SLOPES AND CONSTANT RATES

This unit is about slope and the meaning of slope. Slope can be calculated by counting the rise and run between two points on a line, and then writing the ratio “rise / run”. Slope can also be calculated by using the coordinates of two points on the line and the slope formula. Equations can be expressed in slope-intercept form ($y = mx + b$) where m represents the slope and b represents the y-intercept. Equations can be graphed by finding the x-intercept and the y-intercept or by using the slope and y-intercept method.

Linear equations demonstrate constant rates of change and may be used to make predictions. This combination of ideas will make the analysis of the information easier to interpret. When working with data, (1) examine it closely, (2) imagine or chart the data as a graph, and then (3) make predictions about future values. Analyzing data with charts and graphs has real life applications. This is how investors predict financial plans or how stock traders decide when to sell for profit.

Slope

Graphing a Line Using the Intercepts

Constant Rates of Change and Predicting Solutions

Slope

The slope of a line describes the steepness of the line. The slope is the ratio of vertical rise to horizontal run.

To find the slope of a line graphed on a coordinate plane

-Identify a point on the line.

-From that point move up or down until you are directly across from the next point.

-Move left or right to the next point.

Example 1: From the graph below determine the slope of the line.

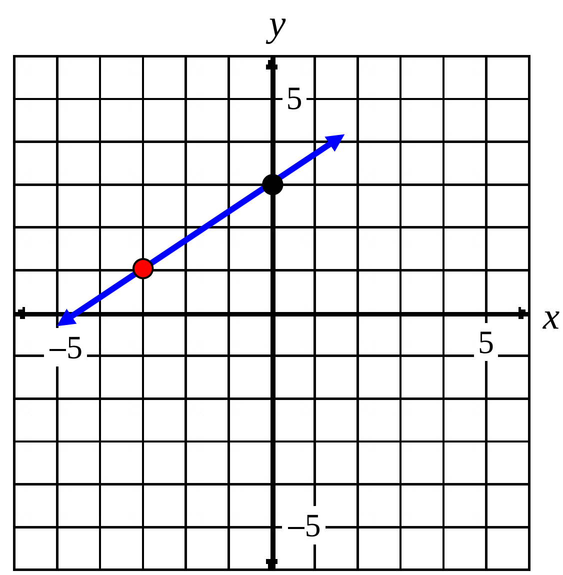
-Put your pencil on the red point.

-Move straight up (vertical rise) until your pencil is in the same line as the black point, (2 units)

-Move right (horizontal run) until you reach the black point. (3 units)

-Slope = $\dfrac{\text{vertical rise}}{\text{horizontal run}}$

You have now determined the slope of the line to be $\dfrac{2}{3}$.



*Just remember: slope $= \dfrac{\text{rise}}{\text{run}}$

Example #2: From the graph below determine the slope of the line.

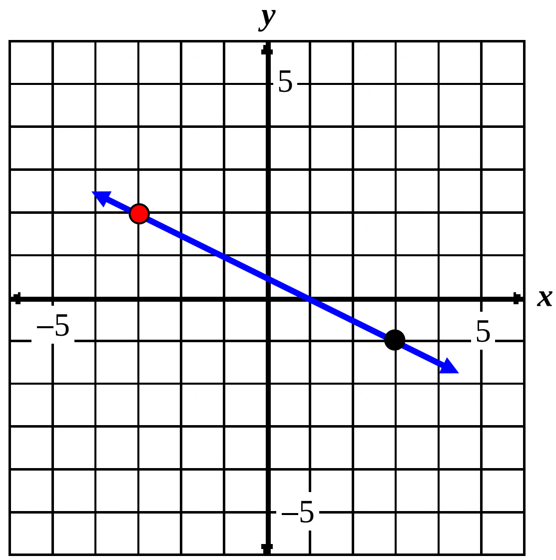
-Put your pencil on the red point.

-Move straight down (vertical rise) until your pencil is in the same line as the black point, (–3 units)

-Move right (horizontal run) until you reach the black point. (6 units)

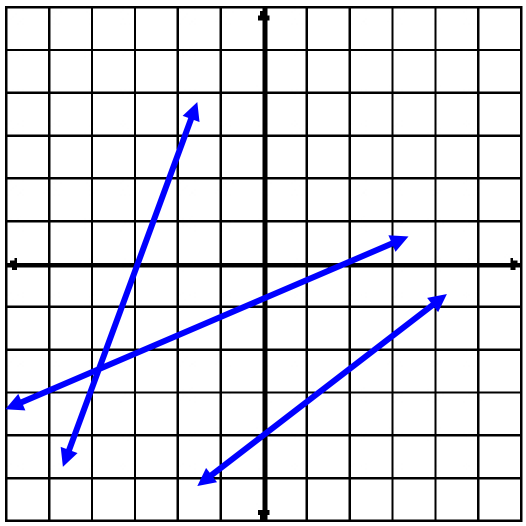
-Slope = $\frac{\text{rise}}{\text{run}}$

You have now determined the slope of the line to be $\frac{-3}{6}$. This slope can be reduced to $-\frac{1}{2}$.

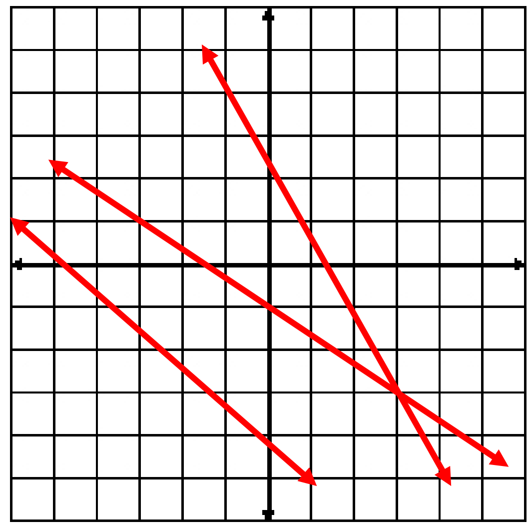


*Notice that if you count down one unit (rise) from the red point and right two units (run), you will be on a point of the line.

On a coordinate plane there are lines that have positive slopes and lines that have negative slopes. Below is an illustration of both.



Lines with positive slopes rise to the right.



Lines with negative slopes rise to the left.

At this point we are going to learn how to find the slope of a line by using two points that lie on the line.

The definition of slope (m) states that given two points (x_1, y_1) and (x_2, y_2), the formula for finding the slope of a line containing these points is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

*Notice, this formula represents the vertical change over the horizontal change.

Example #3: Find the slope of the line containing point A(–2, –6) and B(3, 5).

Let point B = (x_2, y_2) and point A = (x_1, y_1).

$$(x_2, y_2) = (3,5) \quad \rightarrow \quad x_2 = 3, \quad y_2 = 5$$

$$(x_1, y_1) = (-2,-6) \quad \rightarrow \quad x_1 = -2, \quad y_1 = -6$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5-(-6)}{3-(-2)}$$

$$m = \frac{11}{5}$$

The slope (m) of the line is $\frac{11}{5}$.

Example #4: Find the slope of the line containing point C(5, –2) and D(8, –2).

Let point D = (x_2, y_2) and point C = (x_1, y_1).

$(x_2, y_2) = (8, -2) \quad \rightarrow \quad x_2 = 8, \quad y_2 = -2$

$(x_1, y_1) = (5, -2) \quad \rightarrow \quad x_1 = 5, \quad y_1 = -2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2 - (-2)}{8 - 5}$$

$$m = \frac{-2 + 2}{8 - 5}$$

$$m = \frac{0}{3} = 0$$

*Zero divided by a number is zero. $(0 \div 3 = 0)$.

The slope (m) of the line is 0.

This line is a horizontal line. The slopes of horizontal lines are zero.

Example #5: Find the slope of the line containing the point E(9, 4) and F(9, 1).

Let point F = (x_2, y_2) and point E = (x_1, y_1).

$(x_2, y_2) = (9,1) \quad \rightarrow \quad x_2 = 9, \quad y_2 = 1$

$(x_1, y_1) = (9,4) \quad \rightarrow \quad x_1 = 9, \quad y_1 = 4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-4}{9-9}$$

$$m = \frac{-3}{0} = \text{undefined}$$

*Division by zero is undefined ($-3 \div 0 = \mathit{undefined}$).

The slope (*m*) of the line is undefined.

This line is a vertical line. The slopes of vertical lines are undefined.

Graphing a Line Using the Intercepts

Another way to graph linear equations is by using the x-intercept and the y-intercept.

- The x-intercept is the point at which the line crosses the x-axis.
- The y-intercept is the point at which a line crosses the y-axis.

To find the intercepts:

1) To locate the ***y*-intercept** $(0, y)$, replace ***x* with 0** in the equation and solve for y.

2) To locate the ***x*-intercept** $(x, 0)$, replace ***y* with 0** in the equation and solve for x.

3) Plot the two points and connect them with a straight edge.

*Remember: To find the ***x*-intercept, let *y* = 0** and to find the ***y*-intercept, let *x* = 0**.

Example: Graph $2x - 3y = 6$ by using the x- and y-intercepts.

1) To find y-intercept, let $x = 0$.

$2x - 3y = 6$

$2(0) - 3y = 6$

$-3y = 6$

$y = -2$

y-intercept = $(0, -2)$

2) To find x-intercept, let $y = 0$.

$2x - 3y = 6$

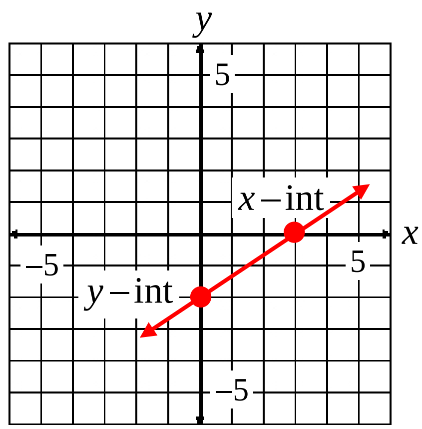
$2x - 3(0) = 6$

$2x = 6$

$x = 3$

x-intercept = $(3, 0)$

The line crosses the *y*-axis at (0, –2) and it crosses the *x*-axis at (3, 0).



The graph of the equation, $2x - 3y = 6$, is a straight line that passes through points (0, –2) and (3, 0).

Constant Rates of Change and Predicting Solutions

In the problem below, we will examine a constant rate of change and use that rate to make a prediction.

On a trip across country, the Wilson family was able to travel an average of 50 miles per hour for several days. They drove nine hours per day and spent the other 15 hours per day sightseeing and sleeping. Establish a table to determine how many miles, days, and hours were spent on the two week trip.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hours driving	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
Miles (d=50 h)	450	900	1350	1800	2250	2700	3150	3600	4050	4500	4950	5400	5850	6300	6750

Based on data in the table, we can write relationships and equations about the data.

To find the number of hours (*H*) spent driving, we can multiply the number of days (*D*) times 9. A formula to represent this relationship is:

$$H = 9D$$

To find the miles traveled (*M*), we can multiply 9 hours per day times the number of days (*D*) time 50, the average rate of speed. A formula to represent this relationship is:

$$M = 9D(50)$$

or

$$M = 9(50)D$$

Using this chart and a little algebra, we can predict much about a long trip or a similar situation like this one.

Example 1: Answer the follow questions about the table and relationships discussed about the Wilson's family trip.

a) Predict how many hours the Wilson family would drive after 20 days?

$$H = 9d$$
$$H = 9(20)$$
$$H = 180$$

The Wilson family would drive 180 hours in 20 days.

b) Predict how many miles the Wilson family would travel in 20 days.

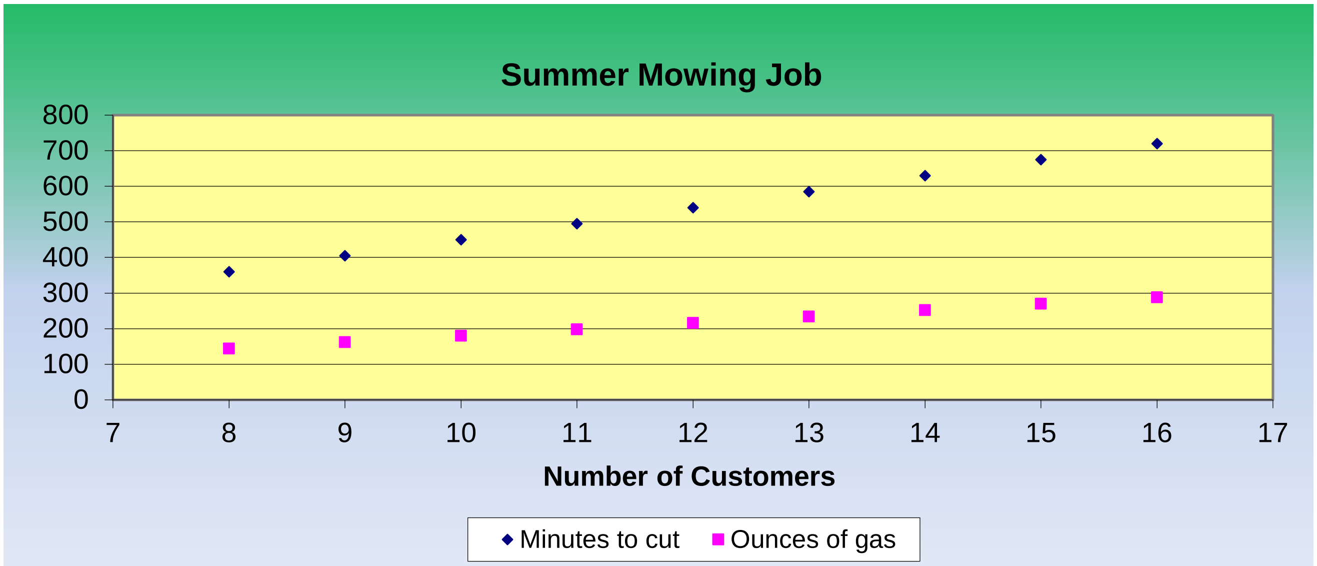
$$M = 9(50)d$$
$$M = 9(50)(20)$$
$$M = 9000$$

The Wilson family would travel 9000 miles in 20 days.

Example 2: Set up a chart to analyze Joe's work on his summer job. Joe mows lawns for customers in his neighborhood. He will spend 45 minutes mowing each lawn and use about 18 ounces of gasoline per lawn. Joe prepared a chart and graph of his work schedule and load for 8 through 16 customers.

Customers	8	9	10	11	12	13	14	15	16
Minutes to cut	360	405	450	495	540	585	630	675	720
Ounces of gas	144	162	180	198	216	234	252	270	288

Joe can now use the chart and graph to decide how many customers to service during the summer.



Notice how the values increase as the number of customers increase. This prediction is a positive correlation as well as a positive slope.

If Joe had 22 customers, (a) how many minutes would he spend mowing grass? (b) How much gasoline would he need?

(a) Since Joe is allowing 45 minutes per customer, write the following relationship: Number of minutes (M) = 45 x number of customers (C).

$$M = 45 \times C$$
$$M = 45\ (22)$$
$$M = 990$$

Joe would spend 990 minutes mowing for 22 customers.

(b) Since Joe is allotting 18 ounces of gasoline per lawn, he could write the following relationship: number of gallons of gasoline (G) = 18 x number of customers (C).

$$G = 18 \times C$$
$$G = 18 \times 22$$
$$G = 396$$

Joe would use 396 ounces of gasoline for 22 customers.