LINEAR AND NONLINEAR EQUATIONS

This unit is an introduction to quadratic (second degree) equations, how to graph quadratic equations, and how to solve simple quadratic equations. In addition, comparisons will be made between linear equations and quadratic equations. It will be helpful to have a graphing calculator. If one is not available, use graph paper to graph the equations accurately.

Differentiate Changes

Graphing Quadratic Functions

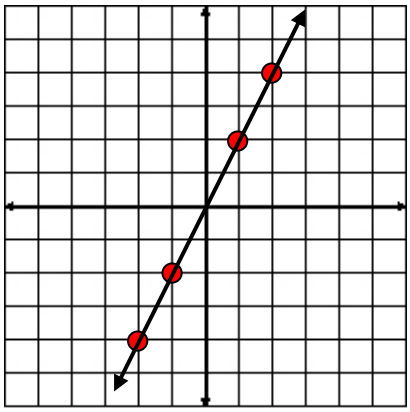
Solving Simple Quadratic Equations

Linear and Nonlinear Equations

Graph Paper

Differentiate Changes

Linear Equations and Graphs



Recall that a relation is as a list of ordered pairs. Often a mathematic expression in the form of an equation can be used to express the relation in a simpler way. For example the relation:

$$(1,2),(2,4),(-1,-2),(-2,-4)$$

is a list of several points on a line that may be expressed by the equation $y = 2x$.

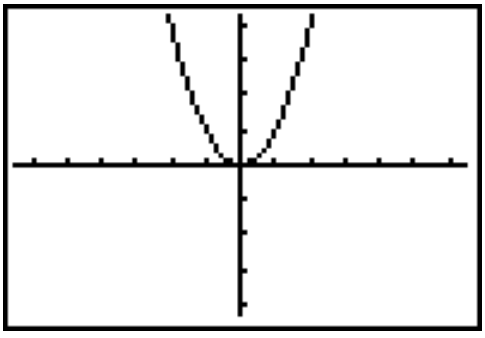
In this equation, the degree (exponent) of x (which can be written as x^1) is the first degree. This relation, when plotted on a coordinate plane, will be a straight line. Equations of degree one are linear equations.

Quadratic Equations and Graphs

If the equation contains an expression of the second degree "x-squared", then the graph will be represented by a parabola, and the equation is called a quadratic equation.

Example: Graph $y = x^2$.

$$y = x^2$$



Just by examining the degree of an equation, the shape of the graph can be predicted before doing any work.

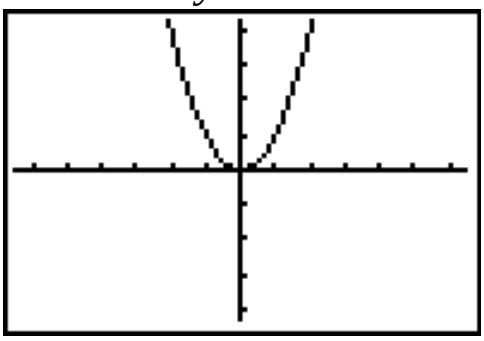
Graphing Quadratic Functions

Quadratic Function

A quadratic function is a function of the form $y = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$.

The graph of a quadratic function is a curve known as a **parabola** and is shown below.

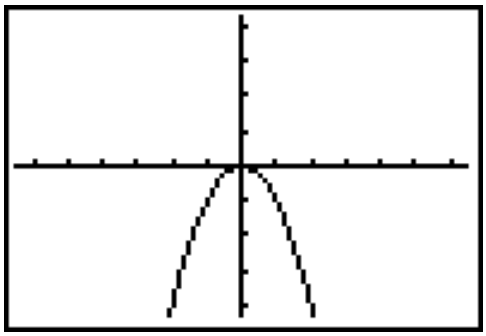
$y = x^2$



- The lowest point on this parabola is the **minimum value** of the function.
- The point (0, 0) on this graph and is called the **vertex** of the parabola.
- In a parabola there is a vertical line called the **axis of symmetry** drawn through the vertex, that reflects the parabola across the line of $x = h$ or in other words splits the parabola into two equal parts. In the case below the axis of symmetry would be the y-axis or $x = 0$.

If the coefficient of *x*-squared is a negative number, the parabola opens downward, and therefore has a maximum value at the vertex.

$$y = -x^2$$



- The equation of this graph can be written as $y = (-1)x^2$. The coefficient of x-square is –1.

- The highest point on this parabola is the **maximum value** of the function.

- The point (0, 0) on this graph and is also called the **vertex** of the parabola.

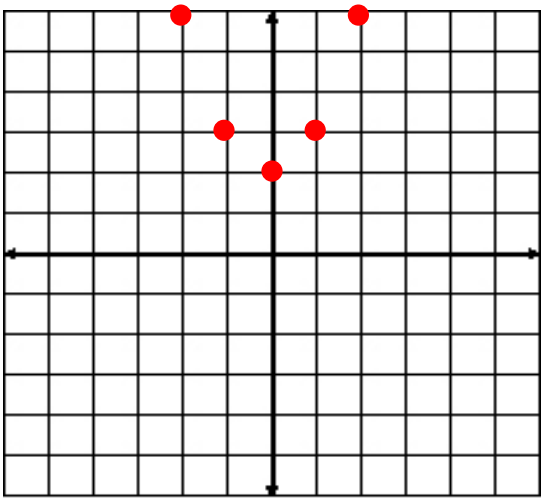
A table of values is very useful when examining the graph a quadratic equation.

Example 1: Graph $y = x^2 + 2$

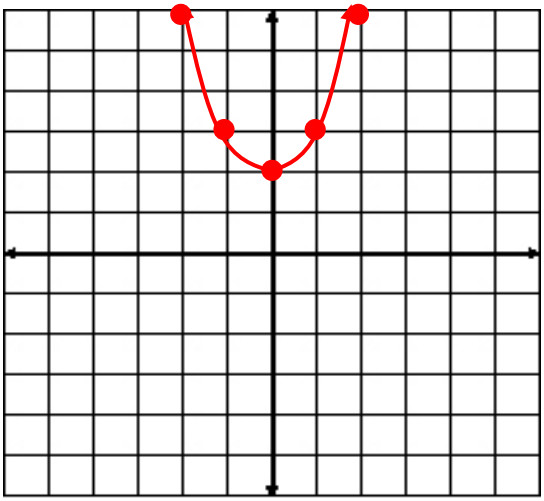
a) Make a table of values using positive and negative *x*-values.

x	$x^2 + 2$	y
–2	$(-2)^2 + 2$	6
–1	$(-1)^2 + 2$	3
0	$(0)^2 + 2$	2
1	$(1)^2 + 2$	3

b) Graph the points from the table on a coordinate plane.

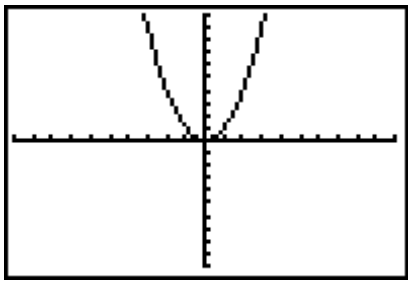


c) Connect the points using a curve (remember that parabolas are curved lines).

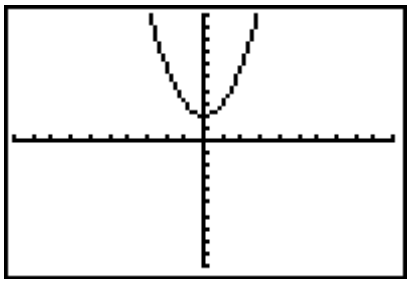


*Notice that the vertex of this parabola is located at (0, 2) as opposed to (0, 0) as shown in the first part of the unit. Let's take a look at both functions and decide why this happened.

a) $y = x^2$



b) $y = x^2 + 2$



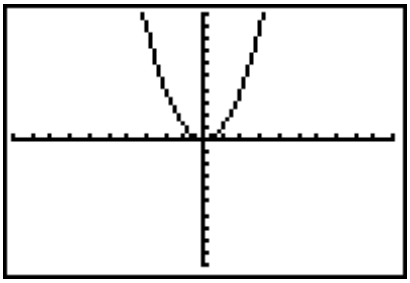
Compare both equations and notice:

- In the first graph's equation $y = x^2$, there is no constant term.
- In the second graph's equation $y = x^2 + 2$, there is a constant of + 2.
- The graph of the second equation has moved up the *y*-axis 2 units.

*It can be concluded that a constant moves the graph vertically along the *y*-axis.

Let's use $y = x^2$ as the parent function (original function) with a vertex at (0, 0) and compare it with $y = x^2 - 3$. What will happen in this case?

$y = x^2$



$y = x^2 - 3$

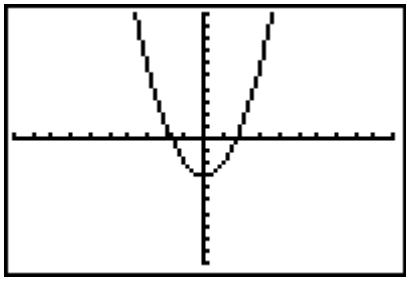


table of values
for $y = x^2 - 3$

x	y
–2	1
–1	–2
0	–3
1	–2
2	1

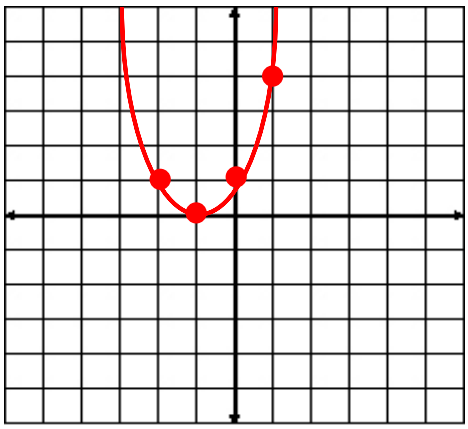
*Due to the pixels in the graphing calculator it may look like the second equation passes through (2, 0) and (–2, 0). This is not the case and at this point we are only concerned about what the constant does to the graph.

By examining the two equations you can see that by subtracting 3 the graph has moved down 3 units from the origin on the *y*-axis.

Example 2: Graph $y = (x+1)^2$

Make a table of values and graph the points.

x	y
–2	1
–1	0
0	1
1	4



Notice that in this example, the number of points on each side of the vertex is not the same. More values for x can be substituted to determine how wide the parabola gets on the left.

*Notice that when the constant term is in a quantity with the variable, the vertex moved horizontally along the x-axis. In the case above it moved one unit to the **left**. What would happen to the vertex of the equation $y = (x - 2)^2$?

If you answered that the vertex would move **2 units to the right**, you are correct.

Solving Simple Quadratic Equations

A quadratic equation is said to have "real" roots where the graph of the equation intersects the *x*-axis. The "real" number roots are the solutions to the equation. The solutions for the equation are the points where the graph crosses the *x*-axis. Recall that when a point is on the *x*-axis, its *y*-coordinate is zero.

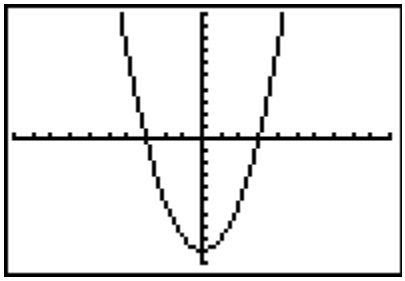
Example 1: Solve by graphing to find the roots of the equation $y = x^2 - 9$.

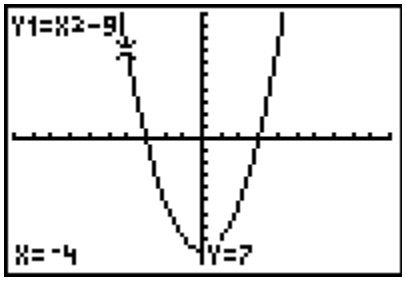
A table of values can be used to help graph and solve the equation.

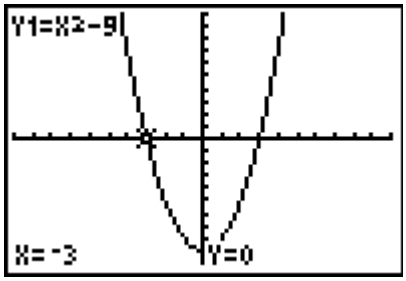
x	y	$y = x^2 - 9$
–4	7	$(-4)^2 - 9$
–3	0	$(-3)^2 - 9$
–2	–5	$(-2)^2 - 9$
–1	–8	$(-1)^2 - 9$
0	–9	$(0)^2 - 9$
1	–8	$(1)^2 - 9$
2	–5	$(2)^2 - 9$
3	0	$(3)^2 - 9$
4	7	$(4)^2 - 9$

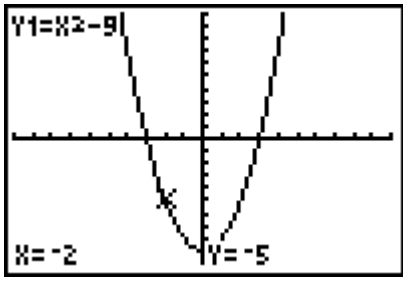
In the graphs below, each point from the table is shown graphed. The graphed point is marked with a star. The resulting curved graph for all the given points plus the points in between is the parabola shown immediately below.

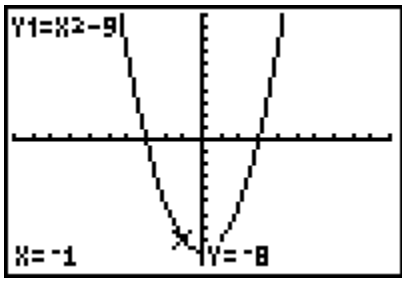
$y = x^2 - 9$

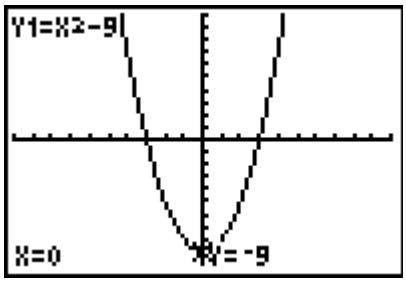


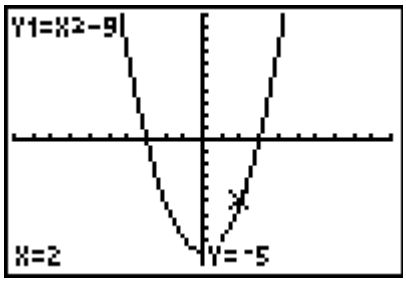


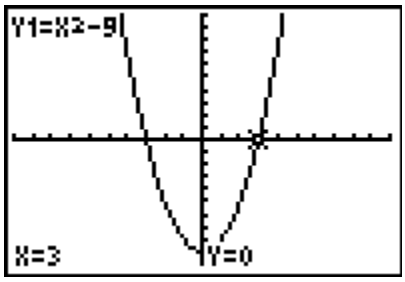


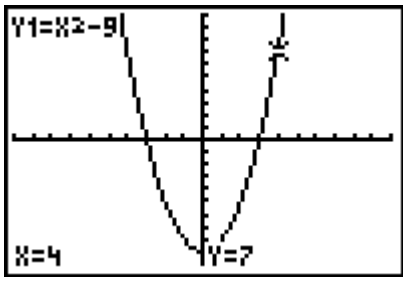






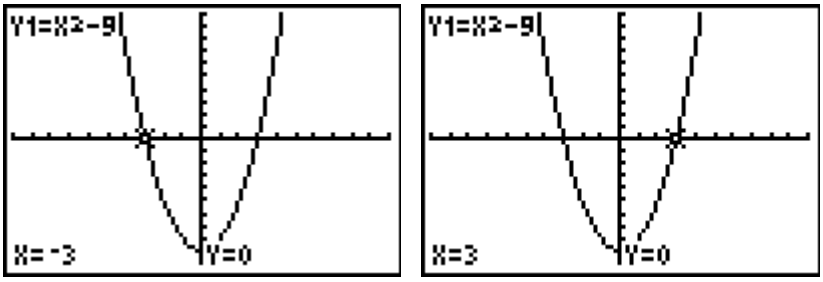






To find the "real" solution for the graph of $y = x^2 - 9$, find where the graph crosses the *x*-axis. The graphs and also the table of values can be used to find the "real" roots of the equation. Notice that in the graphs below, the parabola crosses the *x*-axis at –3 and 3.

* Note that when the graph crosses the *x*-axis, the *y*-value will equal zero.



Thus, the real roots of the equation are $x = -3$ or $x = 3$.

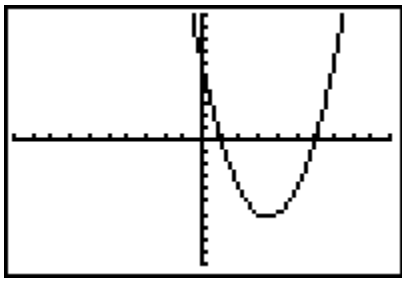
Example 2: Solve by graphing to find the roots of the equation $y = x^2 - 7x + 6$.

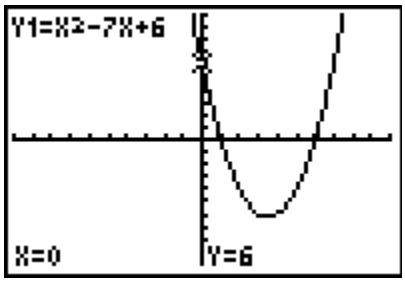
A table of values can be used to help graph and solve the equation.

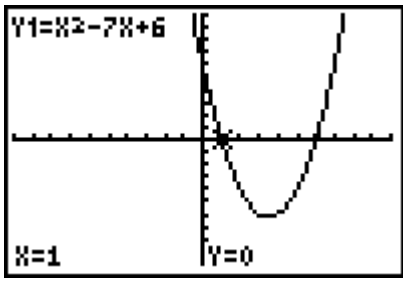
x	y	$y = x^2 - 7x + 6$
0	6	$(0)^2 - 7(0) + 6$
1	0	$(1)^2 - 7(1) + 6$
2	–4	$(2)^2 - 7(2) + 6$
3	–6	$(3)^2 - 7(3) + 6$
4	–6	$(4)^2 - 7(4) + 6$
5	–4	$(5)^2 - 7(5) + 6$
6	0	$(6)^2 - 7(6) + 6$
7	6	$(7)^2 - 7(7) + 6$

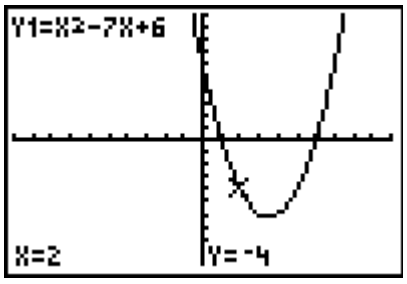
In the graphs below, each point from the table is shown graphed. The graphed point is marked with a star. The resulting curved graph for all the given points plus the points in between is the parabola shown immediately below.

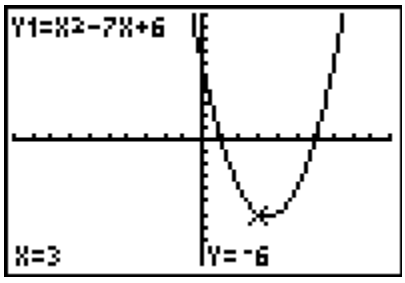
$$y = x^2 - 7x + 6$$

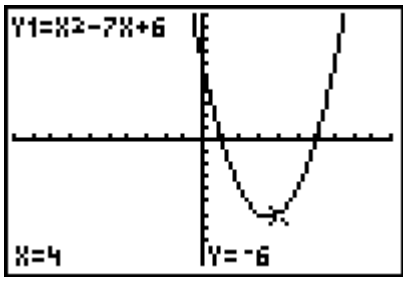


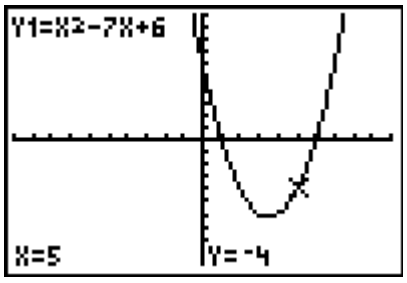

Y1=X²-7X+6
X=0
Y=6

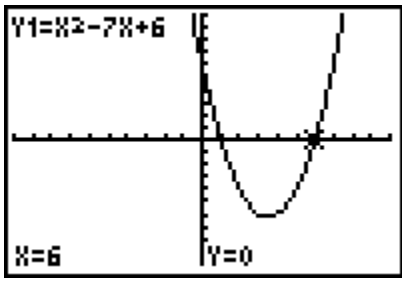

Y1=X²-7X+6
X=1
Y=0

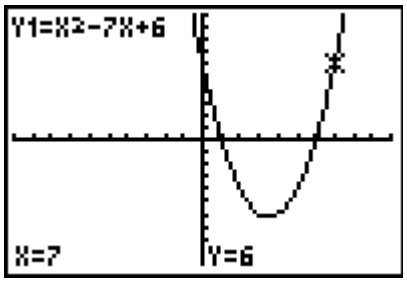

Y1=X²-7X+6
X=2
Y=-4


Y1=X²-7X+6
X=3
Y=-6

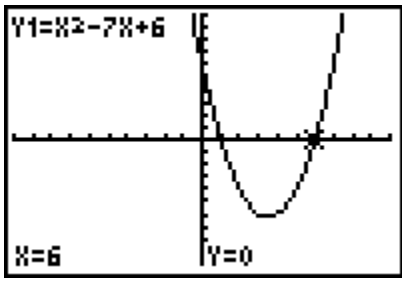

Y1=X²-7X+6
X=4
Y=-6

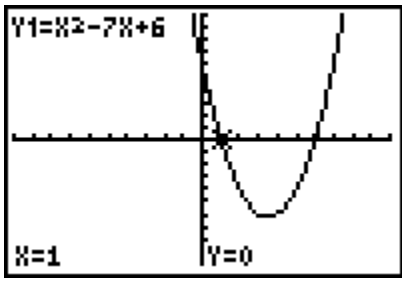

Y1=X²-7X+6
X=5
Y=-4


Y1=X²-7X+6
X=6
Y=0


Y1=X²-7X+6
X=7
Y=6

To find the "real" solution for the graph of $y = x^2 - 7x + 6$, find where the graph crosses the x-axis. The graphs and also the table of values can be used to find the "real" roots of the equation. Notice that in the graphs below, the parabola crosses the x-axis at 1 and 6.





Thus, the solution is $x = 1$ or $x = 6$.

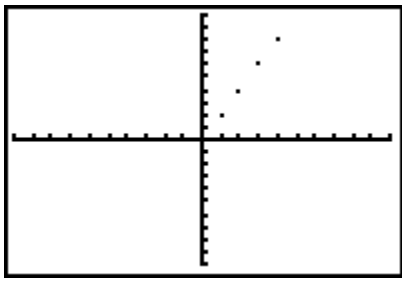
*On some occasions the parabola may not cross the x-axis. In this case, the roots are not "real" numbers. This type of quadratic will be studied in a more advanced math class.

Linear and Nonlinear Equations

Now that we have graphed equations, let's see if we can reverse the process to predict what the equation might look like if we only "see the graph or the table of values". In this section we will examine both a linear graph and a quadratic graph.

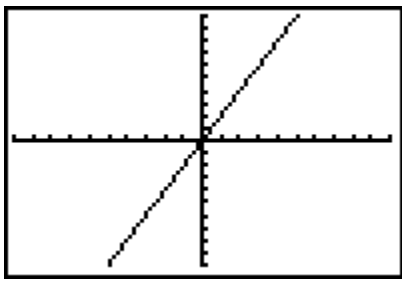
Example 1: Study the table of values and look for a relationship between the *x*-values and the *y*-values.

x	***y***
1	2
2	4
3	6
4	8



The *y*-value is equal to the *x*-value multiplied by two. We can write an equation for the relationship, $y = 2x$. The graphed points from the table suggest a linear equation. Since the graph is a straight line, we can assume the equation is **linear** or *x* is to the first degree.

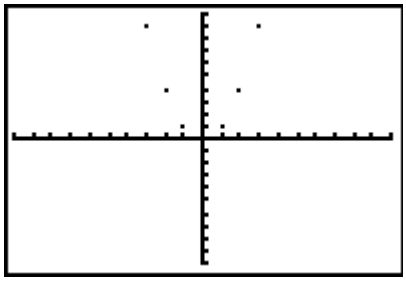
$$\boldsymbol{y = 2x}$$



The equation for the table of values is $y = 2x$ and the graph is **linear**.

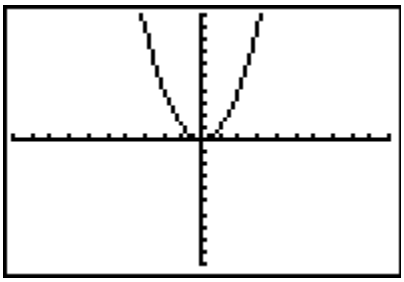
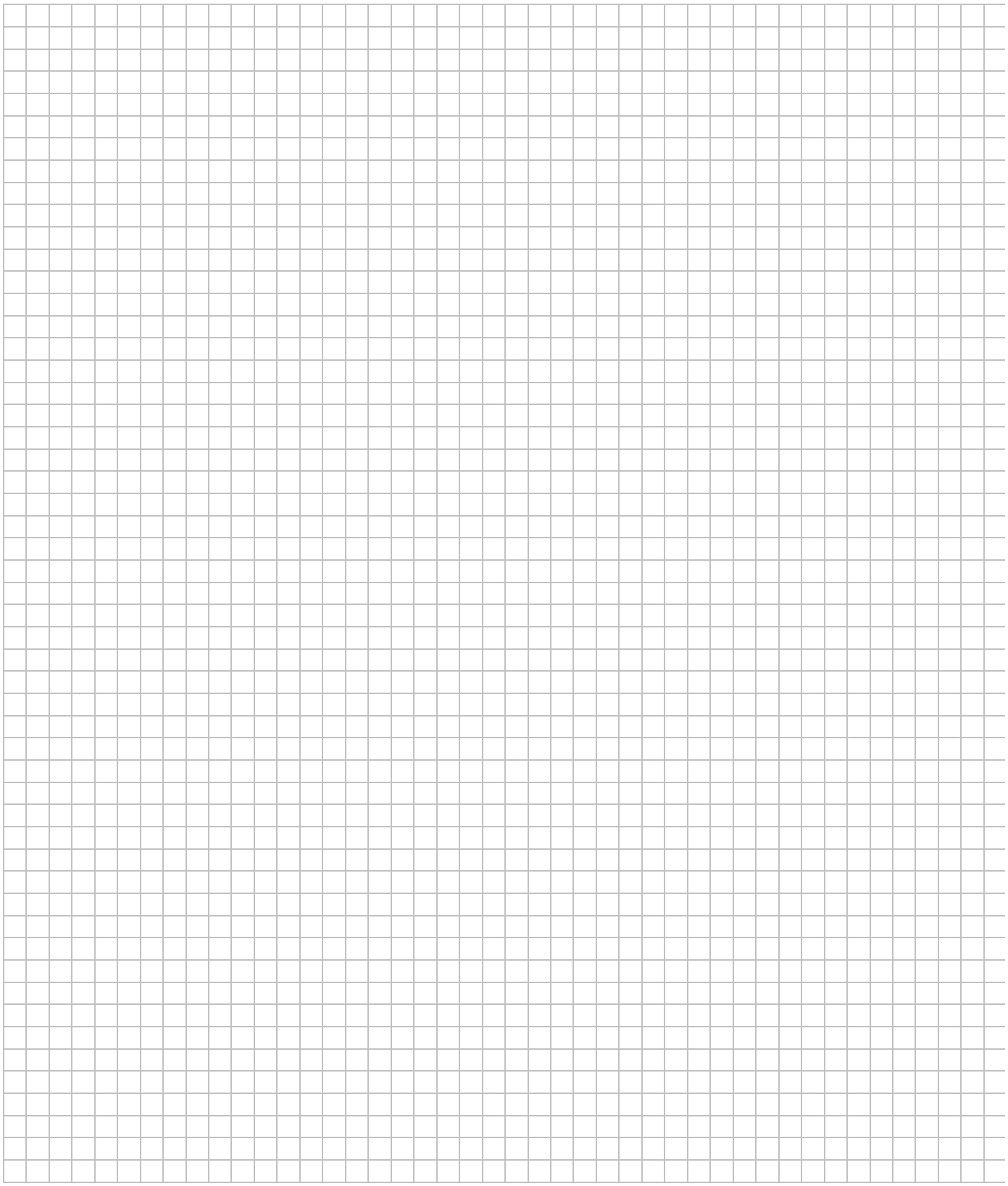
Example 2: Study the table and look for a relationship between the *x*-values and the *y*-values.

x	***y***
–3	9
–2	4
–1	1
0	0
1	3
2	4
3	9



Notice that the negative values are positive *y*-values. Therefore, it can be concluded that the *x*-term is squared and a parabola is the result. The *x*-term is to the second degree or squared.

$$\boldsymbol{y = x^2}$$



The equation for the table of values is $y = x^2$. This graph is **nonlinear** because it forms a curved line and is a parabola, not a straight line.