GRAPHS OF LINEAR EQUATIONS

This unit is about linear equations in which the *y*-value is dependent on the *x*-value for a given set of ordered pairs. The appearance of the graphs of equations is affected by changing the constants of the equations, and this will be explored in this unit. One method for graphing equations is to make tables of *x* and *y* values for the linear equations; then, use the ordered pairs in the table to graph the equation. Some equations develop patterns of change as some values are kept constant while others vary. It will be helpful to have a graphing calculator; but, if one is not available, then use graph paper to graph the equations and study the patterns of change.

Graphing Linear Equations

Compare How Changes in an Equation Affect the Related Graph

Graphing Linear Equations with a Graphing Calculator

Graphing Linear Equations

The graph of a linear equation is a straight line.

Example 1: Graph $y = 2x + 3$.

- First organize the data in a table.
- Choose several values for x. In this example, we chose –2 through 2.
- Substitute the values for x in the equation to find corresponding y-values.
- Make ordered pairs with the results.
- Graph the points.
- Draw a straight line through the points.

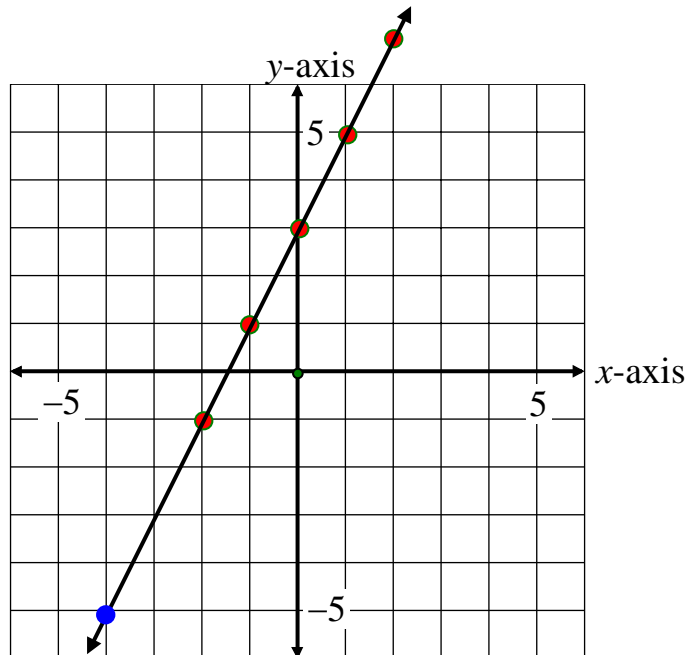| $y = 2x + 3$			
x	Substitution	y	Ordered Pairs $(x\ y)$
–2	$y = 2\times(-2)+3$	–1	(–2,–1)
–1	$y = 2\times(-1)+3$	1	(–1, 1)
0	$y = 2\times(0)+3$	3	(0, 3)
1	$y = 2\times(1)+3$	5	(1, 5)
2	$y = 2\times(2)+3$	7	(2, 7)

Use the ordered pairs to graph the function.
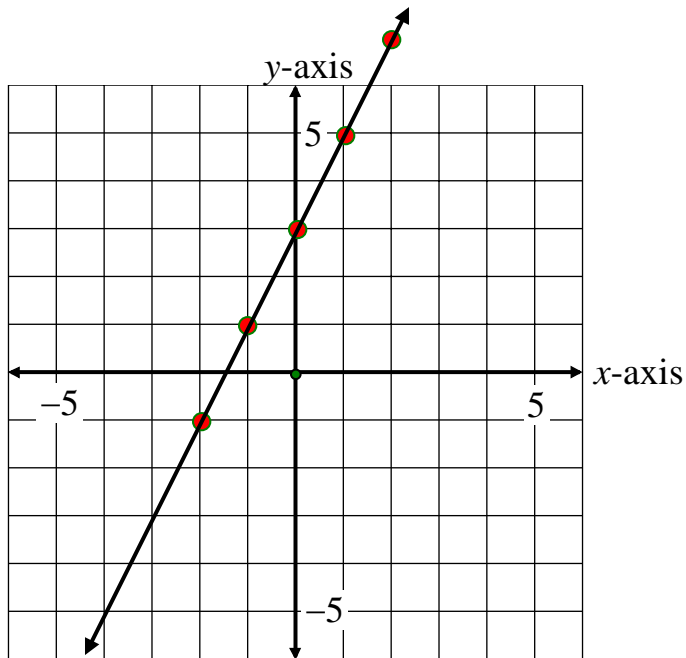
✓Check the linear graph to see if it matches the equation:

- Chose another point on the line that was NOT generated from the table of values.
- Substitute the *x* and *y* values of that point in the equation to see if they test true.

Let's choose (–4, –5). It is highlighted in blue in the graph.

$y = 2x + 3$

$-5 = 2(-4) + 3 \qquad (-4, -5) \;\rightarrow\; x = -4,\; y = -5$

$-5 = -8 + 3$

$-5 = -5$ ✓

The point tests true and is one of the ordered pairs for the equation $y = 2x + 3$.

## Compare How Changes in an Equation Affect the Related Graph

Once we know what to expect from certain relations that are defined by equations, we can experiment with changing the equations and predicting how those changes will appear "graphically".

First, we'll examine changes of the coefficient of x in the equation $y = 2x$.

coefficient - Coefficient is the term used to refer to the number before x where it is multiplied with x.

*Note: The graphs shown below were generated using a graphing calculator. The graphing calculator displays digitized images made up of pixels; thus, the lines appear to be "jagged" rather than smooth and straight. However, for all of the lines that represent equations in this unit, assume the lines are smooth and straight.
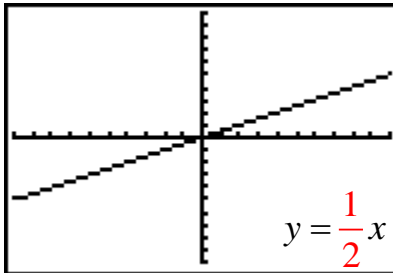
Let's begin with the graph, $y = 2x$.

$$y = 2x$$
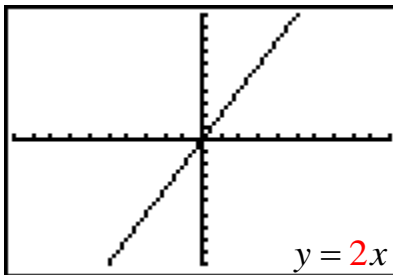
Now, let's increase the coefficient of x from 2 to 5, $y = 5x$. The graph of this equation is a *steeper* line.

$$y = 5x$$
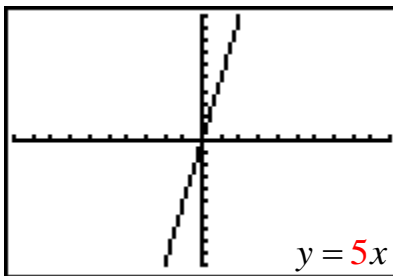
Next, let's decrease the coefficient of x from 5 to 0.5, $y = \dfrac{1}{2}x$. The graph of this equation is a *flatter* (less steep) line.

$$y = \dfrac{1}{2}x$$

Finally, let's change the coefficient from 0.5 to –2, $y = -2x$. The graph of this equation is a line that changes direction, that is, up and to the left. (All the other lines were up and to the right.)

$$y = -2x$$
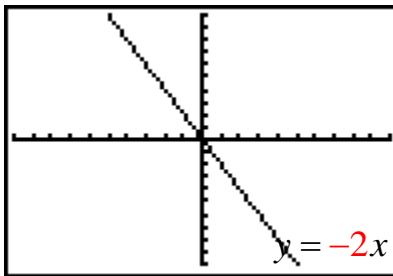
This demonstrates the effect on a line called "slope". We can see larger values greater than one will make steep lines while small fractional values make flatter lines. Negative values will point the line in the opposite direction. All of these changes occurred as we changed the coefficient of x.

Now let's compare four equations as the coefficient of "x" increases.

Example 1: Graph the following equations and make a statement about the change in the graphs when the coefficient of x changes: $y = x, y = 2x, y = 5x, y = 10x$.

$$y = 10x \quad y = 5x \quad y = 2x$$

$y = x$ understood as $y = 1x$

$y = x$
$y = 2x$
$y = 5x$
$y = 10x$
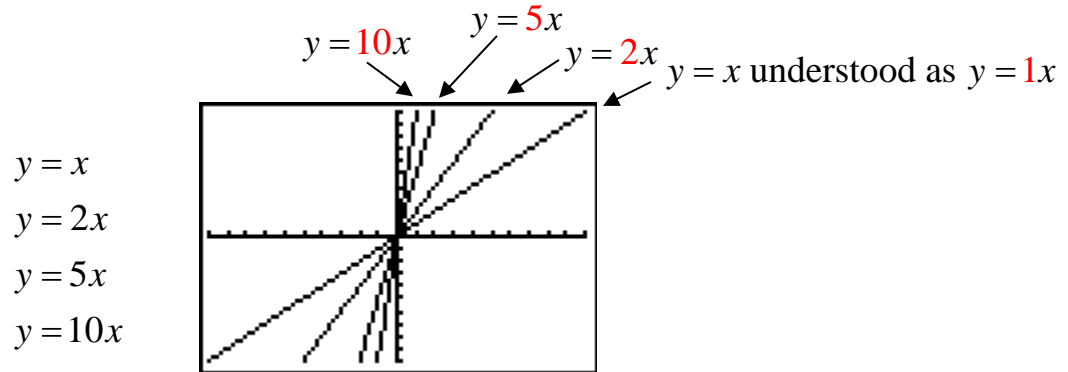
The graphs of these equations show that the lines appear to grow steeper as the coefficient of x increases.

Example 2: Graph the following equations and make a statement about the change in the graphs when the constant that is added to or subtracted from $2x$ changes: $y = 2x + 1, y = 2x + 3, y = 2x - 5$.

$$y = 2x + 3 \quad y = 2x + 1$$

$y = 2x - 5$

$y = 2x + 1$
$y = 2x + 3$
$y = 2x - 5$
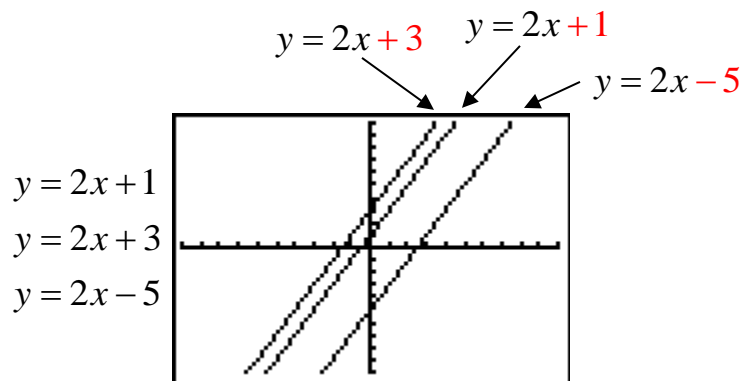
Observe that the lines are parallel and appear to have shifted in position along the y-axis.

All of the equations have the same slope of two, but the line slides up and down the y-axis. The graphs of the equations show that adjusting the constant value will translate the line up or down the y-axis.

Notice that $y = 2x + 3$ is passes through (0, 3) on the *y*-axis.
Notice that $y = 2x + 1$ is passes through (0, 1) on the *y*-axis.
Notice that $y = 2x - 5$ is passes through (0, –5) on the *y*-axis.

Now practice observing changes in linear graphs. If a graphing calculator is available, it would be best to enter the equations and watch as they are graphed. The first equation will be graphed first, and so on. If a graphing calculator is not available, then create tables of values and make the graphs on graph paper.

*Note: An explanation of how to enter equations in a graphing calculator is provided in the content link to "Graphing Linear Equations with a Graphing Calculator".

Two problems are given below as practice. The answers are shown at the bottom of the page.

Problem 1: Graph the following equations:
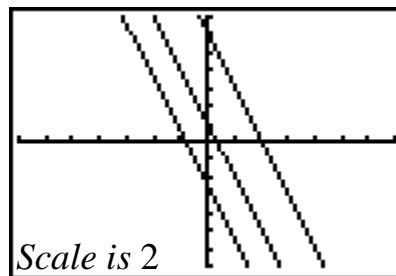$y = -3x + 2, y = -3x - 5, y = -3x + 12.$

*Note: The scale on the graphing calculator is set to two (instead of one) for both the *x*-axis and the *y*-axis.

- What remains the same in the graphs of all three equations?
- What changes occur in the graphs of the three equations?
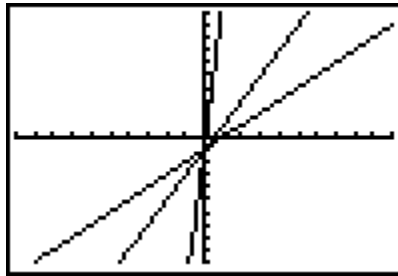
$y = -3x + 2$
$y = -3x - 5$
$y = -3x + 12$

Scale is 2

Problem 2:   Graph the following equations:
$$y = x - 1, y = 2x - 1, y = -12x - 1.$$

*Note: The scale on the graphing calculator is set to one for both the
x-axis and the *y*-axis.

- What remains the same in the graphs of all three equations?
- What changes occur in the graphs of the three equations?
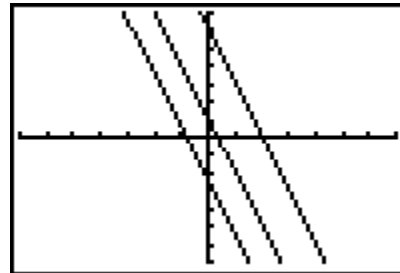
$y = x - 1$

$y = 2x - 1$

$y = 12x - 1$

Answers

Problem 1: The slope is the same, –3. The graph moves up and down the
y-axis depending on the constant value.

$y = -3x + 12$ crosses the *y*-axis at (0, 12).
$y = -3x + 2$ crosses the *y*-axis at (0, 2).
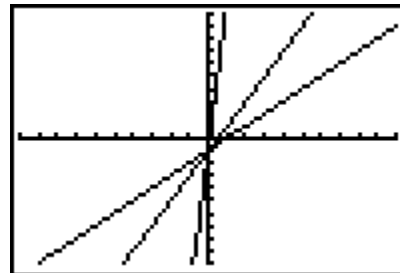$y = -3x - 5$ crosses the *y*-axis at (0,–5).

Problem 2: The *y*-intercept is the same for all three equations, (0,–1). As
the coefficient of *x* (slope) increases, so does the steepness of the line.

$y = x - 1$ is the less steep of the three
equations.

$y = 12x - 1$ is the most steep.

*Note: All of the equations used in this unit were written in the slope intercept form:

$$y = mx + b$$

where *m* represents the slope, the steepness of the line

and *b* represents the *y*-intercept, the point where the line crosses (intercepts) the *y*-axis.

Graphing Linear Equations with a Graphing Calculator

A graphing calculator may be used to graph the linear equations in this unit. These directions show how to graph the equations using a TI-84 graphing calculator.

*Note: Other graphing calculators may be used, but it is the student's responsibility to refer to the calculator's manual to learn how to graph the equations for the calculator.

First, the equation of a line is entered into the calculator in the form which isolates "y"; that is, $y =$ an expression in terms of x.

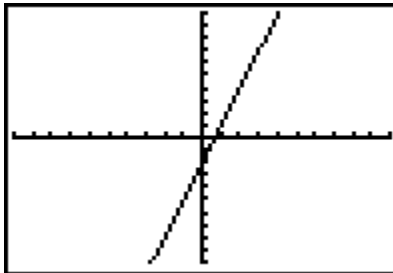> *Example 1*: Graph the equation, $y = 3x - 2$, in the standard viewing window.
>
> *Step 1*: Enter the equation, $y = 3x - 2$, in the Y= Editor.

| Y = | CLEAR | 3 | X,T,Θ,n | — | 2 | ENTER |

> *Step 2*: Display the graph in the standard viewing window.

| ZOOM | 6 |

The graph for $y = 3x - 2$ is displayed in the window of the graphing calculator.

Example 2: Graph the equations $y = x$, $y = 2x$, $y = 5x$, and $y = 10x$ on the standard axes.

Step 1: Enter the graphs $y = x$, $y = 2x$, $y = 5x$ in the Y= editor.

Step 2: Display the graphs in the standard viewing window. Watch as the lines are graphed. How does the steepness of the third line compare to the steepness of the other lines?

Answer: As the coefficient of x (the number before it) increases in value, the slope of the line is steeper.