Course Overview

In this course, students investigate the base-ten number system by reading, writing, representing, comparing and rounding whole numbers and decimals; compute with whole numbers using one and two-digit numbers; develop strategies for performing mental computations; and generate equivalent forms of fractions and decimals to estimate, add, and subtract decimals and fractions with like denominators. Students count money and make change; examine prime and composite numbers; make simple measurement conversions of units; solve multi-step problems; and develop strategies to find perimeter, area, and volume. In geometry, students investigate, classify, and model plane figures and solids. They plot locations in the first quadrant of a coordinate system and make transformations of slides, flips, and turns; use words, tables, and graphs to analyze patterns and relationships to make predictions and solve problems; represent unknowns as variables in equations and inequalities and relate how change in one variable affects the value of a related variable. Students gather and organize data in tables, charts, and graphs and make predictions based on interpretations and appropriate display of data; use mode, median, and range to describe characteristics of data; conduct simple probability experiments and make predictions of possible outcomes ordering events as impossible, unlikely, equal, likely, and certain-to-happen; and make lists to display all possible combinations of different sets of items.

In addition, there are practice worksheets in many of the units that provide more practice on specific topics. To view a practice worksheet, click on the paper/pencil icon. The worksheet will appear on top of the unit's main page. When finished, select File/Close to close the worksheet. The main page of the unit will be visible once again. Students and teachers may work together to decide what worksheet(s) or part of a worksheet(s) can be used as a supplementary activity for the unit. The answer key for each worksheet is provided for the instructor's use.

REVIEW OF DECIMALS AND INTEGERS

Unit Overview

The first six units of this course will provide a review of concepts that are most important for success in this course. Having a sense of decimal operations will help in many ways including estimation and problem-solving. Since the study of mathematics builds on concepts previously learned, it is best to complete the units in the order that they are presented.

Add and Subtract Decimals

Multiply Decimals

Divide Decimals

Integer Operations

Add and Subtract Decimals

To add or subtract decimals, be sure to line up the decimal points so that the place values also line up – tenths with tenths, hundredths with hundredths, and so on.

sum – the answer to an addition problem

difference – the answer to a subtraction problem

Example 1: Find the sum: 8.3+17.82

$$\begin{array}{r} {\scriptstyle 1} \\ 8.3 \\ +17.82 \\ \hline 26.12 \end{array}$$

*In tenths column, 8 + 3 equals 11, so place a one (1) in the answer and carry the other one (1) to ones place.

Example 2: Find the difference: 5.3−3.74

$$\begin{array}{r} {\scriptstyle 4\ 12 10} \\ 5.3\cancel{0} \\ -3.7\ 4 \\ \hline 1.5\ 6 \end{array}$$

*A zero (0) must be put in hundredths place as a place holder. Then, borrow and subtract.

Example 3: Find the difference: 12−5.35

$$\begin{array}{r} {\scriptstyle \ 9} \\ {\scriptstyle 0\ 11\ \cancel{10}\ 10} \\ \cancel{12}.\cancel{0}\cancel{0} \\ -5.3\ 5 \\ \hline 6.6\ 5 \end{array}$$

*Two zeros must be put in tenths and hundredths place as place holders. Then, borrow and subtract.

Example 4: Find the sum.

$13.6+7.5=?$

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 13.6 \\ +\ 7.5 \\ \hline 21.1 \end{array}$$

Example 5: Solve the previous problem using decimal fractions, and then write the answer as a mixed number and a decimal.

$13.6+7.5=?$

$$13.6=13\frac{6}{10}$$

$$+7.5=7\frac{5}{10}$$

$$20\frac{11}{10}=21\frac{1}{10}$$

$$\left\{\begin{aligned} 20\frac{11}{10} &= 20+\frac{10}{10}+\frac{1}{10} \\ &= 20+\ 1\ +\frac{1}{10}=21\frac{1}{10} \end{aligned}\right\}$$

$$21\frac{1}{10}=21.1$$

Example 6: Find the sum.

$18.33+7.5=?$

$$\begin{array}{r} {\scriptstyle 1} \\ 18.33 \\ +7.50 \\ \hline 25.83 \end{array}$$

*Put a zero (0) in hundredths place as a place holder.

Example 7: Solve the previous problem using decimal fractions, and then write the answer as a mixed number and a decimal.

$$18.33+7.5=?$$

$$18.33=18\frac{33}{100}=18\frac{33}{100}$$
$$+7.5 \;=\; 7\frac{5}{10} \;=\; 7\frac{50}{100} \qquad \left\{\frac{5}{10}\times\frac{10}{10}=\frac{50}{100}\right\}$$
$$25\frac{83}{100}=\;25.83$$

Example 8: Solve the problem using decimal fractions, and then check the answer using regular decimal subtraction.

$$25.2-6.9=?$$

$$25.2=25\frac{2}{10}=24\frac{12}{10} \qquad \left\{25\frac{2}{10}=24+1+\frac{2}{10}\;=24+\frac{10}{10}+\frac{2}{10}=24\frac{12}{10}\right\}$$
$$-6.9=\;\;6\frac{9}{10}\;\;=6\frac{9}{10}$$
$$18\frac{3}{10}=18.3$$

☑ *Check*:

$$\begin{array}{ll} & \overset{1\;\;14\;12}{\not{2}\;\not{5}.\not{2}} \\ 25.2 & \\ \underline{-6.9} & \underline{-6.9} \\ 18.3 & 1\;8.\;3 \end{array}$$

Multiply Decimals

Multiplying Decimals Less Than One

To place the decimal point when multiplying decimals, count the decimal places (right of the decimal point) in each factor and total them. The total number is the number of decimal places that will be in the answer.

product - the answer to a multiplication problem

Example 1: Find the product.

Multiply: 0.7 × 0.9

Estimate
1 × 1 = 1

$$\begin{array}{r} 0.7 \\ \times\ 0.9 \\ \hline 0.63 \end{array}$$

*1 decimal place
*1 decimal place
**total* - 2 decimal places
(1+1=2)

Why two decimal places?
Write both decimals as fractions and multiply.

$$\frac{7}{10}\times\frac{9}{10}=\frac{63}{100}=0.63$$

The product of 0.7 and 0.9 is 0.63.

☑ *Quick Check*: The estimate of 1 is close to 0.63.

Example 2: Find the product.

Multiply: 0.12 × 0.36

Estimate
0 × 0 = 0

0.12	*2 decimal places
× 0.36	*2 decimal places
72	
360	*Zero is a place holder.
0.0432	**total* - 4 decimal places (2 + 2 = 4)

Why four decimal places?
Write both decimals as fractions and multiply.

$$\frac{12}{100}\times\frac{36}{100}=\frac{432}{10,000}=0.0432$$

*Note: The zero in front of the four is a place holder to show four decimal places.

The product of 0.12 and 0.36 is 0.0432.

☑*Quick Check*: The estimate of 0 is close to 0.0432.

Click on the tracks below to play a game.

Multiplying Decimals Greater Than One

To place the decimal point when multiplying decimals, count the decimal places (right of the decimal point) in each factor and total them. The total number is the number of decimal places that will be in the answer.

Example 3: Find the product.

Multiply: 5.23 × 7.9

Estimate
5 × 8 = 40

5.23 *2 decimal places
× 7.9 *1 decimal place
4707
36610 *Zero is a place holder.
41.317 **total* - 3 decimal places
(2+1=3)

Why three decimal places?
Write both decimals as mixed numbers and multiply.

$$5\frac{23}{100}\times 7\frac{9}{10}=\frac{523}{100}\times\frac{79}{10}=$$

$$\frac{41{,}317}{1000}=41\frac{317}{1000}=41.317$$

The product of 5.23 and 7.9 is 41.317.

☑*Quick Check*: The estimate of 40 is close to 41.317.

Example 4: Find the product.

Multiply: 46 × 2.8

Estimate
50 × 3 = 150

```
  46      *0 decimal places
× 2.8     *1 decimal place
 ----
  368
  920     *Zero is a place holder.
 ----
 128.8    *total - 1 decimal place
          (0+1=1)
```

Why one decimal place?
Write both numbers in fraction form and multiply.

$$\frac{46}{1}\times 2\frac{8}{10}=\frac{46}{1}\times\frac{28}{10}=$$
$$\frac{1288}{10}=128\frac{8}{10}=128.8$$

The product of 46 and 2.8 is 128.8.

☑*Quick Check*: The estimate of 150 is close to 128.8.

Example 5: Find the product.

Multiply: 5.23 × 3.79

Estimate
5 × 4 = 20

```
  5.23      *2 decimal places
× 3.79      *2 decimal place
 -----
  4707
 36610
156900      *The zeros are place holders.
 -----
19.8217     *total - 4 decimal places
            (2+2=4)
```

Why four decimal places?
Write both decimals as mixed numbers and multiply.

$$5\frac{23}{100}\times 3\frac{79}{100}=\frac{523}{100}\times\frac{379}{100}$$
$$\frac{198,217}{10,000}=19\frac{8,217}{10,000}=19.8217$$

The product of 5.23 and 3.79 is 19.8217.

☑*Quick Check*: The estimate of 20 is close to 19.8217.

Divide Decimals

Dividing Decimals by Whole Numbers

To divide a decimal by a whole number, place the decimal point in the quotient above the decimal point in the dividend.

quotient – the answer to a division problem

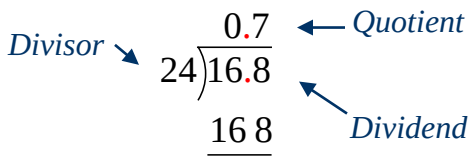
dividend – the number that is to be divided

divisor – the number by which the dividend is being divided

Example 1: Find the quotient.

Divide: 16.8 by 24

Estimate
20 ÷ 20 = 1



The quotient of 16.8 divided by 24 is 0.7.

☑*Quick Check*: The estimate of 1 is close to 0.7.

Dividing By Decimals in Tenths

To divide a decimal by a decimal:

- Make the divisor a whole number.

  Do this by multiplying the divisor by the power of ten that would **move the decimal point to the right of all of the digits in the divisor.**

- Multiply the dividend by the same power of ten used to make the divisor a whole number.

There are five steps in long division:

Divide	÷
Multiply	×
Subtract	–
Compare	*compare*
Bring Down	*bring down*

Example 2: Find the quotient.

Divide: 29.24 by 3.4

Estimate
30 ÷ 3 = 10

$$3.4\overline{)29.24}$$
$$16\,8$$

The divisor and dividend must be multiplied by 10 to make the divisor a whole number. The short cut for multiplying by 10 is move the decimal point one place to the right.

For this problem, repeat the "five steps of long division" until the division "comes out even"; that is, divide until the remainder is zero.

5 Steps of Division

Divide: 34 into 292 to get 8
Multiply: 8 × 34 to get 272
Subtract: 272 from 292 to get 20
Compare: 20 with 34
(20 must be smaller than 34)
Bring Down: 4

Divide, Multiply, Subtract, Compare

$$\begin{array}{r} 8. \\ 34.\overline{)292.4} \\ 272 \\ \hline 20 \end{array}$$

Bring down and start over.

$$\begin{array}{r} 8.6 \\ 34.\overline{)292.4} \\ 272 \\ \hline 20\ 4 \\ 20\ 4 \\ \hline \end{array}$$

The quotient of 29.24 divided by 3.4 is 8.6.

☑*Quick Check*: The estimate of 10 is close to 8.6.

Dividing By Decimals in Hundredths

Example 3: Find the quotient.

Divide: 8.0124 by 1.32

Estimate
8 ÷ 1 = 8

$$1.32\overline{)8.0124}$$

The divisor and dividend must be multiplied by 100 to make the divisor a whole number. The short cut for multiplying by 100 is move the decimal point two places to the right.

For this problem, repeat the "five steps of long division" until the division "comes out even"; that is, divide until the remainder is zero.

$$\begin{array}{r} 6. \\ 132.\overline{)801.24} \\ \underline{792} \\ 92 \end{array} \qquad \begin{array}{r} 6.0 \\ 132.\overline{)\,801.24} \\ \underline{792} \\ 92 \\ \underline{0} \end{array} \qquad \begin{array}{r} 6.07 \\ 132.\overline{)801.24} \\ \underline{792} \\ 92 \\ \underline{0} \\ 924 \\ \underline{924} \end{array}$$

The quotient of 8.0124 divided by 1.32 is 6.07.

☑*Quick Check*: The estimate of 8 is close to 6.07.

Dividing Whole Numbers by Decimals

When the dividend is a whole number, first show the decimal point after the dividend, and then move the decimal point to the right as many places as determined by making the divisor a whole number.

Example 4: Find the quotient.

Divide: 75 by 0.25

$$0.25\overline{)75.00} = 300.$$

75
00
00

The divisor and dividend must be multiplied by 100 to make the divisor a whole number. The short cut for multiplying by 100 is move the decimal point two places to the right.

The quotient of 75 divided by 0.25 is 300.

Click on the bricks below to play a game.

Dividing Decimals and Rounding Quotients

In division, sometimes the answer does not come out even. When this occurs, divide until the quotient has one extra decimal place, and then round to the given place.

*The symbol for approximately equal to is "≈".

Example 5: Find the quotient. Round the answer to the nearest hundredth.

Divide until the quotient is in thousandths, and then round to hundredths.

Divide: 95.8 by 0.24

$$0.24\overline{)95.80}$$

$$399.166 \approx 399.17$$

$$24\overline{)9580.000}$$

72
238
216
220
216
40
24
160
144
160
144
16

The quotient of 95.8 divided by 0.24 is approximately equal to 399.17.

Integer Operations

The integers are a set of numbers that contain the whole numbers and their opposites. There are no decimals or fractions in the set of integers.

Integers: {… –4, –3, –2, –1, 0, 1, 2, 3, 4 …}

Addition of Integers

Rule 1: When the signs are the **same**, **ADD** the values and use the same sign.

Example 1: Find the sums.

$-4+(-5)=-9$ The signs are the same (both are negative), so ADD, and the answer will be negative.

$21+45=66$ The signs are the same (both are positive), so ADD, and the answer will be positive.

Rule 2: When the signs are **not the same**, **SUBTRACT** and take the sign of the number that is farthest from zero on the number line.

Example 2: Find the sums.

$8+(-3)=-5$ The signs are different, so SUBTRACT and the answer will be positive because 8 is farther from 0 than -3 on the number line.

$-7+4=-3$ The signs are different, so SUBTRACT and the answer will be negative because -7 is farther from 0 than 4 on the number line.

$5+(-12)=-7$ The signs are different, so SUBTRACT and the answer will be negative because -12 is farther from 0 than 5 on the number line.

$-11+15=5$ The signs are different, so SUBTRACT and the answer will be positive because 15 is farther from 0 than -11 on the number line.

Subtraction of Integers

Rule: To subtract integers, **ADD the opposite**.

In other words, change the sign of the second integer, and then apply the addition rules of integers.

Example 3: Find the differences.

$-6-(-8) = -6+(+8) = -6+8 = 2$

$-10-4 = -10+(-4) = -14$

$16-(-2) = 16+(+2) = 16+2 = 18$

$5-11 = 5+(-11) = -6$

$13-7 = 6$ (Just use normal whole number subtraction.)

Click on the log below to play a game.

Multiplication and Division of Integers

These two operations have very simple rules.

Rule 1: When the signs of each number are the **same**, the answer is automatically **positive**.

Example 4: Find the products.

$(-8)(-5) = +40 = 40$

$(9)(6) = 54$

$56 \div 7 = 8$

$(-64) \div (-16) = +4 = 4$

Rule 2: When the signs of the two numbers are **different**, the answer is **negative**.

Example 5: Find the quotients.

$(-6)(7) = -42$

$(10)(-3) = -30$

$100 \div (-20) = -5$

$(-72) \div 12 = -6$

Be sure to consider only one pair of numbers at a time. If three numbers are multiplied together, consider the first two, and then the third.

Example 6: Find the product of (–4)(–3)(–5).

$(-4) \times (-3) \times (-5) = 12 \times (-5) = -60$

Click on the bricks below to play a game.