# SURFACE AREA AND VOLUME

In this unit, you will use different formulas and procedures to find the volume and surface area of many solids. It may be helpful to draw some of the shapes as nets and write the formula or procedure to solve them below the drawing. You may use a calculator to help with the computations. Use the calculator carefully and double check all calculations.

Surface Area

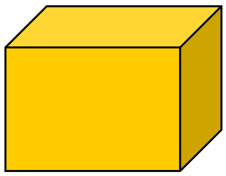
Volume

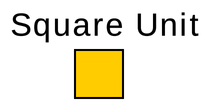
Using Precision When Calculating Measurements

## Surface Area

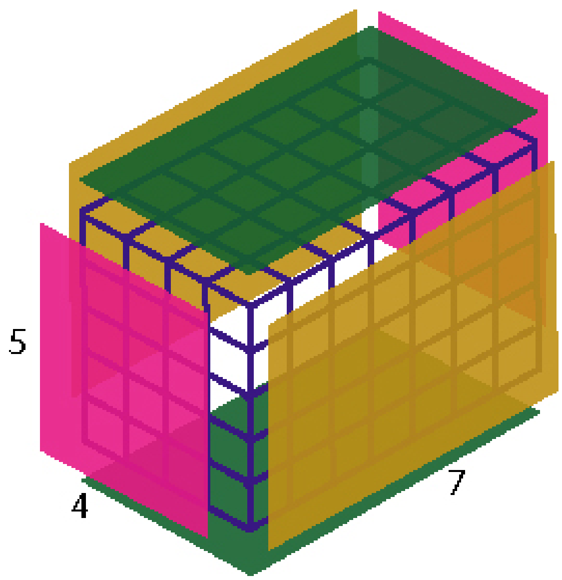
When asked to find surface area, find the areas of the faces of the solid by using formulas previously discovered, and then add the areas of the faces together to find the total.  Surface is measured in square units.

## Surface Area of a Rectangular Prism



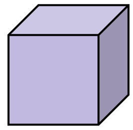



*Example 1*:  Compute the surface area of a rectangular prism with a width of four feet, a length of seven feet, and a height of five feet.

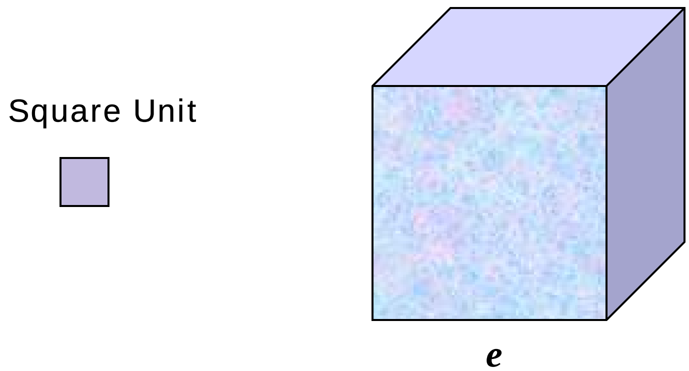


| Face | Computation | Area |
|---|---|---|
| Front<br>7 ft across by 5 ft tall | 7 × 5 | 35 sq ft |
| Back<br>(same as front, hidden from view)<br>7 ft across by 5 ft tall | 7 × 5 | 35 sq ft |
| Bottom<br>(side the box is sitting on)<br>7 ft across by 4 ft wide | 7 × 4 | 28 sq ft |
| Top<br>(same as the bottom)<br>7 ft across by 4 ft wide | 7 × 4 | 28 sq ft |
| Left Side<br>5 ft wide by 4 ft tall | 5 × 4 | 20 sq ft |
| Right Side<br>(same as left side, hidden from view)<br>5 ft wide by 4 ft tall | 5 × 4 | 20 sq ft |
| Total Surface Area<br>(Add together all of the areas of the six faces.) | | **166 sq ft** |

# Surface Area of a Cube



A cube is a special rectangular prism with all of its edges measuring the same and all of its faces having the same area.  The surface area of a cube is the total area of all of the square faces measured in square units.




The area of one face of a cube is a square.

Let $e$ represent the length of one edge.

Then, the area of one face can be represent by $e \times e$ or $e^2$.

Since a cube has six faces, the total surface area of a cube is $6 \times e^2$.

“T” will be used to represent surface area in the formulas developed in this unit.

The formula for the surface area of a cube is:

> The surface area ($T$) of a cube is:
>
> $$T = 6e^2$$

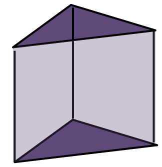
*Example 2*:  Find the surface area of a cube with an edge measuring seven inches.

$$T = 6e^2$$
$$T = 6 \times 7^2$$
$$T = 6 \times 49$$
$$T = 294$$

Check:  Area of one face is 7 × 7 or 49 square inches.

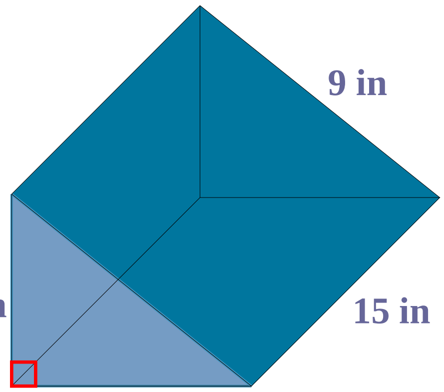
Area of six faces is 49 × 6 or 294 square inches.

## Surface Area of a Triangular Prism



A triangular prism is a prism that has parallel bases shaped like triangles and sides shaped like rectangles.

*Example 3:* Find the surface area of a triangular prism with a height of 15 inches and a base which is a right triangle. The dimensions of the right triangle are a base measuring seven inches and a height measuring five inches.




*Step 1*: Find the area of the two triangular faces.

The formula to find the area of one triangle is $A = \frac{1}{2}bh$.

$$A = \frac{1}{2} \times 7 \times 5$$
$$A = 17.5$$

*Both triangular faces have the same area.

The area of both triangular faces is $2 \times 17.5 = 35$ square inches.

*Step 2*: Find the area of the three rectangular faces.

The formula to find the area of a rectangle is $A = lw$.

*Rectangle 1*: The largest rectangular face measures 15 inches by 9 inches.

$$A = l \times w$$
$$A = 15 \times 9$$
$$A = 135 \text{ sq in}$$

*Rectangle 2*: The rectangular face on which the prism is resting measures 15 inches by 7 inches.

$$A = l \times w$$
$$A = 15 \times 7$$
$$A = 105 \text{ sq in}$$

*Rectangle 3*: The back rectangular face measures 15 inches by 5 inches.

$$A = l \times w$$
$$A = 15 \times 5$$
$$A = 75 \text{ sq in}$$

*Step 3*: Find the **total** area of all five faces.

$$T = 2 \text{ Triangular Areas} + 3 \text{ Rectangular Areas}$$
$$T = 2(17.5) + 135 + 105 + 75$$
$$T = 350 \text{ in}^2$$

The surface area of the triangular prism is 350 square inches.

## Surface Area of a Pyramid

To find the surface area of a pyramid, again, just add the areas of the faces of the solid.  Notice that the sides of a pyramid are triangles.

Please pay particular close attention to the difference between the height of a pyramid and the height along the side of the pyramid which is called the **slant height**.

*The slant height is used when calculating surface area, not the height of the pyramid itself.

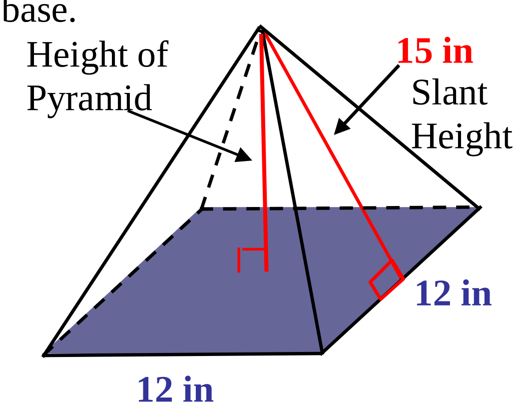
*Example 4:*  Find the surface area of a pyramid with a square base measuring 12 inches on one edge and a slant height of 15 inches.

*Step 1*:  Find the area of the base.

$A = bh$

$A = 12(12)$

$A = 144 \text{ sq in}$




*Step 2*:  Find the area of the triangular faces.

Since each triangular face has a base of 12 inches and a height of 15 inches, the areas of all four triangles are the same.

Find the area of one triangle, and then multiply by four.

| | |
|---|---|
| $A = \frac{1}{2}bh$ | Area of a triangle |
| $A = \frac{1}{2}(12)(15)$ | Substitute (Use slant height as height.) |
| $A = 90$ | Simplify |

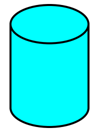
The area of one triangular face is 90 square inches.

The area of all four triangles = 90 × 4 = 360 square inches.

*Step 3*: Find total area of the five faces.

$$T = \text{Area of Square Base} + \text{Area of the Four Triangular Sides}$$
$$T = 144 + 360$$
$$T = 504 \text{ sq in}$$

The surface area of the pyramid is 504 square inches.

# Surface Area of a Cylinder 

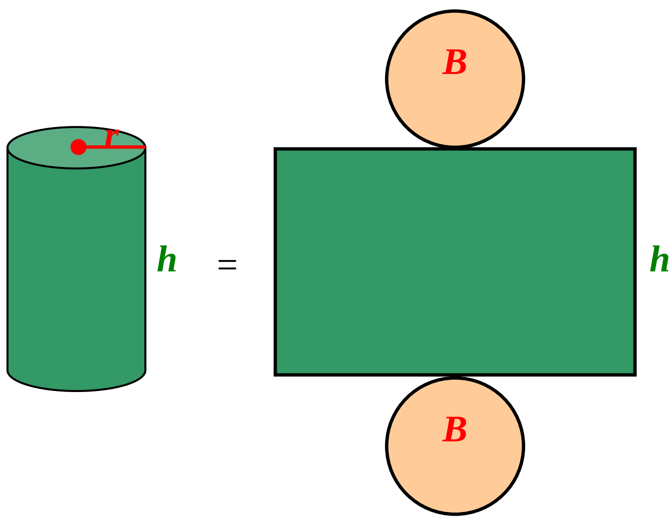
To find the surface area of a cylinder, a little more thinking is involved.

The top and bottom of a cylinder are circles.

The side of a cylinder is one continuous curved surface.  When it is laid flat, it is shaped like a rectangle.


Notice that the length of the rectangle is the same as the circumference (distance around) of the circular base.

The area of the rectangular face is determined by multiplying the circumference of the base circle (length of the rectangle) by the height of the cylinder (width of the rectangle).




The formula for the surface area of a cylinder can be developed as follows:

$$T = \text{Circular Face} + \text{Circular Face} + \text{Curved Surface (Rectangle)}$$

$$T = \pi r^2 + \pi r^2 + C \times h$$

$$T = \pi r^2 + \pi r^2 + 2\pi r \times h$$

$$T = 2\pi r^2 + 2\pi rh$$

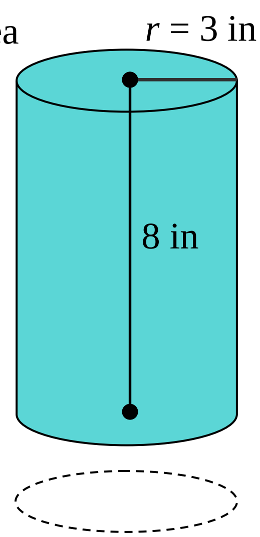
The surface area ($T$) of a cylinder is:

$$\boldsymbol{T = 2\pi r^2 + 2\pi rh}$$

*Example 5:* Find the surface area of a cylinder that has a radius of three inches and a height of eight inches.

*Step 1*: Write the formula for the surface area of a cylinder.

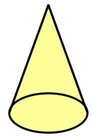
$$T = 2\pi r^2 + 2\pi rh, \text{ where } \pi = 3.14$$




*Step 2*: Substitute the given information in the formula and simplify.

$$T = 2\pi r^2 + 2\pi rh$$
$$T = 2(3.14)(3^2) + 2(3.14)(3)(8)$$
$$T = 2(3.14)(9) + 2(3.14)(3)(8)$$
$$T = 56.52 + 150.72$$
$$T = 207.24$$

The surface area of the cylinder is 207.24 square inches.

## Surface Area of a Cone 

To find the surface area of a cone is the sum of the area of the circular base and the area of the curved face. To find the area of the curved face, multiply "pi" times radius times slant height. The letter *l* is used to represent slant height in the formula.

Please pay particular close attention to the difference between the height of a cone and the height along the side of the cone which is called the **slant height (*l*)**.

*The slant height (*l*) is used when calculating the surface area of a cone, not the height of the cone itself.

The surface area ($T$) of a cone is:

$$\boldsymbol{T = \pi r^2 + \pi r l}$$

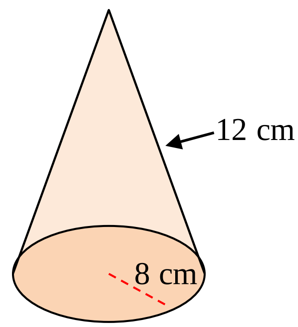
*Example 4:* Find the surface area of a cone with a base that has a radius of eight centimeters and a slant height that is 12 centimeters.

*Step 1*: Write the formula for the surface area of a cone.

$T = \pi r^2 + \pi r l$, where $\pi = 3.14$

*Step 2*: Substitute the given information in the formula and simplify.

$$T = \pi r^2 + \pi r l$$
$$T = (3.14)(8^2) + (3.14)(8)(12)$$
$$T = (3.14)(64) + (3.14)(8)(12)$$
$$T = 200.96 + 301.44$$
$$T = 502.4$$




The surface area of the cone is 502.4 square centimeters.

## Surface Area of a Sphere

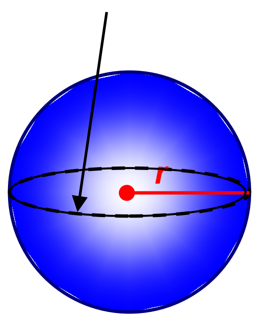
A **sphere** is a three-dimensional figure with all points the same distance from a fixed point, the center.

A **hemisphere** is half of a sphere that is created by a plane that intersects a sphere through its center.

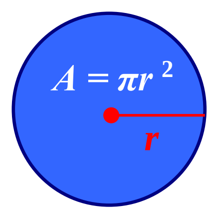
The edge of a hemisphere is a **great circle**.

The surface area of a sphere is four times the **base area**, the area of the circle created by the great circle.

Great Circle




Base Area

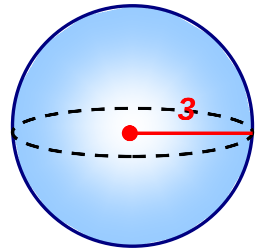



| | | | | |
|---|---|---|---|---|
| Surface area | = | 4 | × | Base Area |
| $T$ | = | 4 | × | $\pi r^2$ |

The surface area ($T$) of a sphere is:

$$T = 4\pi r^2$$

*Example 6:* Find the surface area of a sphere with a radius of two inches round to the nearest square inch.




*Step 1*: Write the formula for the surface area of a sphere.

$$T = 4\pi r^2, \text{ where } \pi = 3.14$$

*Step 2*: Substitute the given information in the formula and simplify.

$$T = 4\pi r^2$$

$$T = 4(3.14)(3^2)$$

$$T = 4(3.14)(9)$$

$$T = 113.04$$

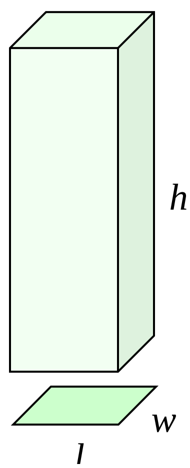
The surface area of the sphere is 113.04 square inches.

## Volume

The volume of a solid is the amount the shape contains. Volume is a measure of capacity and is measure in cubic units.

### Volume of a Rectangular Prism

To calculate the volume of a **rectangular prism**, multiply the area of the base (length × width) times height.

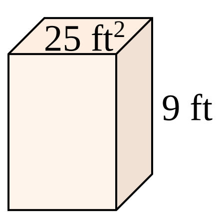



$V$ = Area of Base × Height
$V$ = (length × width) × height
$V = l \times w \times h$

*Example 1*: Compute the volume of a square prism with a base area of 25 square feet and a height of 9 feet.

*A square prism is a prism with a square base.

Since the base area is given, simply multiply the area of the base times the height to calculate the volume.



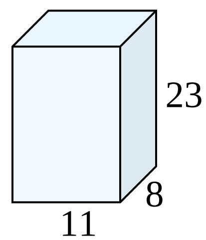

$V$ = Area of base × Height
$V = 25 \times 9$
$V = 225$

**Reminder: Volume is measured in cubic units.*

The volume of the square prism is 225 cubic feet.

*Example 2*: Calculate the volume of a rectangular prism with a length of 11 feet, width of 8 feet, and a height of 23 feet.

Since all dimensions of the prism are given, use the volume formula for a rectangular prism to calculate the volume.

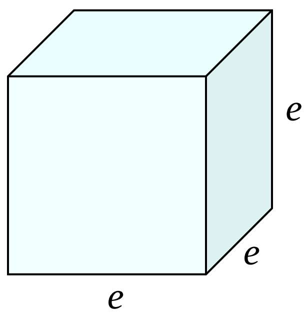



$V = l \times w \times h$
$V = 11 \times 8 \times 23$
$V = 2024$

The volume of the rectangular prism is 2024 cubic feet.

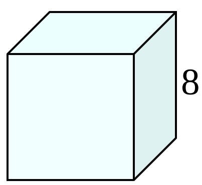
## Volume of a Cube

To calculate the volume of a **cube**, multiply the edge times itself three times.




$V$ = Area of Base × Height
$V$ = (length × width) × height
$V = (e \times e) \times e$
$V = e^3$

*Example 3*: Calculate the volume of a cube with an edge length of eight feet.

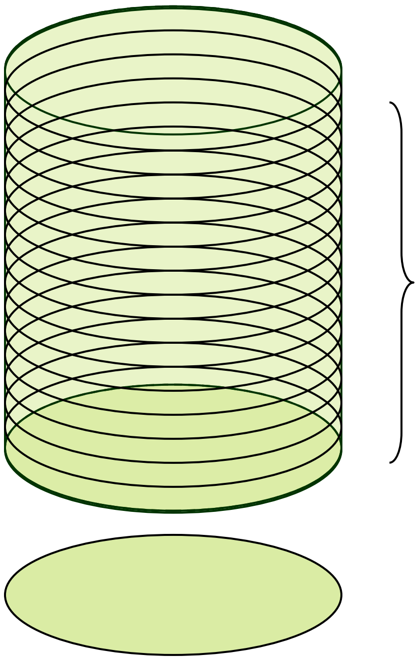



$V = e^3$
$V = 8^3$
$V = 8 \times 8 \times 8$
$V = 512$

The volume of the cube is 512 cubic feet.

## Volume of a Cylinder

The **volume of a cylinder** is the amount a cylinder can hold and is measured in cubic units. To calculate the volume of a cylinder, multiply the area of the base times the height.




Area of Base

$V$ = Area of Base × Height

*The base of a cylinder is a circle. To calculate the area of a circle, use* $A = \pi \times r^2$.

$$V = (\pi \times r^2) \times h$$

*Example 4*: Find the volume of a cylinder with a radius of 12 centimeters and the height is 23 centimeters. (Use 3.14 for "pi".)

$V = \pi \times r^2 \times h$

$V = 3.14 \times 12^2 \times 23$

$V = 3.14 \times 144 \times 23$

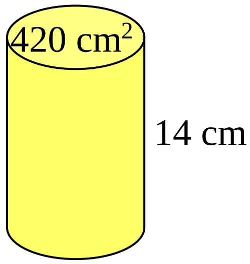
$V = 10{,}399.68$

******Reminder: Volume is measured in cubic units.*

The volume of the cylinder is 10,399.68 cubic centimeters ($cm^3$).

*Example 5*:  Compute the volume of a cylinder with a base area of 420 square centimeters and a height of 14 centimeters.

Since the base area is given, simply multiply the area of the base times the height to calculate the volume.




$V$ = Area of base × Height
$V = 420 \times 14$
$V = 5880$

*****_Reminder:  Volume is measured in cubic units._

The volume of the cylinder is 5880 cubic centimters.

*Example 6*: If the volume of a cylinder is 628 cubic feet and the height of the cylinder is eight feet, what is the radius of the cylinder? (Use 3.14 for "pi".)

$V = \pi \times r^2 \times h$

$628 = 3.14 \times r^2 \times 8$ Substitute $(V = 628 \text{ and } h = 8)$

$628 = 3.14 \times 8 \times r^2$ Apply the commutative property to switch the positions of 8 and $r^2$.

$628 = 25.12 \times r^2$ Simplify ($3.14 \times 8 = 25.12$)

$\frac{628}{25.12} = \frac{\cancel{25.12} \times r^2}{\cancel{25.12}}$ Divide both sides by 25.12.

$25 = r^2$ Simplify.

$r = 5$ Since $r^2 = 25,$ take the square root of 25 to determine $r$.

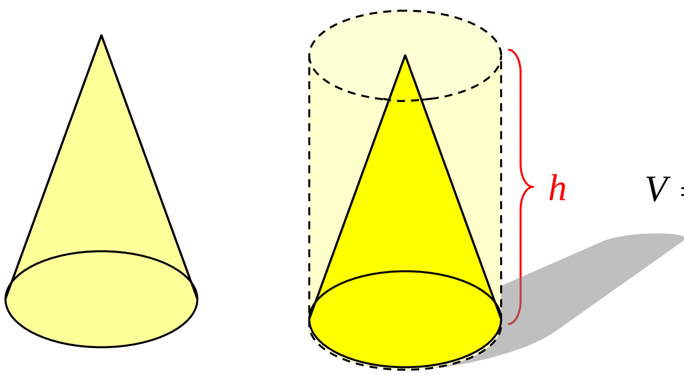
The radius of the cylinder is 5 feet.

*Since the answer is the length the radius, a linear measure, use feet (not cubic feet) to label the answer.

## Volume of a Cone

The volume of a cone can be found in a similar way to finding the volume of a cylinder. The difference is that the volume of a cone is *considerably less* than the volume of a cylinder with the same dimensions (radius and height).

*The volume of a cone is equal to 1/3 the volume of a cylinder with the same dimensions.




$$V = \frac{1}{3}\pi r^2 \times h$$

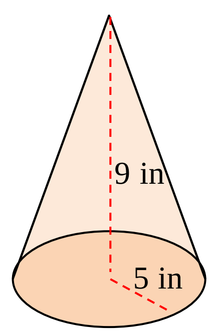
*Example 7*: Find volume of a cone with a radius of five inches and the height is nine inches.

$$V = \frac{1}{3}\pi r^2 \times h$$

$$V = \frac{1}{3} \times 3.14 \times 5^2 \times 9$$

$$V = \frac{1}{3} \times 3.14 \times 25 \times 9$$

$$V = 235.5$$



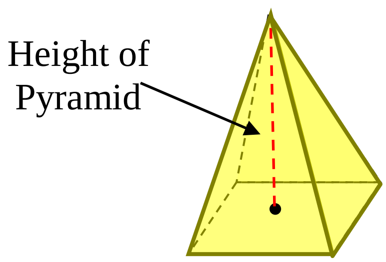

**Reminder:  Volume is measured in cubic units.*

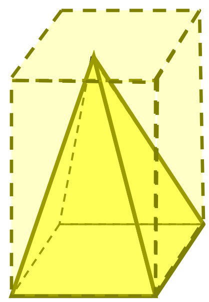
The volume of the cone is 235.5 cubic inches (in$^3$).

## Volume of a Pyramid

The volume of a pyramid is related to a rectangular prism that has the same dimensions (length, width, and height). As with the comparison of the volume of a cylinder and cone that have the same dimensions, the volume of a pyramid is *considerably less* than the volume of the rectangular prism with the same dimensions.

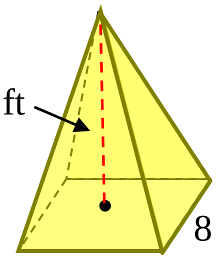
*The volume of a pyramid is equal to 1/3 the volume of a rectangular prism with the same dimensions.



$$V = \frac{1}{3}Bh$$

When calculating the volume of a pyramid, multiply 1/3 times the area of the base (*B*) times the height.




*Example 8*: Find the volume of a square pyramid (base is a square) with the edge of the square base measuring eight feet and the height of the pyramid measuring 12 feet.

$$V = \frac{1}{3}\, Bh$$

$$V = \frac{1}{3} \times (8 \times 8) \times 12$$ Area of square base equals $8^2$ or $8 \times 8$.

$$V = \frac{1}{3} \times 64 \times 12$$ Simplify

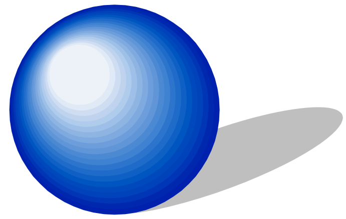
$$V = 256$$

**Reminder: Volume is measured in cubic units.*

The volume of the pyramid is 256 cubic feet (ft$^3$).

## Volume of a Sphere

The volume of a sphere is calculated using the formula $V = \frac{4}{3}\pi r^3$.



*Example 9*: Calculate the volume of a sphere with a radius of seven centimeters, and round the answer to the nearest hundredth.

| | |
|---|---|
| $V = \frac{4}{3}\pi r^3$ | |
| $V = \frac{4}{3}(3.14)(7^3)$ | Subsitute the given radius. |
| $V = \frac{4}{3} \times 3.14 \times (343)$ | $7 \times 7 \times 7 = 343$ |
| $V = 1436.0266...$ | Simplify |
| $V = 1436.03$ | Round to the nearest hundredth. |

**Reminder*: Volume is measured in cubic units.*

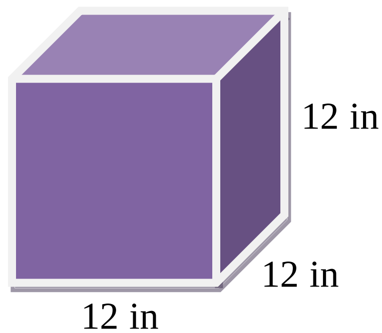
The volume of the sphere is 1436.03 cubic centimeters ($cm^3$).

## Using Precision When Calculating Measurements

When calculating various surface areas or volumes, different units of measure can affect the precision of the final measure.  The smaller unit (more precise) will provide a more accurate measure.

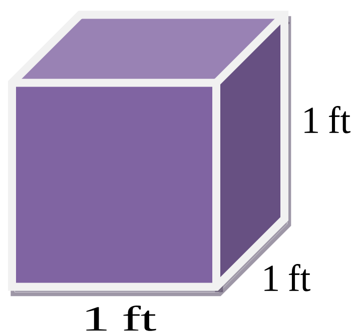
*For example*, if we measure a cube by inches 12 inches on an edge, we can find the volume in cubic inches.

$V = 12 \times 12 \times 12 = 1728 \text{ in}^3$



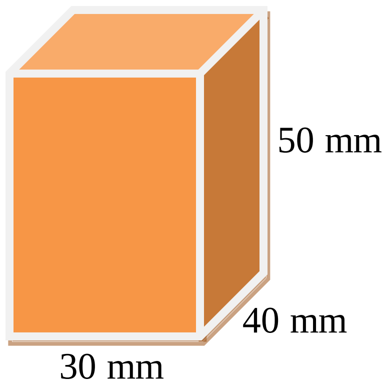

However if we measure the same cube with feet, we find the volume in cubic feet.

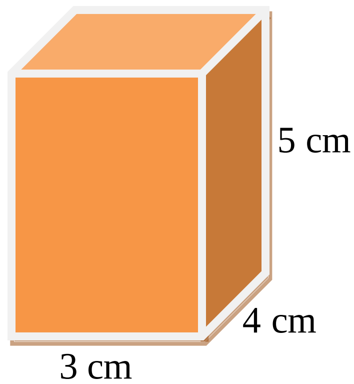
$V = 1 \times 1 \times 1 = 1 \text{ ft}^3$




With the first measurement, a smaller unit, it is possible to experimentally determine the accuracy of the measurement more easily.  In the second, the accuracy of measurement is much less precise.  In other words, think what is more or less than a foot as compared to what is more or less than one inch?

*Example*: Compare the precision of the volume of the same rectangular prism with the measurements shown below. (Recall that 1 cm = 10 mm.)




$V = 30 \times 40 \times 50$
$V = 60,000$

$\text{Volume} = 60,000 \text{ mm}^3$

$V = 3 \times 4 \times 5$
$V = 60$

$\text{Volume} = 60 \text{ cm}^3$

If we are given both measurements (centimeters and millimeters), then we must be careful to select one unit with which to calculate. Remember, if we pick the smaller unit of measure, it will give the more precise result.