ADD AND SUBTRACT FRACTIONS

In this unit you will review addition and subtraction of fractions. Mastery of fractions is important for success at any level of mathematics. Make sure to find least common denominators before adding or subtracting fractions with unlike denominators. Also, simplify answers to lowest terms.

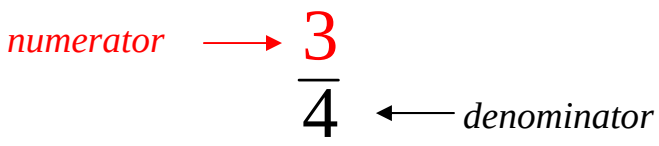
Addition of Fractions

Subtraction of Fractions

Fractions and Decimals with Signs

Addition of Fractions

To add fractions, identify the numerators and the denominators, and then make the adjustments necessary to complete the computation.



When adding fractions and the denominators are the same, just add the numerators. If the denominators are different, write the fractions as equivalent fractions with the same denominator, and then add the numerators.

Same Denominators

Example 1: Add 4/9 + 2/9.

Add the numerators of the fractions.

$$\begin{array}{r} \frac{4}{9} \\ +\frac{2}{9} \\ \hline \frac{6}{9} = \frac{2}{3} \end{array}$$

*To simplify (reduce) the answer, divide the numerator and denominator by three.

$$\frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$$

Example 2: Add 2 5/8 + 9 7/8.

Add the whole numbers and add the numerators of the fractions.

$$\begin{array}{r} 2\frac{5}{8} \\ +9\frac{7}{8} \\ \hline 11\frac{12}{8} = 12\frac{1}{2} \end{array}$$

*Let's take a closer look at how to convert an improper fraction to a mixed number in simplest form. To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator.

$$\frac{12}{8} = 8\overline{)12}\ \begin{array}{l} 1 \\ \underline{8} \\ 4 \end{array} = 1\frac{4}{8} = 1\frac{1}{2} \quad \textit{therefore} \quad 11\frac{12}{8} = 11 + 1\frac{1}{2} = 12\frac{1}{2}$$

*To simplify (reduce) 4/8, divide the numerator and denominator by four.

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

Example 3: Add 3/4 + 8.

$$\begin{array}{r} \frac{3}{4} \\ +8 \\ \hline 8\frac{3}{4} \end{array}$$

Just bring down the 8 since 8 + 0 = 8 and bring down the $\frac{3}{4}$ since $\frac{3}{4}+0=\frac{3}{4}$.

Example 4: Add 5 1/6 + 3 5/6.

Add the whole numbers and add the numerators of the fractions.

$$\begin{array}{r} 5\frac{1}{6} \\ +3\frac{5}{6} \\ \hline 8\frac{6}{6}=9 \end{array}$$

*To simplify the answer, rewrite $\frac{6}{6}$ as one, and then combine the one with the eight.

$$8\frac{6}{6}=8+1=9$$

Different Denominators

In the previous four examples, the denominators were the same. Now let's take a look at adding fractions with different denominators.

Example 5: Add 7/8 + 5/16.

First, find the least common denominator (LCD) for 8 and 16.

List the multiples of 16: 16 = {16, 32, 48, 64, 80…}
List the multiples of 8: 8 = {8, 16, 24, 32, 40…}

*A good "rule of thumb" is to list the multiples of the larger denominator first because the LCD will occur sooner in the multiples of the larger number.

The "least" multiple that is common to both sets is 16; that is, the LCD = 16.

$$\begin{array}{r} \frac{7}{8} = \frac{14}{16} \\ +\frac{5}{16} = \frac{5}{16} \\ \hline \frac{19}{16} = 1\frac{3}{16} \end{array}$$

$\frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$

or say 8 divides into 16, two times, 7 × 2 = 14.

*To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator, and then express the quotient and the remainder as a mixed number.

$$\frac{19}{16} = 16\overline{)19}\begin{array}{l} 1 \\ 16 \\ \hline 3 \end{array} = 1\frac{3}{16}$$

Example 6: Add 3 7/9 + 2 5/6.

First, find the least common denominator (LCD) for 9 and 6.

List the multiples of 9: 9 = {9, 18, 27, 36, 45…}
List the multiples of 6: 6 = {6, 12, 18, 24, 30…}

The "least" multiple that is common to both sets is 18; that is, the LCD = 18.

When changing the fraction part of the mixed number to the same denominator, just rewrite the whole number with the fraction.

$$\begin{array}{r} 3\frac{7}{9} = 3\frac{14}{18} \\ +2\frac{5}{6} = 2\frac{15}{18} \\ \hline 5\frac{29}{18} = 6\frac{11}{18} \end{array} \qquad \left(\begin{array}{l} \frac{7}{9}\times\frac{2}{2} = \frac{14}{18} \\ \\ \frac{5}{6}\times\frac{3}{3} = \frac{15}{18} \end{array} \right)$$

*To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator, and then express the quotient and the remainder as a mixed number. Finally, combine the two whole numbers.

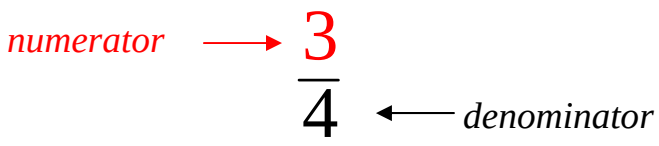
$$\frac{29}{18} = \begin{array}{r} 1 \\ 18\overline{)29} \\ \underline{18} \\ 11 \end{array} = 1\frac{11}{18}$$

$$5\frac{29}{18} = 5 + 1\frac{11}{18} = 6\frac{11}{18}$$

Equivalent Fractions Activity

Subtraction of Fractions

To subtract fractions, identify the numerators and the denominators, and then make the adjustments necessary to complete the operation.



When subtracting fractions and the denominators are the same, subtract the numerators. If the denominators are not the same, write the fractions as equivalent fractions with the same denominator, and then subtract the numerators. In some instances, borrowing will be necessary.

Example 1: Subtract 11/12 – 5/12.

Subtract the numerators of the fractions.

$$\begin{array}{r} \frac{11}{12} \\ -\frac{5}{12} \\ \hline \frac{6}{12} = \frac{1}{2} \end{array}$$

*To simplify (reduce) 6/12, divide the numerator and denominator by six.

$$\frac{6}{12} \div \frac{6}{6} = \frac{1}{2}$$

Example 2: Subtract 7/8 – 5/6.

First, find the least common denominator (LCD) for 8 and 6.

List the multiples of 8: 8 = {8, 16, 24, 32, 40…}
List the multiples of 6: 6 = {6, 12, 18, 24, 30…}

*A good "rule of thumb" is to list the multiples of the larger denominator first because the LCD will occur sooner in the multiples of the larger number.

The "least" multiple that is common to both sets is 24; that is, the LCD = 24.

$$\begin{array}{r} \frac{7}{8}=\frac{21}{24} \\ -\frac{5}{6}=\frac{20}{24} \\ \hline \frac{1}{24} \end{array}$$

$$\frac{7}{8}\times\frac{3}{3}=\frac{21}{24}$$

$$\frac{5}{6}\times\frac{4}{4}=\frac{20}{24}$$

Example 3: Subtract 8 14/15 – 2 3/5.

First, find the least common denominator (LCD) for 15 and 5.

List the multiples of 15: 15 = {15, 30, 45, 60, 75…}
List the multiples of 5: 5 = {5, 10, 15, 20, 25…}

The "least" multiple that is common to both sets is 15; that is, the LCD = 15.

When changing the fraction part of the mixed number to the same denominator, just rewrite the whole number with the fraction.

$$8\frac{14}{15} = 8\frac{14}{15}$$
$$-2\ \frac{3}{5} = 2\frac{9}{15} \quad \left(\frac{3}{5}\times\frac{3}{3}=\frac{9}{15}\right)$$
$$6\frac{5}{15} = 6\frac{1}{3}$$

*To simplify (reduce) 5/15, divide the numerator and denominator by five.

$$\frac{5}{15}\div\frac{5}{5}=\frac{1}{3}$$

Example 4: Subtract 7 – 2 4/13.

To subtract a mixed number from a whole number, borrowing is needed. In this problem, **borrow** a one from seven and write the one in thirteenths (13/13).

$$\begin{array}{r} \cancel{7}^{\,6} = 6\frac{13}{13} \quad \left(7 = 6+1 = 6+\frac{13}{13} = 6\frac{13}{13}\right) \\ -2\ \frac{4}{13} = 2\frac{4}{13} \\ \hline 4\frac{9}{13} \end{array}$$

Example 5: Subtract 9 3/8 – 4 7/8.

In this problem, not only will we need to **borrow** a one and change it to eighths; but, we will also have to combine the eighths that are equal to one (8/8) with the eighths that are part of the mixed number (3/8).

$$\begin{array}{r} \cancel{9}^{\,8}\,\frac{3}{8} = 8\frac{11}{8} \\ -4\ \ \frac{7}{8} = 4\ \frac{7}{8} \\ \hline 4\frac{4}{8} = 4\frac{1}{2} \end{array}$$

$$\begin{aligned} 9\frac{3}{8} &= 8+1+\frac{3}{8} \\ &= 8+\frac{8}{8}+\frac{3}{8} \\ &= 8\ \ \frac{11}{8} \end{aligned}$$

A shortcut to find 11 in $\frac{11}{8}$ is to add the 3 + 8 in $\frac{3}{8}$.

Example 6: Subtract 5 5/12 – 2 11/18.

In this problem, we must first find the LCD and rewrite the mixed numbers. From there, we will need to borrow as shown in the previous problem.

First, find the least common denominator (LCD) for 12 and 18.

List the multiples of 18: 18 = {18, 36, 54, 72, 90…}
List the multiples of 12: 12 = {12, 24, 36, 48, 60…}

The "least" multiple that is common to both sets is 36; that is, the LCD = 36.

$$5\frac{5}{12} = 5\frac{15}{36} = 4\frac{51}{36}$$
$$-2\frac{11}{18} = 2\frac{22}{36} = 2\frac{22}{36}$$
$$2\frac{29}{36}$$

Use the shortcut and add 15 + 36.

$$\frac{15+36}{36} = \frac{51}{36}$$

Fractions and Decimals with Signs

Fractions and decimals may have signs. The rules for determining the signs of the answers to computations with integers also apply to computations with fractions and decimals. Let's take a look at a few examples and how to apply the rules.

Example 1: Find the sum of –2 3/4 + –5 7/8.

> Addition Rule: When the signs are the same (both negative), "add" and use the same sign.

$$\begin{array}{r} -2\frac{3}{4}=2\frac{6}{8} \\ +\ -5\frac{7}{8}=5\frac{7}{8} \\ \hline -7\frac{13}{8}=-8\frac{5}{8} \end{array}$$

The sum of –2 3/4 + –5 7/8 is –8 5/8.

Example 2: Find the sum of 5 2/3 + –7 1/9.

Addition Rule: When the signs are different, "subtract" and take the sign of the number farther from zero on the number line.

*Note: Since –7 1/9 is farther from zero than 5 2/3 on the number line, the sign of the answer is negative. Also, write the –7 1/9 above the 5 2/3 to "subtract".

$$\begin{array}{rr} & -7\frac{1}{9} = \not{7}^{6}\frac{1}{9} = 6\frac{10}{9} \\ + & 5\frac{2}{3} = 5\frac{6}{9} = 5\frac{6}{9} \\ \hline & -1\frac{4}{9} \end{array}$$

The sum of 5 2/3 + –7 1/9 is –1 4/9.

Example 3: Find the difference of 2/3 – 7/8.

$$\begin{array}{r} \frac{2}{3}=\frac{16}{24} \\ -\frac{7}{8}=\frac{21}{24} \\ \hline \end{array}$$

*Note: Since 21/24 is larger than 16/24, rewrite the problem and apply the rules for subtraction of integers.

Subtraction Rule: Rewrite the problem to "add the opposite".

2/3 – 7/8 = 2/3 + –7/8 *The opposite of 7/8 is –7/8.

Since the signs are different, "subtract" and take the sign of the number farther from zero (7/8) on the number line. To subtract, write 7/8 as the top number.

$$\begin{array}{r} -\frac{7}{8}=\frac{21}{24} \\ +\ \frac{2}{3}=\frac{16}{24} \\ \hline -\frac{5}{24} \end{array}$$

*Subtract and take the sign of the number farther from zero on the number line.

The difference of 2/3 – 7/8 is –5/24.

Example 4: Find the difference of –2 3/10 – (–5 3/4).

Subtraction Rule: Rewrite the problem to "add the opposite".

–2 3/10 + (+5 3/4)

*The opposite of –5 3/4 is + 5/3/4.

$$5\frac{3}{4} = 5\frac{15}{20}$$
$$+\!-\ 2\frac{3}{10} = 2\frac{6}{20}$$
$$3\frac{9}{20}$$

*Subtract and take the sign of the number farther from zero.

The difference of –2 3/10 – (–5 3/4) is 3 9/20.

Example 5: Find the product of 6.3×-5.8.

Multiplication and Division Rule: If the signs are different, the answer is negative.

$$6.3\times -5.8 = -36.54$$

The product of 6.3×-5.8 is –36.54.

Example 6: Find the quotient of $-2.7712\div -0.32$.

Multiplication and Division Rule: If the signs are the same, the answer is positive.

$$-2.7712\div -0.32 = +8.66$$

The quotient of $-2.7712\div -0.32$ is 8.66.