PERIMETER, CIRCUMFERENCE AND AREA

In this unit, you will work with two-dimensional shapes. You will measure perimeter, circumference and area. You will use established formulas to make computations.

Perimeter

Circumference

Area of a Rectangle and a Triangle

Area of a Square and a Parallelogram

Area of a Trapezoid

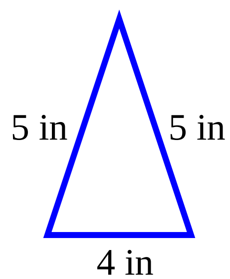
Area of Circle

Formula Chart for Perimeter, Circumference, and Area

Perimeter

perimeter – Perimeter is the distance around an object.

Example 1: Find the perimeter of the triangle.

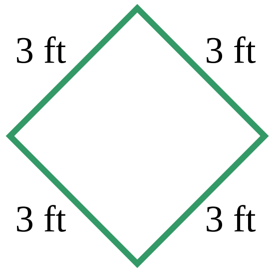


To determine the perimeter, total all the sides of the triangle.

$P = 5 + 5 + 4 = 14$

The perimeter of the triangle is 14 inches.

Example 2: Find the perimeter of the square.



Since a square has all sides equal in length, a formula may be written for finding the perimeter.

Let s represent one side of a square.

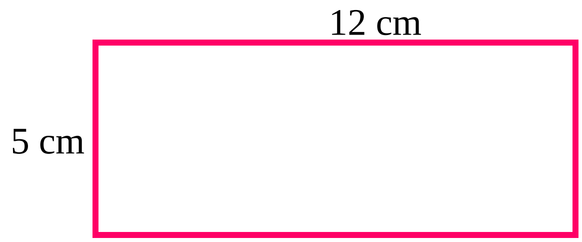
$P = s + s + s + s$	Add up all the sides.
$P = 4s$	Collect the s's.

Calculate the perimeter.

$P = 4s$	Formula for the perimeter of a square.
$P = 4(3)$	Substitute 3 for s.
$P = 12$	Simplify

The perimeter of the square is 12 feet.

Example 3: Find the perimeter of the rectangle.



In a rectangle, the opposite sides have the same measure, so a formula may be written to find the perimeter.

Let l represent the length of the rectangle and w represent the width of the rectangle.

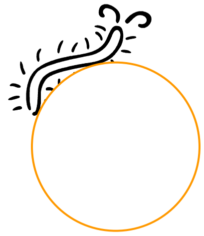
$P = l + l + w + w$	Add up all the sides.
$P = 2l + 2w$	Collect the l's and the w's.

Calculate the perimeter.

$P = 2l + 2w$	Formula for the perimeter of a rectangle.
$P = 2(12) + 2(5)$	Substitute 12 for l and 5 for w.
$P = 24 + 10$	Simplify
$P = 34$	Add

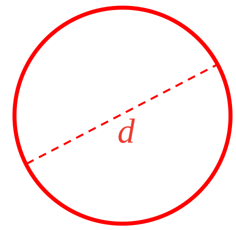
The perimeter of the rectangle is 34 feet.

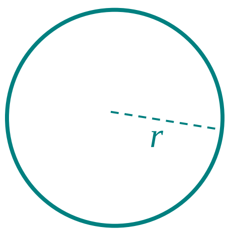
Circumference



circumference – Circumference is the total distance around a circle.

diameter – The diameter of a circle is the length across the widest part of the circle. It is represented by a line segment that extends from one point on the circle to the opposite side of the circle and passes through the center of the circle.





radius – The radius of a circle is half the length of the diameter of a circle. It is represented by a line segment that extends from the center of the circle to a point on the circle.

radii– Radii is the plural of radius.

Pi – Pi is an irrational number that represents the ratio of the circumference of a circle to the diameter $(\frac{C}{d})$ of a circle. Pi is a decimal that never ends and never develops into a repeating pattern.

π = 3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679
8214808651 3282306647 0938446095 5058223172 5359408128
4811174502 8410270193 8521105559 6446229489 5493038196
4428810975 6659334461 2847564823 3786783165 2712019091
4564856692 3460348610 4543266482 1339360726 0249141273
7245870066 0631558817 4881520920 9628292540 9171536436
7892590360 0113305305 4882046652 1384146951 9415116094
3305727036 5759591953 0921861173 8193261179…

Calculating Circumference

To determine the circumference of a circle, multiply the diameter of the circle times “pi”. To simplify the calculation, round “pi” to 3.14.

$$C = \pi d$$

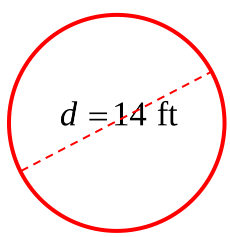
Since a radius is half the length of a diameter, we can say that a diameter equals two radii. Thus, a second formula can be written for calculating circumference.

$$C = \pi \times d \quad \rightarrow \quad C = \pi \times (2 \times r) \quad or \quad C = 2 \times \pi \times r$$

To determine the circumference of a circle when given the radius, multiply two times “pi” times the radius of the circle.

$$C = 2\pi r$$

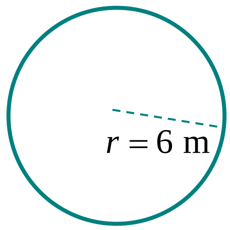
Example 1: Find the circumference of a circle that has a diameter equal to 14 feet.



$$C = \pi d$$
$$C = 3.14 \times 14$$
$$C = 43.96 \text{ ft}$$

The circumference of the circle is 43.96 feet.

Example 2: Find the circumference of a circle that has a radius equal to 6 meters.



$$C = 2\pi r$$
$$C = 2 \times 3.14 \times 6$$
$$C = 37.68 \text{ m}$$

The circumference of the circle is 37.68 meters.

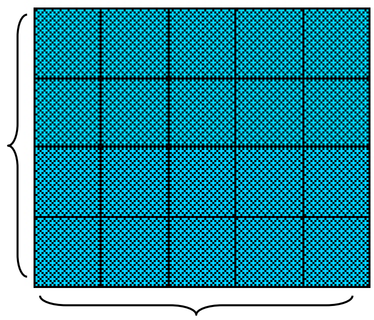
Area of a Rectangle and a Triangle

The **area of a rectangle** is the product of the length and the width.

Area is a measurement of coverage and is measured in **square units**.

$$A = lw$$

Example 1: Find the area of a rectangle that measures 5 units by 4 units.

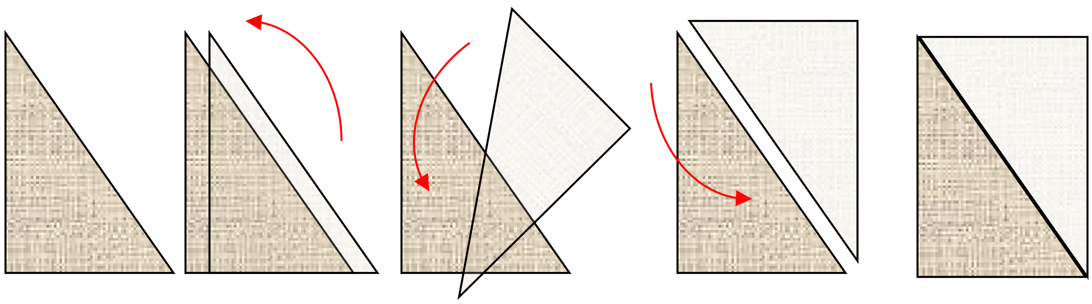


$$A = lw$$
$$A = 5 \times 4$$
$$A = 20 \text{ square units}$$

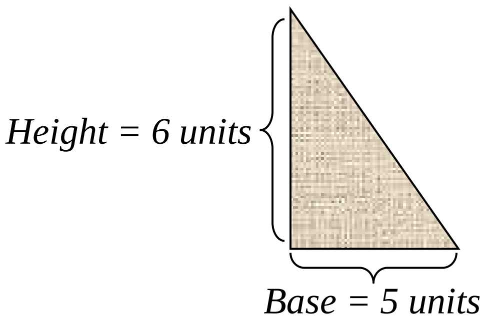
The area of the rectangle is 20 square units.

The **area of a triangle** is equal to half the area of a rectangle with the same base and height. Study the figure below and follow the arrows to see that the area is only half as much.

$$A = \frac{1}{2}bh$$



Example 2: Find the area of a triangle that measures 5 units by 6 units.



$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 5 \times 6$$

$$A = \frac{1}{2} \times 30$$

$$A = 15 \text{ square units}$$

The area of the triangle is 15 square units.

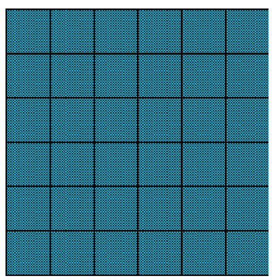
Area of a Square and a Parallelogram

Area is a measurement of coverage and is measured in **square units**.

The **area of a square** is the product of its length and width. Since squares have sides of equal length, the area of a square is the product of its length (side) and its width (side).

$$A = lw$$
$$A = s \times s$$
$$A = s^2$$

Example 1: Find the area of a square that is 6 units in length on each side.



Side = 6 units

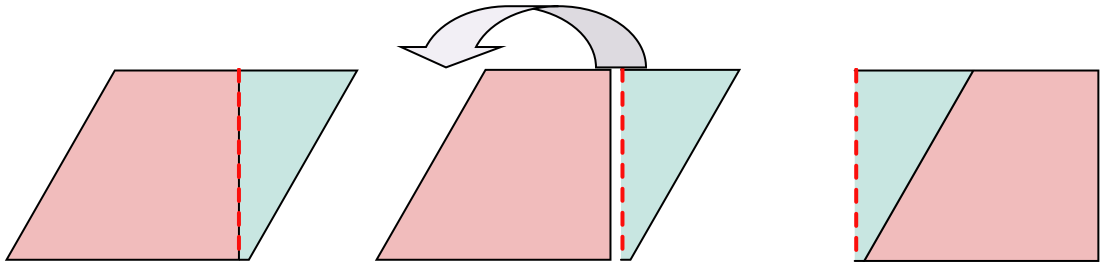
$$A = s^2$$

$$A = 6^2$$

$$A = 36 \text{ square units}$$

The area of the square is 36 square units.

The **area of a parallelogram** can be rearranged into the shape of a rectangle if the parallelogram is cut along a perpendicular height from the top to its base. Thus, a formula for the area of a parallelogram can be written based on the formula for the area of a rectangle.

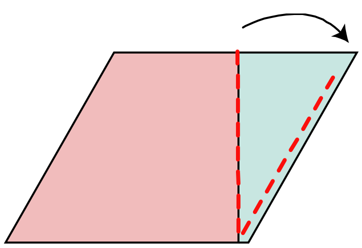


Move the triangular piece to the left side.

The **area of a parallelogram** is the product of its base and height. The height of a parallelogram is the length of a perpendicular line from the top of the parallelogram to the base.

$$A = lw$$
$$A = bh$$

*Notice that the height of a parallelogram is shorter than the length of its side. When calculating the area of a parallelogram, be sure to use the height of the parallelogram rather than the length of the side.

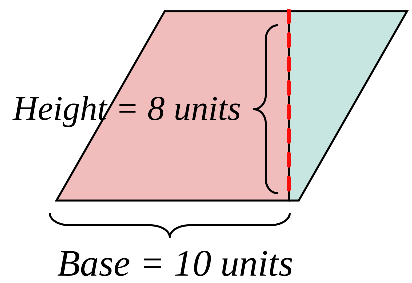


Example 2: Find the area of a parallelogram that has a base of 10 units and a height of 8 units.

$$A = bh$$

$$A = 10 \times 8$$

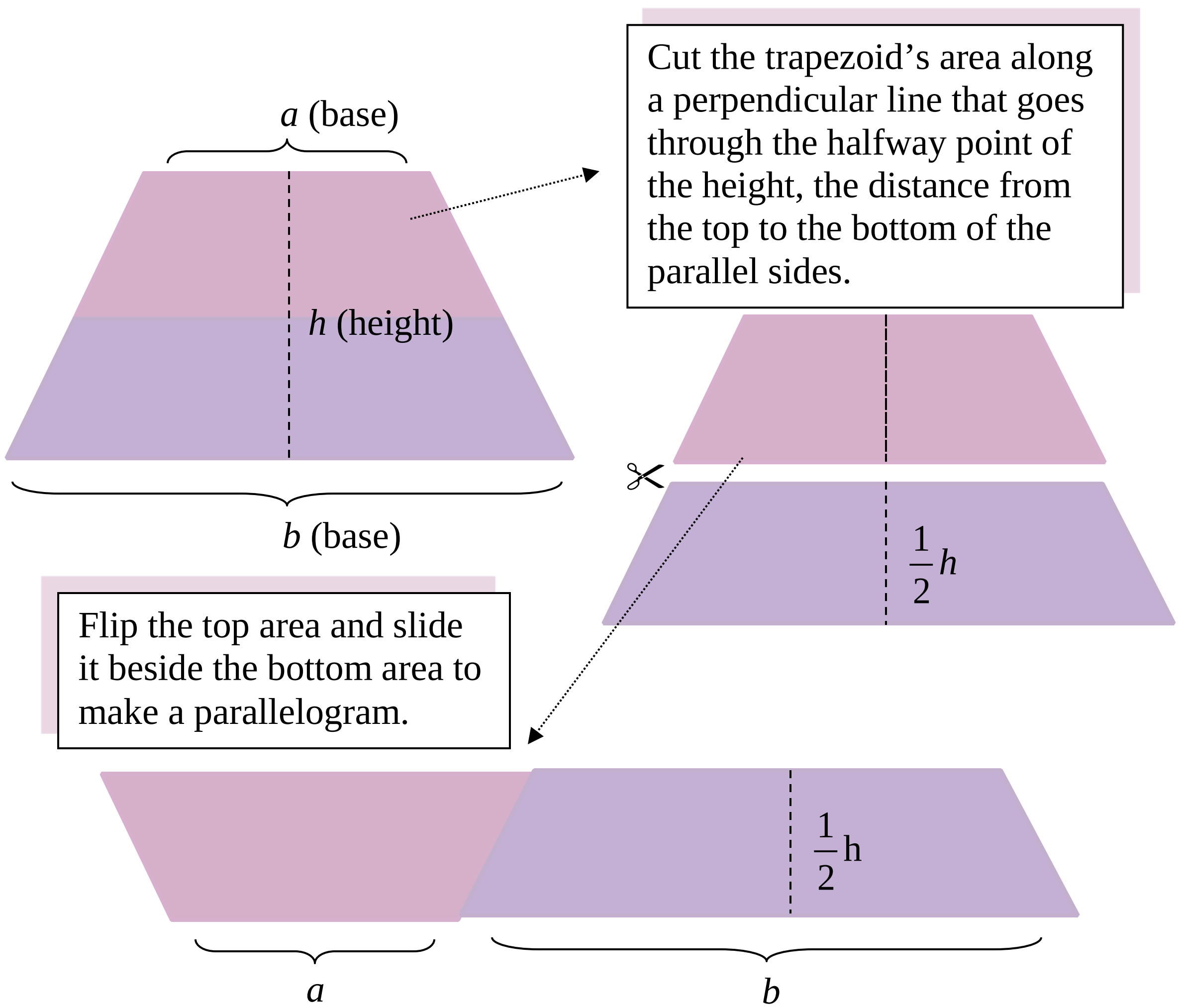
$$A = 80 \text{ square units}$$



Area of a Trapezoid

Area is a measurement of coverage and is measured in **square units**.

The **area of a trapezoid** can be rearranged into the shape of a parallelogram. Let's take a look at how this can happen.



$\text{Base} = (a + b)$ $\qquad$ $\text{Height} = \frac{1}{2}h$

Build the formula for the area of a trapezoid based on the formula for the area of a parallelogram.

$$A = bh$$

$$A = (a+b)(\frac{1}{2}h)$$

$$A = (\frac{1}{2}h)(a+b)$$ Apply the commutative property.

$$\boxed{A = \frac{1}{2}h(a+b)}$$

The area of a trapezoid equals one-half of the height times the sum of the bases.

*Note: The bases of a trapezoid are the parallel sides.

Example: Find the area of a trapezoid where the parallel sides measure 4 feet and 10 feet and the height of the trapezoid is 15 feet.

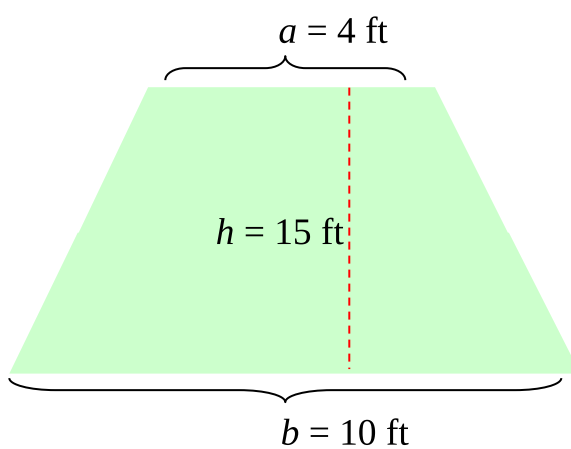
$$A = \frac{1}{2}h(a+b)$$

$$A = \frac{1}{2} \times 15 \times (4+10)$$

$$A = \frac{1}{2} \times 15 \times 14$$

$$A = \frac{1}{2} \times 210$$

$$A = 105 \text{ sq ft}$$



The area of the trapezoid is 105 square feet.

Area of a Circle

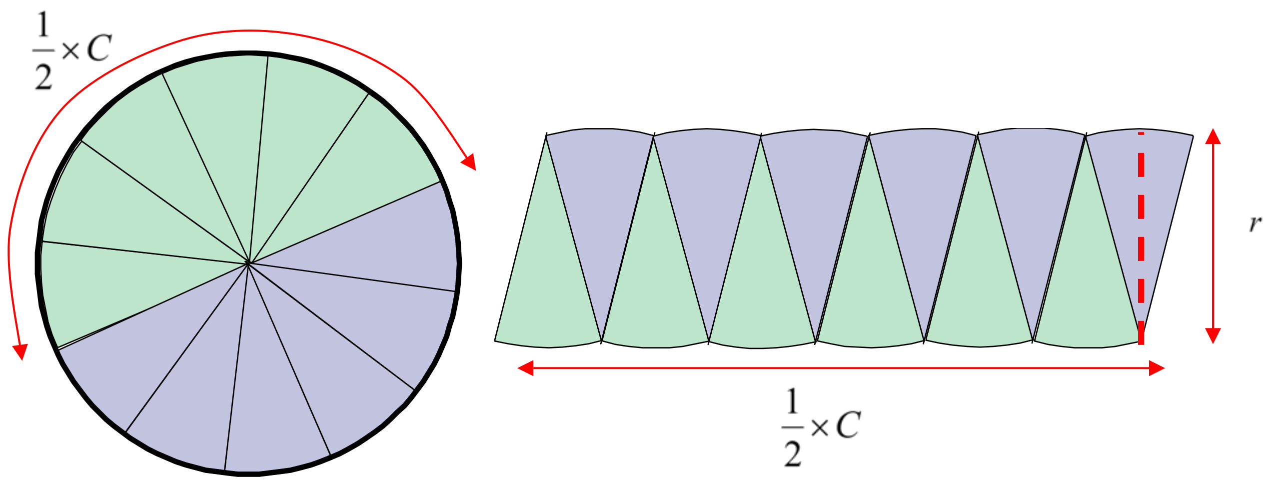
Area is a measurement of coverage and is measured in **square units**.

The area of a circle can be rearranged into a shape that approximates a parallelogram.

The length of the parallelogram is the same length as half the circle's circumference. The height of the parallelogram is the same as the radius of the circle.

Let's take a look at how this can happen.

The circle shown below is divided into 12 congruent pieces. The pieces are then laid out to make a shape that looks similar to a parallelogram.



Notice that the length of the "parallelogram" is half of the length of the circumference of the circle.

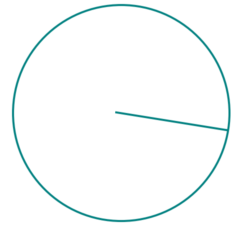
Notice that the height of the parallelogram is close to the radius of the circle.

For this theory to truly work, the circle would be divided into many, many, more pieces. When that is done, then the bottom of the parallelogram is close to a straight line and the height of the parallelogram is closer to a perpendicular line.

Now, we'll build the formula based on this theory.

Statement	Reason
$A = bh$	Formula for area of a parallelogram.
$A = (\frac{1}{2}C) \times r$	base $= \frac{1}{2}C$ height $= r$
$A = (\frac{1}{2} \times 2\pi r) \times r$	$C = 2\pi r$
$A = 1 \times \pi r \times r$	$\frac{1}{2} \times 2 = 1$
$A = \pi r \times r$	Identity Property (Any number times 1 is the number.)
$A = \pi \times (r \times r)$	Associative Property (Regrouping is allowed in multiplication.)
$A = \pi \times r^2$	$r \times r = r^2$

Example 1: Find the area of a circle that has a radius of five inches.

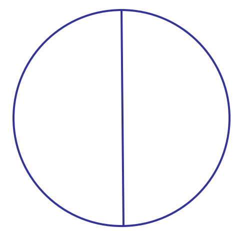


r = 5 in

$$A = \pi r^2$$
$$A = 3.14 \times 5^2$$
$$A = 3.14 \times 25$$
$$A = 78.5 \text{ square inches}$$

The area of the circle is 78.5 square inches.

Example 2: Find the area of a circle that has a diameter of twenty feet.



Diameter = 20 ft

*Since a diameter is given, and a diameter equals two radii, take half of 20 to determine the radius.

$$A = \pi r^2$$
$$A = 3.14 \times 10^2$$
$$A = 3.14 \times 100$$
$$A = 314 \text{ square feet}$$

The area of the circle is 314 square feet.

Formula Chart for Perimeter, Circumference, and Area

The chart below is a list of many shapes and the corresponding formulas for calculating the perimeter and area of the shapes.

Shape	Perimeter / Circumference	Area
Triangle	$P = s + s + s$	$A = \frac{1}{2}bh$
Rectangle	$P = 2l + 2w$	$A = lw$
Square	$P = 4s$	$A = s^2$
Parallelogram	$P = 2l + 2w$	$A = bh$
Trapezoid	$P = s + s + s + s$	$A = \frac{1}{2}h(a + b)$
Circle	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$
Regular Polygon n-sides	$P = n \times s$	