RELATIONS AND FUNCTIONS

In this unit, you will become an investigator by trying to piece together some data given in a chart. You will decide if the data is a relation or a function and write an equation to relate the data.

Relations

Relations and Functions

Relations

A list of ordered pairs like this $(1,2),(2,4),(3,6),(4,8)$ is called a relation.

Take a careful look at each abscissa (first part of the pairs) $\{1, 2, 3, 4\}$ and notice that if each number is multiplied by 2, the result is $\{2, 4, 6, 8\}$, the ordinate (the second corresponding part of each pair in the relation) .

If the ordered pair in this relation is represented generally as (x, y), then we can conclude that $y = 2x,$ and we then can predict more pairs in the relation.

> *Example*: What is the ordered pair in this relation, $y = 2x,$ when the abscissa is a 9?
>
> $y = 2x$ Relationship of the ordered pairs.
> $y = 2(9)$ Substitute 9 for "x".
> $y = 18$ Simplify.
> The ordinate is 18.
>
> The ordered pair is (9,18).

Now look at this relation: $(1,1),\ (2,4),\ (3,9),\ (4,16)$

$$(1,1),\ (2,4),\ (3,9),\ (4,16)$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$(1,1^2),(2,2^2),(3,3^2),(4,4^2)$$

Can you see that $y = x^2$?

Good! 

How about this relation? $(1,0), (2,1), (3,2), (4,3)$

$$\begin{array}{cccc} (1,0), & (2,1), & (3,2), & (4,3) \\ \downarrow & \downarrow & \downarrow & \downarrow \\ (1,1-1), & (2,2-1), & (3,3-1), & (4,4-1) \end{array}$$

Can you see that $y = x - 1$? ✓

Looking for a pattern helps us to predict ordered pairs of a relation.

Relations and Functions

Some relations can be thought of as **functions.** In a function, the output is related to the input through a function rule.

Look at the function $y = 5x$.

Input a number (x) through the function rule ($5x$) to get the output (y).

$y = 5x$		
y = 5 times x		
Input (x)	Function Rule ($5x$)	Output (y)
1	5 times 1	*5*
2	5 times 2	*10*
3	5 times 3	*15*
4	5 times 4	*20*
5	5 times 5	*25*

Example 1: What values are the output for the function $\boldsymbol{y = x + 4}$ when x equals 12, 13, 14, 15 and 16? Make a table to organize and display the results.

Input a number (***x***) through the function rule (***x* + 4**) to get the output (***y***).

***y* = *x* + 4**		
***y* = *x* plus 4**		
Input (x)	Function Rule (***x* + 4**)	Output (y)
12	12 + 4	*16*
13	13 + 4	*17*
14	14 + 4	*18*
15	15 + 4	*19*
16	16 + 4	*20*

Functions may be graphed in a coordinate plane. Use the input as the x-coordinate and the output as the y-coordinate and write ordered pairs (x, y).

Let's take a look at the graph of a *linear* function; that is, a function that has a straight line as its graph.

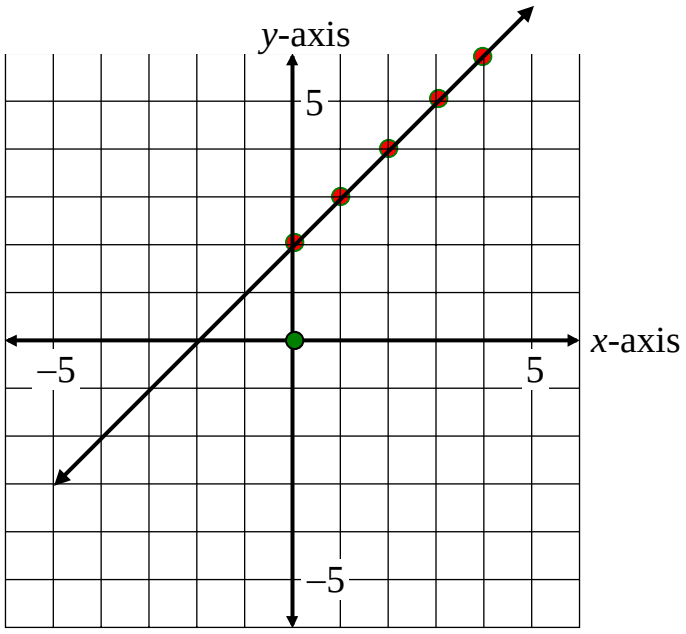
Example 2: Determine the graph of the function $y = 2 + x$ using the following values for x: 0, 1, 2, 3, and 4. Make a table to organize and display the results.

Input a number (***x***) through the function rule (**2 + *x***) to get the output (***y***).

Write the input and the output as a set of ordered pairs to prepare for graphing the function.

y* = 2 + *x			
y* equals 2 plus *x			
Input (x)	Function Rule (**2 + *x***)	Output (y)	Ordered Pairs
0	2 + 0	*2*	(0,2)
1	2 + 1	*3*	(1,3)
2	2 + 2	*4*	(2,4)
3	2 + 3	*5*	(3,5)
4	2 + 4	*6*	(4,6)

Use the ordered pairs to plot the points. Draw a straight line through the points.



Since the graph of this function forms a straight line, the function is considered **a linear function.**

Now let's look at the differences between a relation and function and define each one of them.

relation: a pairing of a set of numbers generally represented as a set of ordered pairs.

Example 3: Write the data that represents the relation shown below as a set of ordered pairs.

Height (inches)	Weight
68	125
64	118
65	112
72	145
64	126
67	130
66	128

The set of ordered pairs is:

{(68, 125), (64, 118), (65, 112), (72, 145), (64, 126), (67,130), (66, 128)}

*Notice in this example that there are repeated numbers in the height column. Because of this, the chart and set of ordered pairs only represents a **relation**.

function: a pairing between two sets of numbers in which each element in the first set is paired with **exactly** one element of the second set.

Example 4: Write the data that represents the function shown below as a set of ordered pairs.

11	63
12	64
13	65
14	70
15	72
16	72

The set of ordered pairs is:

{(11, 63), (12, 64), (13, 65), (14, 70), (15, 72), (16, 72)}

*Notice in this example that there are no repeated values in the first column. Because of this, the chart and the set of ordered pairs represent a **function**.

In common terms,

a **relation** is a set of ordered pairs

and

a **function** is a set of ordered pairs where the first coordinates (the *x*-coordinates) are all different.