POLYGONS

In this unit, you will investigate the features of regular polygons. You will examine the interior angles and exterior angles of polygons and special formulas that are related to these angles.

Polygons

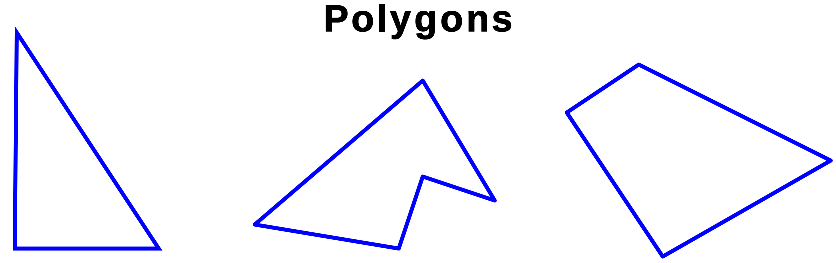
Interior and Exterior Angles of Regular Polygons

Polygons

polygon - A polygon is a closed figure that is made up of line segments that lie in the same plane. Each side of a polygon intersects with two other sides at its endpoints.

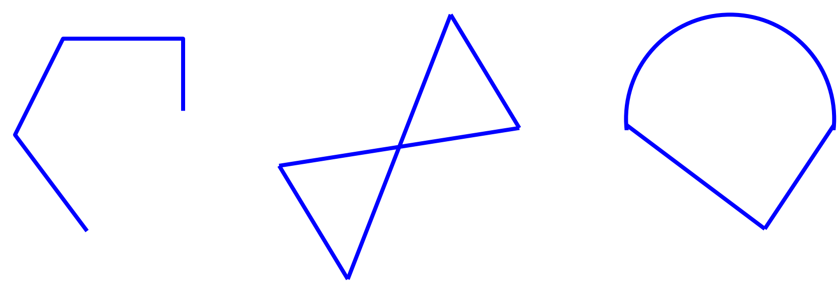
Here are some examples of polygons:

Polygons



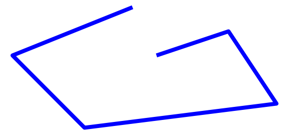
Here are some examples of figures that are NOT classified as polygons. The reason the figure is not a polygon is shown below it.

Not Polygons



The path is open.	Two sides of the figure intersect at a point other than the endpoints.	One side of the figure is curved.

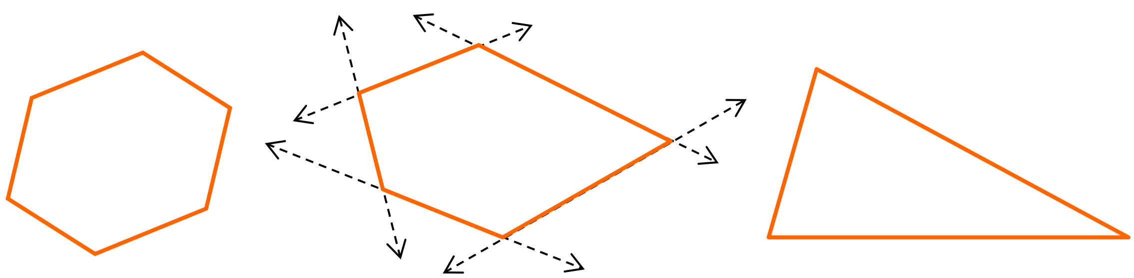
Example 1: Is the shape a polygon? Explain why or why not.



The shape is not a polygon because the path is open.

convex polygon – A convex polygon is a polygon where **none** of the sides lie in the interior of the polygon.

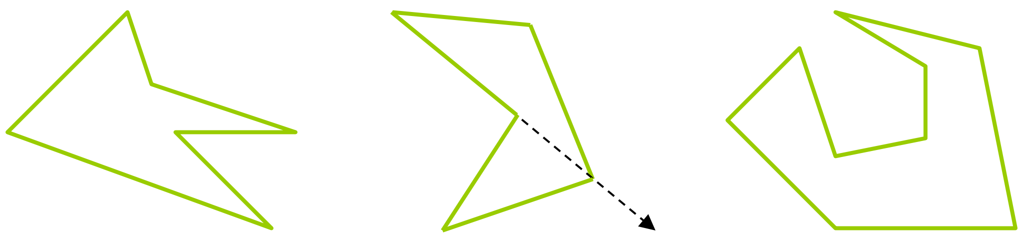
Convex Polygons



When the sides of the polygon are extended as lines, none of the lines fall in the interior of the polygons.

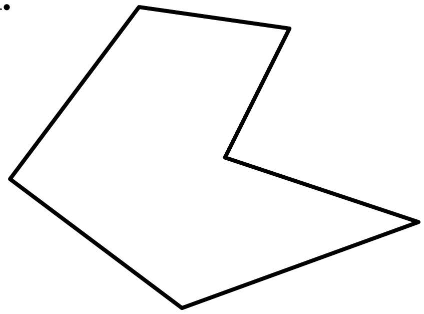
concave polygon – A concave polygon is a polygon where at least one of the sides, when extended as a line, lies within the interior of the polygon.

Concave Polygons

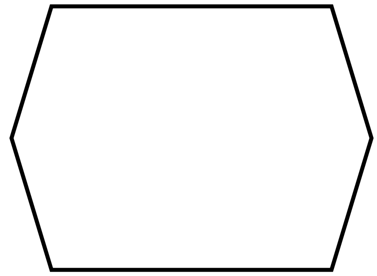


Example 2: Which polygon is convex? Please explain.

A.



B.



Choice B is the convex polygon. A convex polygon is a polygon where **none** of the sides lie in the interior of the polygon.

Note: Choice A is a concave polygon.

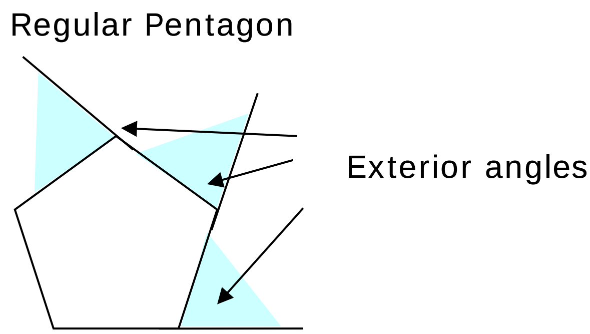
In general, a polygon with *n* sides is called an *n-gon*.   Several common polygons have been given names based on the number of sides.

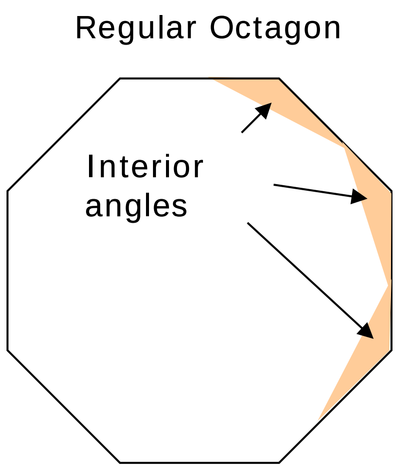
Number of Sides	**Polygon**
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	*n*-gon

Interior and Exterior Angles of Regular Polygons

regular polygons – Regular polygons are polygons where all sides and angles are congruent.

In the regular pentagon at the right, lines are extended from the sides of the polygon to show exterior angles. Three of the five exterior angles are shown.





In the regular octagon on the left, three of the eight interior angles are shown.

There are established formulas to find the measures of the angles in a regular polygon.

Theorem 1

The sum of the angles of a convex polygon with “n sides is:

$$(n-2)\times 180$$

Example 1: Find the sum of the interior angles of a regular hexagon.

Number of sides in a regular hexagon: 6

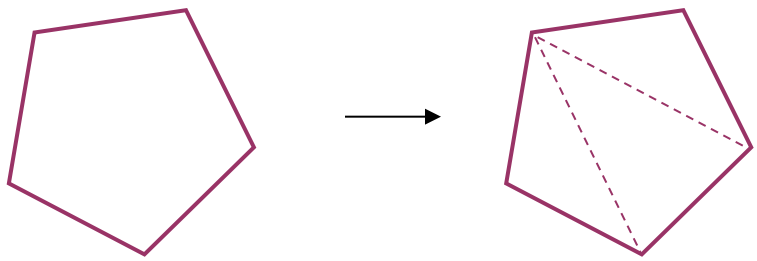
$(n-2)\times 180°$	Formula
$(6-2)\times 180°$	Substitute 6 for n in the formula.
$(4)\times 180°$	Simplify
$720°$	Simplify

The sum of the interior angles of a regular hexagon is 720 degrees.

Let's examine why Theorem 1 works.

Example 2: Draw a regular pentagon and draw all possible non-overlapping diagonals from one vertex, and then answer the following questions.

(a) How many triangles are formed?



Three triangles are formed.

(b) Recall that the angles of a triangle total 180°.

Three triangles are formed by the non-overlapping diagonals. The sum of the angles in the three triangles is

$$180 \times 3 = 540$$

Since all of the angles of the pentagon are included in these triangles, then the sum of the five angles in a pentagon is 540 degrees.

(c) Check the solution using the formula given in Theorem 1.

Number of sides in a regular pentagon: 5

$\left(n-2\right)\times 180°$	Formula
$\left(5-2\right)\times 180°$	Substitute 5 for n in the formula.
$(3)\times 180°$	Simplify
✓ $540°$	Simplify

Example 3: What is the measure of one interior angle of a regular hexagon?

In Example 1, the formula was used to determine that the sum of the six interior angles of a regular polygon is 720 degrees. Since the six angles of a regular polygon are equal in measure, divide $720°$ by six to find the measure of one interior angle.

$$720° \div 6$$

$$120°$$

Each interior angle of a regular hexagon measures $120°$.

Theorem 2

The sum of the measures of the exterior angles of a convex polygon (one at each vertex) is:

$$360°$$

In this theorem, it is simply stated that the measures of the exterior angles of ANY regular polygon total 360 degrees!

Example 4: What is the sum of the exterior angles of a hexagon?

Answer: $360°$

This is true of any **convex** polygon.

Now let's look at a different way to find the measure of an interior angle of any regular polygon.

Example 5: Apply Theorem 2 to find the measure of one interior angle of a regular hexagon?

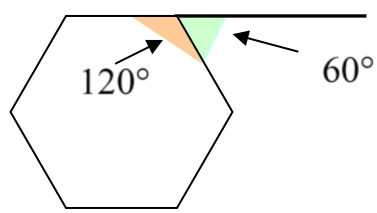
First recall Theorem 2: The sum of the exterior angles of any polygon is $360°$.

Find the measure of one exterior angle of a regular hexagon by dividing by 6. (The exterior angles of a **regular** polygon are equal in measure.)

$$360° \div 6 = 60°$$

One exterior angle of a regular hexagon measures 60°.

Notice that the exterior angle and the interior angle together form a straight line (thus, a straight angle).



Recall that straight angles measure 180 degrees.

$$180° - 60° = 120°$$

Therefore, the interior angle of a regular hexagon measures 120°.

There are many ways to solve a math problem; and, it is interesting how the many solutions all connect!!!