ESTIMATION AND SCIENTIFIC NOTATION

In this unit, you will solve problems where you must decide if you need an estimated answer or an exact answer. You will also decide if the answers are reasonable for the problem.

You will also learn to write and compute numbers in scientific notation. When exploring in science, numbers are either very small or very large such as 10 million miles or one billionth of an inch. It is difficult to write these types of numbers on paper or enter them into a calculator or computer because they have so many digits. There is a better way to write the numbers and that is scientific notation.

Determining if an Estimate is Sufficient

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Determining if an Estimate is Sufficient

The ability to estimate is a very important math skill. Imagine being able to find large sums, differences, products, or quotients using only your brainpower. Think how impressed your friends might be if you could be accurate in all of these computations. There are many strategies to make these estimates accurate, but it is also important to decide within a problem if making an estimate is appropriate. When estimating you must also decide if an estimate is “close enough or if the estimate is too low or too high. The accuracy of the estimate is as important as the actual estimate itself. All of these skills take some thought and practice as we will try to simulate.

Read the following problem.

> How long would it take you to repay a loan from a friend if no interest is charged? You borrow $1,498 and will repay $78.55 per week.

Would you decide to estimate?

Can you see the strategy to estimate quickly?

You can quickly estimate to see if the payments are within your budget.

(1) Round $1498 up to $1500.

(2) Round $78.55 down to $75 since 75 divides into 150 evenly.

(3) Divide to estimate: 1500 / 75 = 20 weeks

(4) Measuring time, 20 weeks would equal 5 months.

Is the estimate too low or too high?

The actual estimate will be too high, but close enough to make a decision.

> $1498 / $78.55 = 19.07 weeks (rounded to nearest hundredth)

The actual value will be paid off after 19 weeks with the final payment of $78.55 plus $5.55 extra.

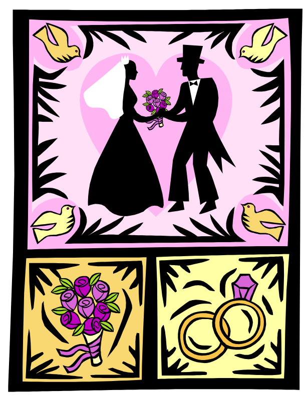
Judging the Reasonableness of a Solution

In each of the problems in this section, you are to make an estimate based on the information in the problem and write it down. After making an estimate, redo the problem with exact values to determine the actual value. Finally, compare the actual answer to the estimate to decide if the answer is reasonable.

Once again, each problem has three responses:

1. Make an estimate.
2. Redo the problem with exact values.
3. Compare the estimate with exact value and explain if the answer is reasonable.

Example 1: For a wedding reception a caterer must prepare chicken breasts for 500 guests. How many cases of chicken breasts, 80 per case, should the caterer purchase in preparation for the reception?



Would you decide to estimate? Since the chicken comes in cases that have 80 chicken breasts per case, more than enough must be ordered to prepare for the reception. Thus, estimating high is appropriate.

Can you see the strategy to estimate quickly?

$$500 \div 80 = \frac{500}{80} = \frac{50\not{0}}{8\not{0}} = 50 \div 8$$

$50 \div 8$ is more than 6 but less than 7

$48 \div 8 = 6 \qquad 56 \div 8 = 7$

Is the estimate too low or too high?

6 cases = 480 chicken breasts $(6 \times 80 = 480)$

7 cases = 560 chicken breasts $(7 \times 80 = 560)$

Will either estimate work?

No, 480 will not be enough to serve all of the 500 guests.

Explain the reasonable estimate.

The exact amount of chicken breasts needed is 6 1/4 boxes which is not an option since the chicken breasts are sold in whole boxes.

$$500 \div 80 = \frac{500}{80} = \frac{50\cancel{0}}{8\cancel{0}} = \frac{50}{8} = 6\frac{2}{8} = 6\frac{1}{4}$$

Certainly six cases (480 chicken breasts) is closer to the number of 500 guests; however, not having enough food at a wedding reception is not a wise decision. Seven cases (560) provide too many chicken breasts; but for this occasion, it is wise to be over the estimated amount.

Example 2: How much money do you need to take on a trip to a Major League baseball game held in a nearby city? You will need money for the following purchases:

(1) tickets for admittance to the game - $48
(2) a game program - $5
(3) nachos - $4.75
(4) a soda - $3.75
(5) a souvenir - ranges from $5 to $25
(6) parking fees - around $10 (You are riding with someone else; but, plan to pay the parking fees.)

Would you decide to estimate? In this problem the total amount you wish to spend is really the question; so estimating the total would be appropriate.

Can you see the strategy to estimate quickly?

The method of rounding all values higher to make the arithmetic mental math is appropriate for this problem.

Item	Actual Cost		Rounded Cost
Ticket	$48.00	rounds to	50
Program	$5.00	rounds to	5
Nachos	$4.75	rounds to	5
Soda	$3.75	rounds to	5
Souvenir	$5 to $25	rounds to	25
Parking	$10.00	rounds to	10
Total	$96.50		100

Will your estimate be low or high?

Since you are rounding every amount up, the estimate will be high; however, you want to have enough money and it is okay to have money left over.

Could you estimate low?

You can, but estimating too low would lead to an embarrassing situation when you did not have enough money to cover expenses.

An estimate that is too low would not be appropriate; but, a high estimate will work nicely. One must consider the consequences when making these types of estimates.

Standard Form for Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:

- a number that is less than 10, but greater than or equal to 1
- a power of ten

$$\mathbf{5.6 \times 10^4 = 56{,}000}$$

In this section, we will take numbers given in scientific notation and write them in standard form.

Example 1: What is the standard form for 7×10^4?

$7 \times 10^4 =$

$= 7 \times (10 \times 10 \times 10 \times 10)$ $\quad 10^4 = 10 \times 10 \times 10 \times 10$

$= 7 \times 10{,}000$ $\quad 10 \times 10 \times 10 \times 10 = 10{,}000$

$= 70{,}000$

The standard form for 7×10^4 is 70,000.

Example 2: What is the standard form for 8.2×10^5?

$8.2 \times 10^5 =$

$= 8.2 \times (10 \times 10 \times 10 \times 10 \times 10)$

$= 8.2 \times 100{,}000$

$= 820{,}000$

$$\begin{array}{r} 8.2 \\ \times 100000 \\ \hline 820000.\not{0} = 820{,}000 \end{array}$$

The standard form for 8.2×10^5 is $820{,}000$.

A shortcut to multiply by a power of ten is to start at the decimal point's location and move it **right** as many places as the given power.

$$8.2 \times 10^5 = 820000. = 820,000$$

*Use zeros as place holders. In this case, the 2 takes up one place, so four zeros are needed to complete the move of 5 places to the right for the power of 5.

Express Large Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:

- a number that is less than 10, but greater than or equal to 1
- a power of ten

$$56,000 = 5.6 \times 10^4$$

To express a number in scientific notation, determine the first part that is a number between 1 and 10, and then determine the power of 10 that will make the expression equal to the given number.

Example 1: What is the scientific notation for 9,000,000?

Look at the first nonzero digits of the number and write as a **number between 1 and 10**.

$$9 \times 10^6$$

Multiply the number by a **power of ten** that will give the standard number.

$$\begin{aligned} 9,000,000 &= \\ &= 9 \times 1,000,000 \\ &= 9 \times 10^6 \end{aligned}$$

A shortcut to calculate the power is to place a decimal point after the first digit of the number, and then count the number of places **right** to the end of the whole number.

$$9,000,000 = 9.000000 = 9 \times 10^6$$

Count six places to the end of the number, so the power is 6. Counting right gives a positive power.

The scientific notation for 9,000,000 is 9×10^6.

Example 2: What is the scientific notation for 673,000?

Look at the nonzero digits at the beginning of the number and write as a **number between 1 and 10** by placing a decimal point after the first digit.

6.73×10^5

Multiply the number by a **power of ten** that will give the standard number.

$$\begin{aligned} 673{,}000 &= \\ &= 6.73 \times 100{,}000 \\ &= 6.73 \times 10^5 \end{aligned}$$

A shortcut to calculate the power is to place a decimal point after the first digit of the number, and then count the number of places **right** to the end of the whole number.

$$673{,}000 = 6.\underline{73000} = 6.73 \times 10^5$$

Count five places to the end of the number, so the power is 5. Counting right gives a positive power.

The scientific notation for 673,000 is 6.73×10^5.

Express Small Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:

- a number that is less than 10, but greater than or equal to 1
- a power of ten

$$\mathbf{0.00347 = 3.47 \times 10^{-3}}$$

To express a number in scientific notation, determine the first part that is a number between 1 and 10, and then determine the power of 10 that will make the expression equal to the given number.

Example 1: What is the scientific notation for 0.00000004?

Look at the nonzero digit at the end of the number and write as a **number between 1 and 10**.

$$4 \times 10^{-8}$$

Multiply the number by a **power of ten** that will give the standard number.

$$\begin{aligned} 0.00000004 &= \\ &= 4 \times 0.00000001 \\ &= 4 \times 10^{-8} \end{aligned}$$

A shortcut to calculate the power is to place a decimal point after the first non-zero digit of the number, and then count the number of places **left** to the original position of the decimal point.

$$\not{0}.00000004 = \underbrace{00000004}.= 4\times10^{-8}$$

Count eight places to the left to the original position of the decimal point, so the power is –8. Counting left gives a negative power.

*The zero located left of the decimal point has no value as it is just a place-holder used to indicate there is no whole number in the decimal number; thus, it is disregarded when calculating the scientific notation.

The scientific notation for 0.00000004 is 4×10^{-8}.

Example 2: What is the scientific notation for 0.0000295?

Look at the nonzero digits at the end of the number and write as a **number between 1 and 10** by placing a decimal point after the first digit.

$$2.95\times10^{-5}$$

Multiply the number by a **power of ten** that will give the standard number.

$$\begin{aligned}0.0000295 &= \\ &= 2.95\times0.00001 \\ &= 2.95\times10^{-5}\end{aligned}$$

A shortcut to calculate the power is to place a decimal point after the first non-zero digit of the number, and then count the number of places **left** to the original position of the decimal point.

$$\not{0}.0000295 = \underset{\smile\smile\smile\smile\smile}{00002}.95 = 2.95 \times 10^{-5}$$

Count five places to the left to the original position of the decimal point, so the power is –5. Counting left gives a negative power.

*The zero located left of the decimal point has no value as it is just a place-holder used to indicate there is no whole number in the decimal number; thus, it is disregarded when calculating the scientific notation.

The scientific notation for 0.0000295 is 2.95×10^{-5}.

Compute in Scientific Notation

In this section, we will first review scientific notation and then, examine how to add, subtract, multiply, and divide numbers when they are written in scientific notation. Please recall that numbers in scientific notation have two parts; one part has a value between one and ten and the other part is a power of ten.

Example 1: Explain the meaning of 2.334×10^5.

The first value, 2.334, is a number between 1 and 10 while the second part is 10^5, a power of ten. The number written in standard form is 233,400.

Example 2: Which is larger, 4.34×10^5 or 4.34×10^4?

A number like 4.34×10^5 will be larger than 4.34×10^4 because the power of 5 places the first number in a larger place value.

In standard form, 434,000 > 43,400.

Using scientific notation will make some calculations easier, especially multiplication and division.

Multiplying in Scientific Notation

Example 3: Consider 4,250,000,000 times 3,000. Use scientific notation to simplify the problem.

To enter the large number into a normal handheld calculator may be difficult in some instances. In other instances, the results will be shown in scientific notation.

Let's examine the problem this way.

Step 1: Write each number in scientific notation.

$$4{,}250{,}000{,}000 \quad \times \quad 3{,}000$$
$$4.25\times10^{9} \quad \times \quad 3\times10^{3}$$

Step 2: Multiply the first values of each scientific notation.

$$4.25 \quad \times \quad 3 = 12.75$$

Step 3: Multiply the powers of ten.

$$10^{9} \times 10^{3} = 10^{12}$$

*Note: Since this is exponent multiplication and the base is the same (10), we only have to add the exponents. Nine factors of ten and 3 factors of ten make 12 factors of ten or10^{12}.

Step 4: Put the parts together.

$$12.75 \times 10^{12}$$

The answer is correct, but not in scientific notation. (The first value must fall between one and ten.)

Step 5: We must move the decimal…

$12.75 \rightarrow 1.275$ smaller by one place value (ie. divide by 10)

Step 6: We will adjust the 10's…

$10^{12} \rightarrow 10^{13}$ which is larger by one place value (ie. multiply by 10)

*Multiplying by 10 in *Step 6* balances dividing by 10 in *Step 5*, and thus, the value of the answer remains the same.

By adjusting one part of the number up and one part down, we have balanced and kept the answer the same while also being correct with our scientific notation rules.

The product of 4,250,000,000 times 3,000 is 1.275×10^{13}.

Summarizing the work:

$$
\begin{aligned}
4{,}250{,}000{,}000 \text{ times } 3{,}000 &= 4.25 \times 10^{9} \times 3 \times 10^{3} \\
&= 4.25 \times 10^{9} \times 3 \times 10^{3} \\
&= 4.25 \times 3 \times 10^{9+3} \\
&= 12.75 \times 10^{12} \\
&= 1.275 \times 10^{13} \\
&= 12{,}750{,}000{,}000{,}000
\end{aligned}
$$

The product of 4,250,000,000 times 3,000 is 12,750,000,000,000.

Adding and Subtracting in Scientific Notation

When we try addition (or subtraction), it is important to note the powers of ten should match. This is needed in order for the short cut to work.

Example 4: Compute $(4.1 \times 10^{4}) + (5.3 \times 10^{4})$ using scientific notation.

*Note: the powers are the same and do not need adjusted.

Step 1: Values first… $4.1 + 5.3 = 9.4$

Step 2: Powers next, but since we are adding, the place value will remain the same; thus the power gets copied… 10^{4}.

Step 3: Put the parts together…

$$
\begin{aligned}
(4.1 \times 10^{4}) + (5.3 \times 10^{4}) &= \\
&= (4.1 \times 10^{4}) + (5.3 \times 10^{4}) \\
&= 9.4 \times 10^{4}
\end{aligned}
$$

*Note: This answer does not need adjusted as it is in scientific notation because 9.4 is a number between 1 and 10.

For subtraction follow the same format as addition, only subtract.

Example 5: Compute $(8.5 \times 10^6) - (2.9 \times 10^6)$ using scientific notation.

*Note: the powers are the same and do not need adjusted.

Step 1: Values first… $8.5 - 2.9 = 5.6$

Step 2: Powers next, but since we are subtracting, the place value will remain the same; thus the power gets copied… 10^6.

Step 3: Put the parts together…

$$(8.5 \times 10^6) - (2.9 \times 10^6) =$$
$$= (8.5 \times 10^6) - (2.9 \times 10^6)$$
$$= 5.6 \times 10^6$$

*Note: This answer does not need adjusted as it is in scientific notation because 5.6 is a number between 1 and 10.

Dividing in Scientific Notation

For division follow the same format as multiplication, only divide the values and subtract the powers.

Example 6: Consider $\frac{9.42 \times 10^{16}}{3 \times 10^{12}}$ or $(9.42 \times 10^{16}) \div (3 \times 10^{12})$.

Step 1: Numbers… $9.42 \div 3 = 3.14$

Step 2: Powers… $10^{16} \div 10^{12} = 10^{16-12} = 10^4$

Step 3: Answer… 3.14×10^4

Step 4: Putting it all together…

$$\frac{9.42 \times 10^{16}}{3 \times 10^{12}} =$$

$$= \frac{9.42 \times 10^{16}}{3 \times 10^{12}}$$

$$= \frac{9.42}{3} \times 10^{16-12}$$

$$= 3.14 \times 10^{4}$$

*Note: This answer does not need adjusted as it is in scientific notation because 3.14 is a number between 1 and 10.