There are practice worksheets in many of the units that provide more practice on specific topics. To view a practice worksheet, click on the paper/pencil icon. The worksheet will appear on top of the unit's main page. When finished, select File/Close to close the worksheet. The main page of the unit will be visible once again. Students and teachers may work together to decide what worksheet(s) or part of a worksheet(s) can be used as a supplementary activity for the unit. The answer key for each worksheet is provided for the instructor's use.

In this course, a recurring review of previous topics learned is provided throughout the units. Links to these review topics are listed under "Content Explanations for Review Problems". The appropriate review content links will be included in this unit and all of the remaining units of the course.

SIMILAR SHAPES

This unit is about similar triangles and other polygons, and the applications of proportional reasoning. The knowledge of similar shapes is very useful when solving problems that involve indirect measurement. In many of the problems, setting up a proportion and using cross multiplication will be very helpful. Sketch drawings for the problems on paper as accurately as possible, and be certain to label the figures appropriately.

Similar Triangles

Similar Polygons

Indirect Measurements

Similar Triangles

Similar triangles are triangles that have the same shape. That is, the corresponding sides of the triangles are proportional and the corresponding angles are congruent (same size). Let's examine each of these properties separately.

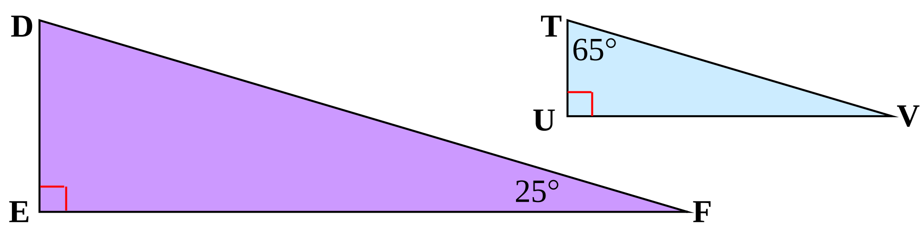
If two triangles are similar, then their corresponding angles are congruent.

*The math symbol to denote **similarity** is "**~**".

*The math symbol to denote **congruency** is "**≅**".

*$\triangle DEF \sim \triangle TUV$ is read "Triangle DEF is similar to triangle TUV."

Example 1: $\triangle DEF \sim \triangle TUV$. State the corresponding congruent angles, and then find the measures of angles D and V.



*The red square in each triangle denotes a right angle. Angle E and angle U are right angles.

The corresponding congruent angles are:

$E \cong \angle U$ $\angle D \cong \angle T$ $\angle F \cong \angle V$

*$\angle E \cong \angle U$ is read "angle E is congruent to angle U."

Angle E and Angle U

Angle E and angle U are right angles; therefore, they have the same measure.

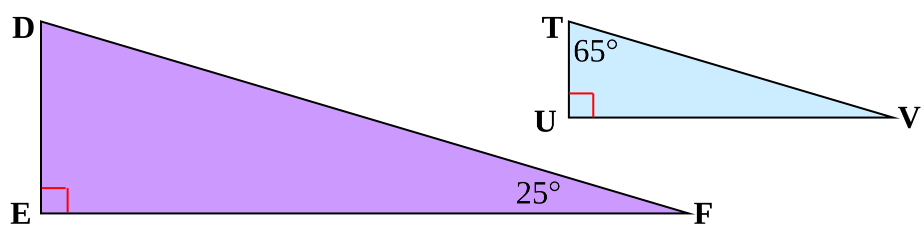
We can state: $m\angle\text{E} = m\angle\text{U}$

This statement is read, "The measurement of angle E equals the measurement of angle U."

Since we know that right angles measure 90 degrees, we can state:

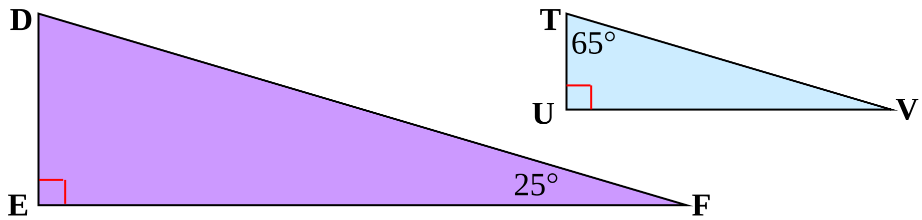
$m\angle\text{E} = 90°\quad m\angle\text{U} = 90°$

We could also simply state: $m\angle\text{E} = m\angle\text{U} = 90°$



Angle F and Angle V

Angle F and angle V are corresponding angles; thus, they are congruent and they have equal measures.



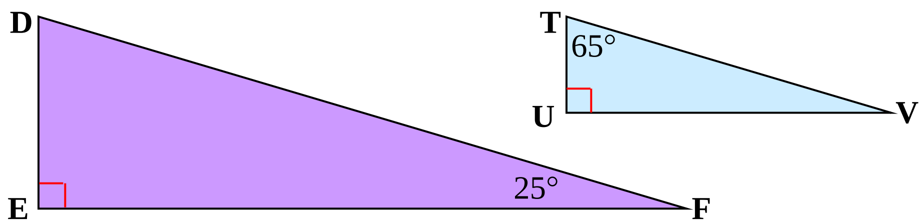
$\angle F \cong \angle V$, therefore $m\angle F = m\angle V$

Given, $m\angle V = 25°$

Therefore, $m\angle F = 25°$

Angle D and Angle T

Angle D and angle T are corresponding angles; thus, they are congruent and they have equal measures.



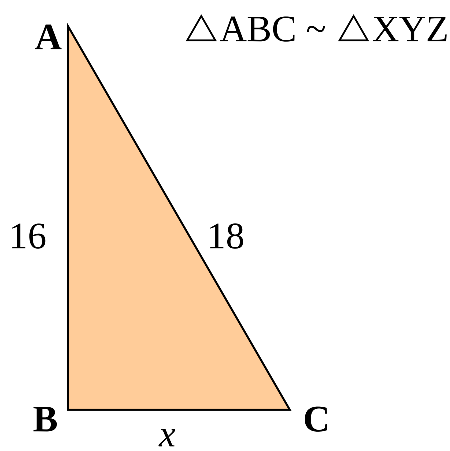
$\angle D \cong \angle T$, therefore $m\angle D = m\angle T$

Given, $m\angle T = 65°$

Therefore, $m\angle D = 65°$

If two triangles are similar, then their corresponding sides are proportional.

Example 2: △ABC ~ △XYZ. Find the length of segment BC that is labeled with *x* given the measurements shown in the figure.



Since the triangles are similar triangles, the corresponding sides are proportional and there exists a common ratio between the pairs of corresponding sides. This common ratio is called the **scale factor** and is the **same** between any pair of the corresponding sides in the two given similar triangles.

Therefore, the corresponding sides can be written as equivalent ratios.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

$$\frac{16}{8} = \frac{x}{4} = \frac{18}{9}$$

To solve for *x*, set up a proportion based on two of the ratios. One proportion could be $\frac{AB}{XY} = \frac{BC}{YZ}$. Another one could be $\frac{BC}{YZ} = \frac{AC}{XZ}$.

First, we'll use $\frac{AB}{XY} = \frac{BC}{YZ}$.

$$\frac{AB}{XY} = \frac{BC}{YZ}$$

$\frac{16}{8} = \frac{x}{4}$ Substitute the given values.

$8x = 64$ Cross multiply.

$x = 8$ Divide.

Now, we'll use the second proportion to check the answer.

$$\frac{BC}{YZ} = \frac{AC}{XZ}$$

$\frac{x}{4} = \frac{18}{9}$ Substitute the given values.

$9x = 72$ Cross multiply.

$x = 8$✓ Divide.

The length of the missing side (x) is 8 units.

Similar Polygons

similar figures – Similar figures are figures that have the same shape, but are different in size.

similar polygons – Similar polygons are polygons that have congruent corresponding angles and the measures of their corresponding sides are proportional.

*The math symbol to denote **similarity** is "~".

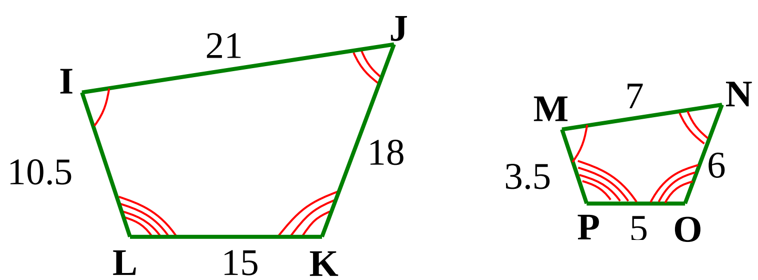
*The math symbol for **congruency** is "≅".

Example 1: Quadrilateral IJKL and MNOP are similar.

(a) State the congruent angles.

(b) Write the corresponding sides as proportional ratios.

Quadrilateral IJKL ~ Quadrilateral MNOP



*The corresponding congruent angles are marked with curved "hash marks".

a.) The corresponding congruent angles are:

$\angle I \cong \angle M$ denoted by one curved hash mark in the figures

$\angle J \cong \angle N$ denoted by two curved hash marks in the figures

$\angle K \cong \angle O$ denoted by three curved hash marks in the figures

$\angle L \cong \angle P$ denoted by four curved hash marks in the figures

b.) The corresponding proportional sides are:

$$\frac{IJ}{MN} = \frac{JK}{NO} = \frac{LK}{PO} = \frac{IL}{MP}$$

$$\frac{21}{7} = \frac{18}{6} = \frac{15}{5} = \frac{10.5}{3.5} = \frac{3}{1}$$

$$\begin{bmatrix} \frac{21}{7} \div \frac{7}{7} = \frac{3}{1} \\ \\ \frac{18}{6} \div \frac{6}{6} = \frac{3}{1} \\ \\ \frac{15}{5} \div \frac{5}{5} = \frac{3}{1} \\ \\ \frac{10.5}{3.5} \div \frac{3.5}{3.5} = \frac{3}{1} \end{bmatrix}$$

All the ratios reduce to the same scale factor, 3/1.

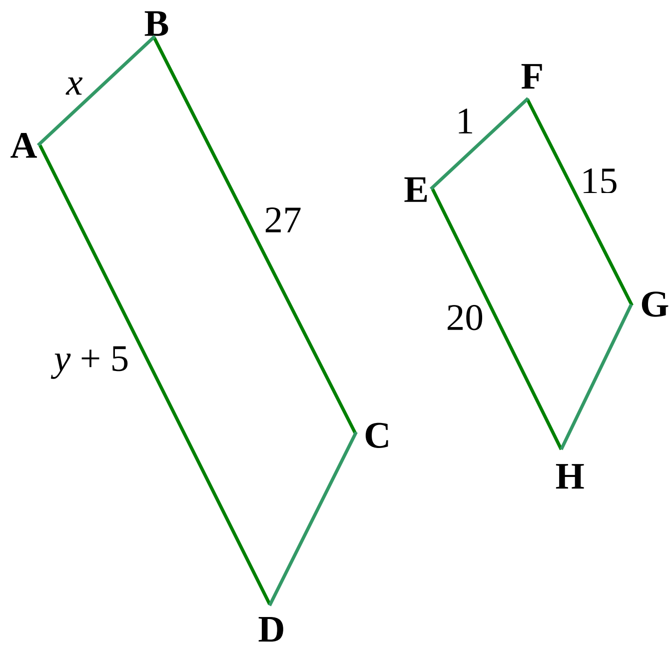
scale factor – The scale factor for two similar polygons is the ratio of the lengths of any two corresponding sides.

The scale factor of quadrilateral IJKL *to* quadrilateral MNOP is $\frac{3}{1}$ or 3.

When comparing the other way around, the scale factor of quadrilateral MNOP *to* quadrilateral IJKL is $\frac{1}{3}$.

Refer to the figure below to solve the next three examples.

Given: Quadrilateral ABCD ~ Quadrilateral EFGH



Example 1: Find the scale factor.

Use a pair of corresponding sides that are given. For this problem, use sides BC and FG to write the ratio and simplify.

$$\frac{BC}{FG} = \frac{27}{15} = \frac{9}{5}$$

The scale factor is $\frac{9}{5}$.

Example 2: Find the value of x.

Write a proportion using the values of corresponding sides that are given and the variable x.

Sides BC and FG are corresponding sides and the values are given. The side (EF) that corresponds with x is also given.

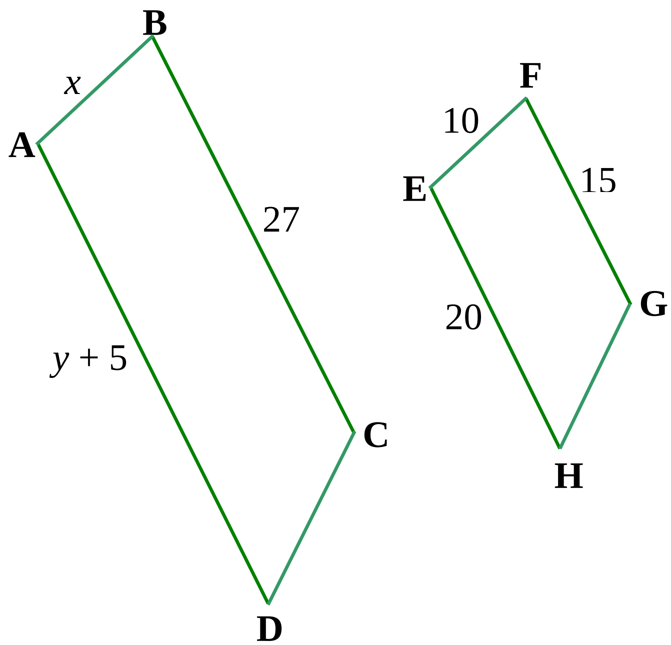
$$\frac{AB}{EF} = \frac{BC}{FG}$$

$$\frac{x}{10} = \frac{27}{15}$$

$$x(15) = 10(27)$$

$$15x = 270$$

$$x = 18$$



$Check: \quad \frac{18}{10} = \frac{9}{5}$ which is the scale factor.✓

Example 3: Find the value of *y*.

Write a proportion using the values of corresponding sides that are given and the variable *y*.

Sides *BC* and *FG* are corresponding sides and the values are given. The side (*EH*) corresponding with *y* is also given.

$$\frac{AD}{EH} = \frac{BC}{FG}$$

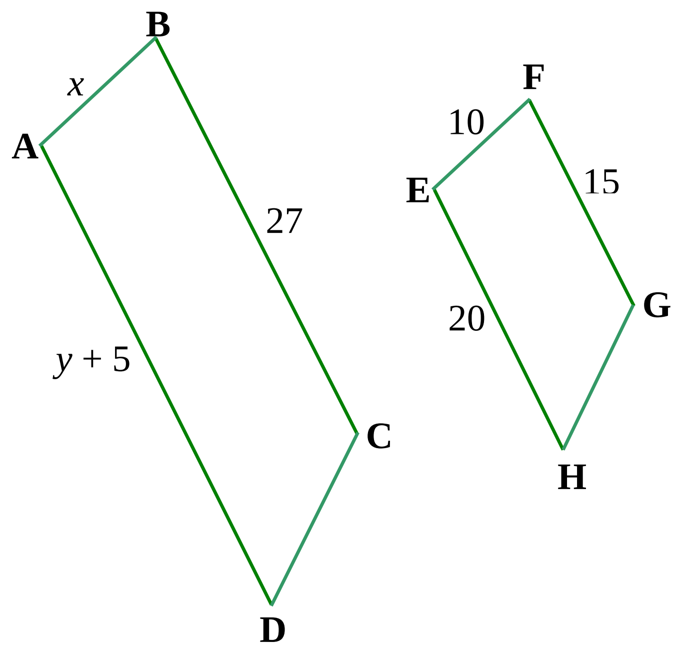
$$\frac{y+5}{20} = \frac{27}{15}$$

$$(y+5)(15) = 20(27)$$

$$15y + 75 = 540$$

$$15y = 465$$

$$y = 31$$

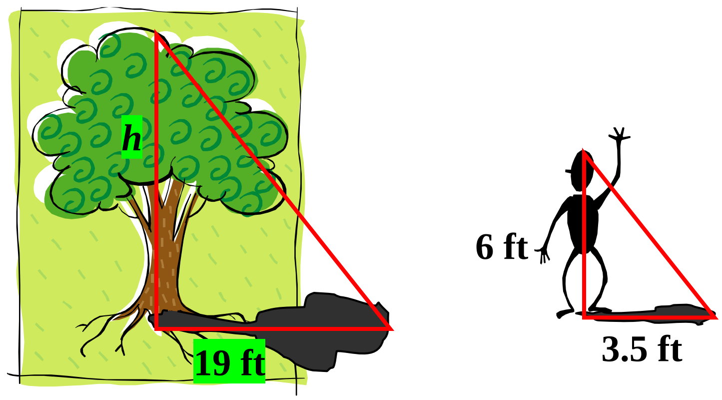


$\textit{Check}: \quad \frac{31+5}{20} = \frac{36}{20} = \frac{9}{5}$ which is the scale factor.✓

Indirect Measurements

Take a look at two scenarios where similar triangles are applied to find lengths and distances indirectly.

Scenario 1: Bill is preparing to cut down a tree in the yard. He doesn't want to actually fell the tree until he has a pretty good estimate of the height. Then he notices that both he and the tree are casting a shadow. His shadow is 3.5 feet long, while the shadow of the tree is 19 feet long. If Bill is six feet tall, how tall is the tree?



Since similar triangles are formed by the tree casting a shadow and Bill casting a shadow, the height of the tree can be determined indirectly. Use the values from the similar triangles to set up a proportion to solve for the height (*h*) of the tree.

$$\frac{\text{height of tree}}{\text{Bill's height}} = \frac{\text{shadow of tree}}{\text{Bill's shadow}}$$

$$\frac{h}{6} = \frac{19}{3.5}$$

Cross multiply and solve.

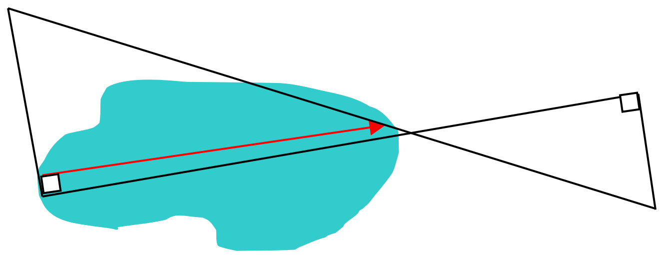
$$3.5h = 19 \times 6$$

$$3.5h = 114$$

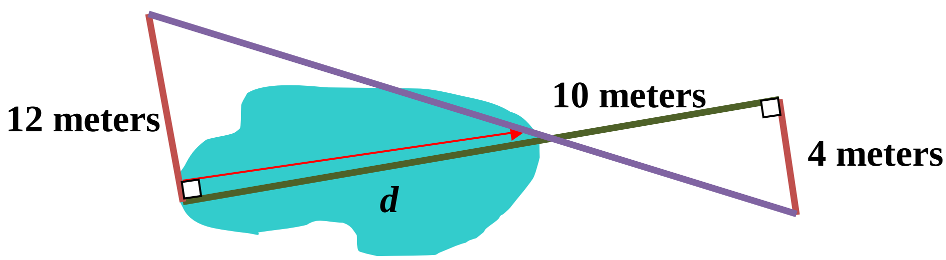
$$h = 33 \text{ (rounded to nearest whole number)}$$

The height of the tree is approximately 33 feet.

Now, let's take a look at the problem of finding the distance across a lake which is difficult to measure under normal circumstances. Measurements can be made across dry land and the angles can be measured with a surveyor tool. If enough information is gathered, the distance across the lake can be determined indirectly.



Scenario 2: Use the measurements shown below and similar triangles to find the distance (d) across the lake.



*Notice that the triangles are similar, but turned in different directions. The corresponding sides are denoted by different colors.

Now, set up a proportion of values to solve for the distance (*d*).

We will use the two legs of the right triangles, not the hypotenuses, to write a proportion in words first, and then solve.

$$\frac{\text{shorter leg of large right }\square}{\text{shorter leg of small right }\square} = \frac{\text{longer leg of large right }\square}{\text{longer leg of small right }\square}$$

$$\frac{12}{4} = \frac{d}{10}$$

Cross multiply and solve.

$$4d = 12 \times 10$$
$$4d = 120$$
$$d = \frac{120}{4}$$
$$d = 30$$

The distance across the lake is approximately 30 meters.

Use these ideas to determine proportions using indirect measurement that will give solutions geometrically that cannot easily be done otherwise.