned} m-16 &= -10 \\ 6-16 &= -10 \\ -10 &= -10 \checkmark \end{aligned}$$

Example 3: Solve $24+p=-19$ for *p*.

Isolate *p*.

$$\begin{array}{r} \cancel{24}+p=-19 \\ \underline{\cancel{-24}\qquad -24} \\ p=-43 \end{array} \qquad -19-24=-19+(-24)$$

For this problem $p=-43$.

Check: Substitute –43 in for *p* in the original equation and simplify.

$$\begin{aligned} 24+\quad p\quad &=-19 \\ 24+(-43)&=-19 \\ -19&=-19 \checkmark \end{aligned}$$

Example 4: Solve $-32=k-7$ for *k*.

Isolate *k*. In this example, the variable *k* is on the right side of the equation. So, look to the right side to determine what we must "undo" to isolate the variable. Thus, we begin by adding 7 to the right side, and then follow by adding 7 to the left side.

$$\begin{array}{l} -32=k\cancel{-7} \\ \underline{+7\qquad \cancel{+7}} \\ -25=k \end{array} \qquad \text{Add 7 to BOTH sides.}$$

For this problem $k=-25$.

Check: Substitute –25 in for *k* in the original equation and simplify.

$$\begin{aligned} -32&=k-7 \\ -32&=-25-7 \\ -32&=-32 \checkmark \end{aligned}$$

Example 5: Solve $n+\frac{2}{3}=8\frac{5}{9}$ for n.

Isolate n.

$$n+\frac{2}{3}=8\frac{5}{9}$$ Determine the LCD (9)

$$n+\cancel{\frac{6}{9}}=8\frac{5}{9}$$ Change 2/3 to 9ths. $\left(\frac{2}{3}\times\frac{3}{3}=\frac{6}{9}\right)$

$$\underline{-\cancel{\frac{6}{9}}\quad -\frac{6}{9}}$$ Subtract 6/9 from BOTH sides.

$$n=7\frac{8}{9}$$

$$\left(\begin{array}{r} 8\frac{5}{9}=7\frac{14}{9} \\ \underline{-\frac{6}{9}=\quad\frac{6}{9}} \\ 7\frac{8}{9} \end{array}\right)$$

For this problem $n=7\frac{8}{9}$.

Check: Substitute 7 8/9 in for n in the original equation and simplify.

$$n+\frac{2}{3}=8\frac{5}{9}$$

$$7\frac{8}{9}+\frac{6}{9}=8\frac{5}{9}$$

$$7\frac{14}{9}=8\frac{5}{9}$$

$$8\frac{5}{9}=8\frac{5}{9}\ \checkmark$$

Example 6: Solve $c-2\frac{3}{4}=6\frac{7}{10}$ for c.

Isolate c.

Determine the LCD (20).

Change 2 3/4 and 6 7/10 to 20ths.

$$\left(2\frac{3}{4}\times\frac{5}{5}=2\frac{15}{20}\right) \qquad \left(6\frac{7}{10}\times\frac{2}{2}=6\frac{14}{20}\right)$$

Now solve for c.

$$c-2\frac{3}{4} = 6\frac{7}{10}$$

$$c-\cancel{2\frac{15}{20}} = 6\frac{14}{20}$$ Write the equation in 20ths.

$$\cancel{+2\frac{15}{20}} \quad +2\frac{15}{20}$$ Add 2 15/20 to BOTH sides.

$$c=8\frac{29}{20}$$ Simplify the mixed fraction.

$$c=9\frac{9}{20}$$

For this problem $c=9\frac{9}{20}$.

Check: Substitute 9 9/20 in for *c* in the original equation and simplify.

$$
\begin{aligned}
c - 2\frac{3}{4} &= 6\frac{7}{10} \\
9\frac{9}{20} - 2\frac{3}{4} &= 6\frac{7}{10} \\
9\frac{9}{20} - 2\frac{15}{20} &= 6\frac{7}{10} \\
8\frac{29}{20} - 2\frac{15}{20} &= 6\frac{7}{10} \\
6\frac{14}{20} &= 6\frac{7}{10} \\
6\frac{7}{10} &= 6\frac{7}{10} \checkmark
\end{aligned}
$$

Example 7: Maria scored 15 more points than Amy in the final basketball game of the season. Maria totaled 23 points in the game. How many points did Amy score? (a) Write an equation to determine Amy's score. (b) What was Amy's score?

Let *a* represent Amy's score.

Use the information given in the problem to write an equation.

Maria scored 15 more points than Amy.		Maria totaled 23 points in the game.
↓		↓
$a+15$	$=$	23

$$\begin{aligned} a \cancel{+15} &= 23 \\ \cancel{-15} & \quad -15 \\ \hline a &= 8 \end{aligned}$$

Subtract 15 from BOTH sides.

Amy scored eight (8) points in the game.

Check: 8 + 15 = 23 ✓

Simple Equations: Multiplication and Division

Keep Equations in Balance

Equations are balanced on the equals sign, so to keep the sides in balance, whatever math operation is completed on one side of the equation must also be completed on the other side of the equation.

Example 1: Solve $9x = -54$ for *x*.

We need to **isolate** *x* on the left side of the equals sign; that is, we must undo the " 9 times *x*" by dividing by 9. To keep the equation in balance, we must also divide by 9 on the right side.

$$\begin{aligned} 9x &= -54 \\ \frac{\cancel{9}x}{\cancel{9}} &= \frac{-54}{9} \\ x &= -6 \end{aligned}$$

Divide BOTH sides by 9.

*Dividing by 9 on the left side of the equation cancels out the "multiplied by 9" which leaves just *x* on the left side.

For this problem $x = -6$.

Check: Substitute –6 in for *x* in the original equation and simplify.

$$\begin{aligned} 9x &= -54 \\ 9(-6) &= -54 \\ -54 &= -54 \checkmark \end{aligned}$$

Example 2: Solve $\frac{y}{8}=32$ for *y*.

We need to **isolate** *y* on the left side of the equals sign; that is, we must undo the "y divided by 8" by multiplying by 8. To keep the equation in balance, we must also multiply by 8 on the right side.

$$\frac{y}{8}=32$$

$$\cancel{(8)}\frac{y}{\cancel{8}}=32(8) \qquad \text{Multiply BOTH sides by 8.}$$

$$y=256$$

*Multiplying by 8 on the left side of the equation cancels out the "divided by 8" which leaves just *y* on the left side.

For this problem $y=256$.

Check: Substitute 256 in for *y* in the original equation and simplify.

$$\frac{y}{8}=32$$

$$\frac{256}{8}=32 \qquad \begin{array}{r} 32 \\ 8\overline{)256} \end{array}$$

$$32=32\ \checkmark$$

Example 3: Solve $-144=-8p$ for *p*.

Isolate *p*. In this example, the variable *p* is on the right side of the equation. So, look to the right side to determine what we must "undo" to isolate the variable. Thus, we begin by dividing by –8 on the right side, and then follow by dividing by –8 on the left side.

$$-144 = -8p$$

$$\frac{-144}{-8} = \frac{\cancel{-8}}{\cancel{-8}}p \qquad \text{Divide BOTH sides by } -8.$$

$$18 = p$$

For this problem $p=18$.

Check: Substitute 18 in for *p* in the original equation and simplify.

$$-144=-8p$$
$$-144=-8(18)$$
$$-144=-144 \checkmark$$

Example 4: Solve $\frac{m}{-5} = 200$ for *m*.

$$\frac{m}{-5} = 200$$

$$\cancel{(-5)}\frac{m}{\cancel{-5}} = 200(-5) \qquad \text{Multiply BOTH sides by } -5.$$

$$m = -1000$$

For this problem $m = -1000$.

Check: Substitute -1000 in for *m* in the original equation and simplify.

$$\frac{m}{-5} = 200$$

$$\frac{-1000}{-5} = 200$$

$$200 = 200 \checkmark$$

Example 5: The speed of a cheetah is 1.75 times as fast as the speed of a zebra. The speed of a cheetah is 70 MPH. (a) Write an equation to determine the speed of a zebra. (b) What is the speed of a zebra?

Let z represent the speed of a zebra.

Use the information given in the problem to write an equation.

The speed of a cheetah is 1.75 times as fast as the zebra.	The speed of a cheetah is 70 MPH.
↓	↓

$$1.75z \quad = \quad 70$$

$$\begin{aligned} 1.75z &= 70 \\ \frac{\cancel{1.75}z}{\cancel{1.75}} &= \frac{70}{1.75} \\ z &= 40 \end{aligned}$$

Divide BOTH sides by 1.75.

The speed of a zebra is 40 MPH.

Check: 1.75 (40) = 70 ✓