MULTIPLY AND DIVIDE FRACTIONS

In this unit, you will compute with multiplication and division of fractions and mixed numbers.   You will also review reciprocals that are used to divide fractions and practice canceling to simplify answers.

Multiplication of Fractions

Reciprocals

Division of Fractions

Multiplication of Fractions

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators. Simplify the fractions when necessary.

numerator – A numerator is the top part of a fraction. In the fraction 2/3, the numerator is two $\left(\dfrac{2}{3}\right)$.

denominator – A denominator is the bottom part of a fraction. In the fraction 2/3, the denominator is three $\left(\dfrac{2}{3}\right)$.

proper fraction – A proper fraction is a fraction where the numerator is less than the denominator. An example of a proper fraction is $\dfrac{7}{8}$.

*Recall that the answer to a multiplication problem is called the **product**.

Example 1: Find the product of the proper fractions, 2/3 × 8/9.

$$\frac{2}{3} \times \frac{8}{9} \;=\; \frac{2 \times 8}{3 \times 9} \;=\; \frac{16}{27}$$

> Multiply the numerators.
> Multiply the denominators.

Multiplication of fractions can be made easier by using canceling to simplify first, and then multiplying the numerators and the denominators.

Canceling

Look for a numerator and a denominator that will simplify.

Example 2: Find the product of proper fractions, $3/4 \times 8/11$.

In canceling, one number must be in the numerator and the other number must be in the denominator.

$$\frac{3}{4} \times \frac{8}{11} = \frac{3}{\cancel{4}_1} \times \frac{\cancel{8}^2}{11} = \frac{3 \times 2}{1 \times 11} = \frac{6}{11}$$

Cancel the 4 and 8 by 4.

Example 3: Find the product of 2/3 of 9.

Make the whole number 9 a fraction by placing it over 1. $9 = 9/1$.

$$\frac{2}{3} \times \frac{9}{1} = \frac{2}{\cancel{3}_1} \times \frac{\cancel{9}^3}{1} = \frac{2 \times 3}{1 \times 1} = \frac{6}{1} = 6$$

Cancel the 3 and 9 by 3.

Multiplying Mixed Numbers

improper fraction – An improper fraction is a fraction where the numerator is larger than or equal to the denominator. An example of an improper fraction is $\frac{12}{5}$.

mixed number – A mixed number is a number that is a combination of a whole number and a fraction. An example of a mixed number is $2\frac{2}{5}$.

*To multiply mixed numbers, first change the mixed numbers to improper fractions.

Example 4: Find the improper fractions for 1 1/11 and 2 4/9.

*Multiply the denominator by the whole number, and then add on the numerator. Put that number over the denominator.

$$1\frac{1}{11}=\frac{11\times1+1}{11}=\frac{12}{11} \qquad\qquad 2\frac{4}{9}=\frac{9\times2+4}{9}=\frac{22}{9}$$

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Mixed Fractions to Improper Fractions

Example 5: Find the product of the mixed numbers, 1 1/11 × 2 4/9.

In the previous problem, the two mixed numbers are expressed as improper fractions. (1 1/11 = 12/11 and 2 4/9 = 22/9)

$$\frac{12}{11}\times\frac{22}{9} \;=\; \frac{\cancel{12}^{\,4}}{\cancel{11}_{\,1}}\times\frac{\cancel{22}^{\,2}}{\cancel{9}_{\,3}} \;=\; \frac{8}{3} \;=\; 2\frac{2}{3}$$

Cancel the 11 and 22 by 11.

Cancel the 12 and 9 by 3.
(Think of a number that will divide into 12 and 9 evenly. That number is 3.

Multiplying Multiple Fractions

Example 6: Find the product of the proper fractions $9/16 \times 5/8 \times 2/3$.

Simplify through canceling, and then multiply the numerators and denominators.

*With multiple fractions, cancel any numerator with any denominator.

Look for a numerator and a denominator that will simplify.

$$\frac{\cancel{9}^3}{20} \times \frac{5}{8} \times \frac{2}{\cancel{3}^1}$$ $\left[\text{ First cancel the 9 and 3. }\right]$

$$\frac{\cancel{9}^3}{\cancel{20}^4} \times \frac{\cancel{5}^1}{8} \times \frac{2}{\cancel{3}^1}$$ $\left[\text{ Now cancel the 5 and 20. }\right]$

$$\frac{\cancel{9}^3}{\cancel{20}^4} \times \frac{\cancel{5}^1}{\cancel{8}^4} \times \frac{\cancel{2}^1}{\cancel{3}^1}$$ $\left[\text{ Last cancel the 2 and 8. }\right]$

$$\frac{\cancel{9}^3}{\cancel{20}^4} \times \frac{\cancel{5}^1}{\cancel{8}^4} \times \frac{\cancel{2}^1}{\cancel{3}^1} = \frac{3}{16}$$ $\left(\begin{array}{l}\text{Multiply the cancelled} \\ \text{numerators and} \\ \text{denominators.}\end{array}\right)$

Multiplying Fractions with Signs

Example 7: Find the product of –2/3 × –9/16.

> Multiplication and Division Rule: If the signs are the same,
> the answer is positive.

$$-\frac{2}{3} \times -\frac{9}{16} =$$

$$= -\frac{\cancel{2}^{1}}{\cancel{3}^{1}} \times -\frac{\cancel{9}^{3}}{\cancel{16}^{8}}$$

$$= +\frac{3}{8}$$

The product of –2/3 × –9/16 is 3/8.

Example 8: Find the product of –6 3/4 × 2 4/15.

> Multiplication and Division Rule: If the signs are different,
> the answer is negative.

$$-6\frac{3}{4} \times 2\frac{4}{15} =$$

$$= -\frac{27}{4} \times \frac{34}{15}$$

$$= -\frac{\cancel{27}^{9}}{\cancel{4}^{2}} \times \frac{\cancel{34}^{17}}{\cancel{15}^{5}}$$

$$= -\frac{153}{10} = -15\frac{3}{10}$$

The product of –6 3/4 × 2 4/15 is –15 3/10.

Reciprocals

The **reciprocal** (inverse) of a fraction is the fraction that is multiplied by the original fraction to equal one (1).

For example, the **reciprocal** of $\frac{1}{2}$ is $\frac{2}{1}$ because $\frac{1}{2} \times \frac{2}{1} = \frac{1}{\cancel{2}^1} \times \frac{\cancel{2}^1}{1} = \frac{1}{1} = 1$.

*To find the reciprocal of a proper fraction, just "flip" the fraction.

> *Example 1*: What is the reciprocal of 3/5?
>
> Flip the fraction: $\frac{3}{5} \rightarrow \frac{5}{3}$.
>
> The reciprocal of 3/5 is 5/3.

*To find the reciprocal of a mixed number, first write the mixed number as an improper fraction, and then "flip" the fraction.

> *Example 2*: What is the reciprocal of 2 3/4?
>
> First, write the fraction as an improper fraction.
>
> $$2\frac{3}{4} = \frac{4 \times 2 + 3}{4} = \frac{11}{4}$$
>
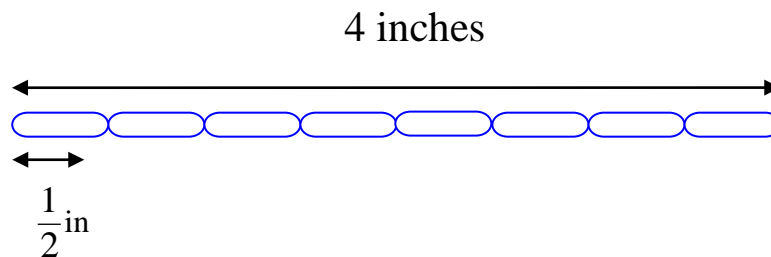> Then, flip the fraction: $\frac{11}{4} \rightarrow \frac{4}{11}$.
>
> The reciprocal of 2 3/4 is 4/11.

Division of Fractions

Introduction to Division of Fractions

A child's bracelet is four (4) inches long. Each of the links is a half-inch long. How many links are in the chain?

4 inches

$\frac{1}{2}$ in

Count the 1/2-inch links to determine the number of links in the bracelet. There are eight. Four divided by one-half equals eight.

$$4 \div \frac{1}{2} = 8$$

It takes eight half-inch links to make up a bracelet four inches long.

*Recall that the answer to a division problem is called the **quotient**, the number being divided is the **dividend**, and the number that the dividend is being divided by is the **divisor**.

$$4 \div \frac{1}{2} = 8$$

*Notice, that in this division problem of fractions, the dividend (4) and the divisor (1/2) are both smaller than the quotient (8). Interesting!!!

To divide fractions, multiply by the **reciprocal or inverse** of the second fraction.

Let's revisit the problem above, and solve it using the algorithm (mathematical process) for dividing fractions.

Example 1: Find the quotient of 4 divided by 1/2.

*Note: When entering a division problem via the keyboard, it would look like 4 / (1/2) . The first slash (/) represents division and the second slash is part of the fraction.

$$4 \div \frac{1}{2} =$$

$$= \frac{4}{1} \div \frac{1}{2} \qquad \left[\text{Write 4 as a fraction } (\frac{4}{1}). \right]$$

$$= \frac{4}{1} \times \frac{2}{1} \qquad \left[\text{Multiply by the reciprocal.} \right]$$

$$= \frac{4 \times 2}{1} = \frac{8}{1} = 8 \qquad \left[\begin{array}{l} \text{Multiply the numerators} \\ \text{and the denominators.} \end{array} \right]$$

Dividing Fractions and Mixed Numbers

Now, we'll take a look at several examples of division of fractions and mixed numbers.

Example 2: Find the quotient of 3/4 divided by 4/5.

$$\frac{3}{4} \div \frac{4}{5} =$$

$$= \frac{3}{4} \times \frac{5}{4} \qquad \left[\text{Multiply by the reciprocal.} \right]$$

$$= \frac{3 \times 5}{4 \times 4} \qquad \left[\begin{array}{l} \text{Multiply the numerators} \\ \text{and the denominators.} \end{array} \right]$$

$$= \frac{15}{16}$$

Example 3: Find the quotient of 2/3 divided by 8/9.

$$\frac{2}{3} \div \frac{8}{9} =$$

$$= \frac{2}{3} \times \frac{9}{8} \qquad \boxed{\text{Multiply by the reciprocal.}}$$

$$= \frac{\cancel{2}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{9}^{3}}{\cancel{8}^{4}} \qquad \boxed{\text{Use cancelling.}}$$

$$= \frac{1 \times 3}{1 \times 4} = \frac{3}{4}$$

Example 4: Find the quotient of 12 divided by 3/5.

$$12 \div \frac{3}{5} =$$

$$= \frac{12}{1} \times \frac{5}{3} \qquad \boxed{\text{Multiply by the reciprocal.}}$$

$$= \frac{\cancel{12}^{4}}{1} \times \frac{5}{\cancel{3}^{1}} \qquad \boxed{\text{Use cancelling.}}$$

$$= \frac{4 \times 5}{1 \times 1} = \frac{20}{1} = 20$$

Example 5: Find the quotient of 2 2/7 divided by 6 2/3.

$$2\frac{2}{7} \div 6\frac{2}{3} \ =$$

$$= \frac{16}{7} \div \frac{20}{3}$$

Change both mixed numbers to improper fractions.

$$= \frac{16}{7} \times \frac{3}{20}$$

Multiply by the reciprocal.

$$= \frac{\cancel{16}^{4}}{7} \times \frac{3}{\cancel{20}^{5}}$$

Use cancelling.

$$= \frac{4 \times 3}{7 \times 5} = \frac{12}{35}$$

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Divide Fractions

Dividing Fractions with Signs

Example 6: Find the quotient of 3 3/4 divided by –2 1/10.

> Multiplication and Division Rule: If the signs are different, the answer is negative.

$$3\frac{3}{4} \div -2\frac{1}{10} =$$

$$= \frac{15}{4} \div -\frac{21}{10}$$

$$= \frac{15}{4} \times -\frac{10}{21}$$

$$= \frac{\cancel{15}^{\,5}}{\cancel{4}^{\,2}} \times -\frac{\cancel{10}^{\,5}}{\cancel{21}^{\,7}}$$

$$= -\frac{25}{14} = -1\frac{11}{14}$$

The quotient of 3 3/4 divided by –2 1/10 is –1 11/14.

Example 7: Find the quotient of –2 11/20 divided by –2 4/15.

> Multiplication and Division Rule: If the signs are the same, the answer is positive.

$$-2\frac{11}{20} \div -2\frac{4}{15} \quad =$$

$$= -\frac{51}{20} \div -\frac{34}{15}$$

$$= -\frac{51}{20} \times -\frac{15}{34}$$

$$= -\frac{\cancel{51}^{3}}{\cancel{20}_{4}} \times -\frac{\cancel{15}^{3}}{\cancel{34}_{2}}$$

$$= +\frac{9}{8} = 1\frac{1}{8}$$

The quotient of –2 11/20 divided by –2 4/15 is 1 1/8.

Click on the gears below to play a game.

Multiply and Divide Fractions