THE REAL NUMBER SYSTEM AND ORDER OF OPERATIONS

In this unit, you will learn about the real number system. The real numbers are the counting numbers, whole numbers, integers, rational numbers and irrational numbers. You will examine how these numbers are connected to each other.

You will also solve problems that have a definite order to them. It is important to become familiar with the accepted order of operations so that you can solve the problems in math correctly. Remember to work multiplication and division from left to right as one operation does not take priority over the other. The same is true with addition and subtraction; work these operations from left to right.

Real Numbers

Order of Operations

Real Numbers

Real numbers are the numbers that can be represented on the number line. They include the whole numbers, their opposites, and all the other numbers in between them.

The real numbers are a union of the rational and irrational numbers.

Rational Numbers - Rational numbers are numbers that can be written as a quotient of two integers.

The natural numbers, whole numbers, and integers are subsets of the rational numbers. Note: A member of any of these sets can be expressed as a quotient of two integers.

Natural Numbers: {1, 2, 3, 4, 5...}

Whole Numbers: {0, 1, 2, 3, 4, 5...}

Integers: {...–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5...}

Also included in the rational numbers are **fractions** (where the numerator and denominators are integers) and **repeating and terminating decimals**. Fractions may not have zero (0) as the denominator.

Irrational Numbers – Irrational numbers are numbers that **cannot** be expressed as the ratio of two integers. Examples of irrational numbers are (a) square roots of non-perfect squares, (b) pi (π), and (c) decimals that do not develop into a repeating pattern.

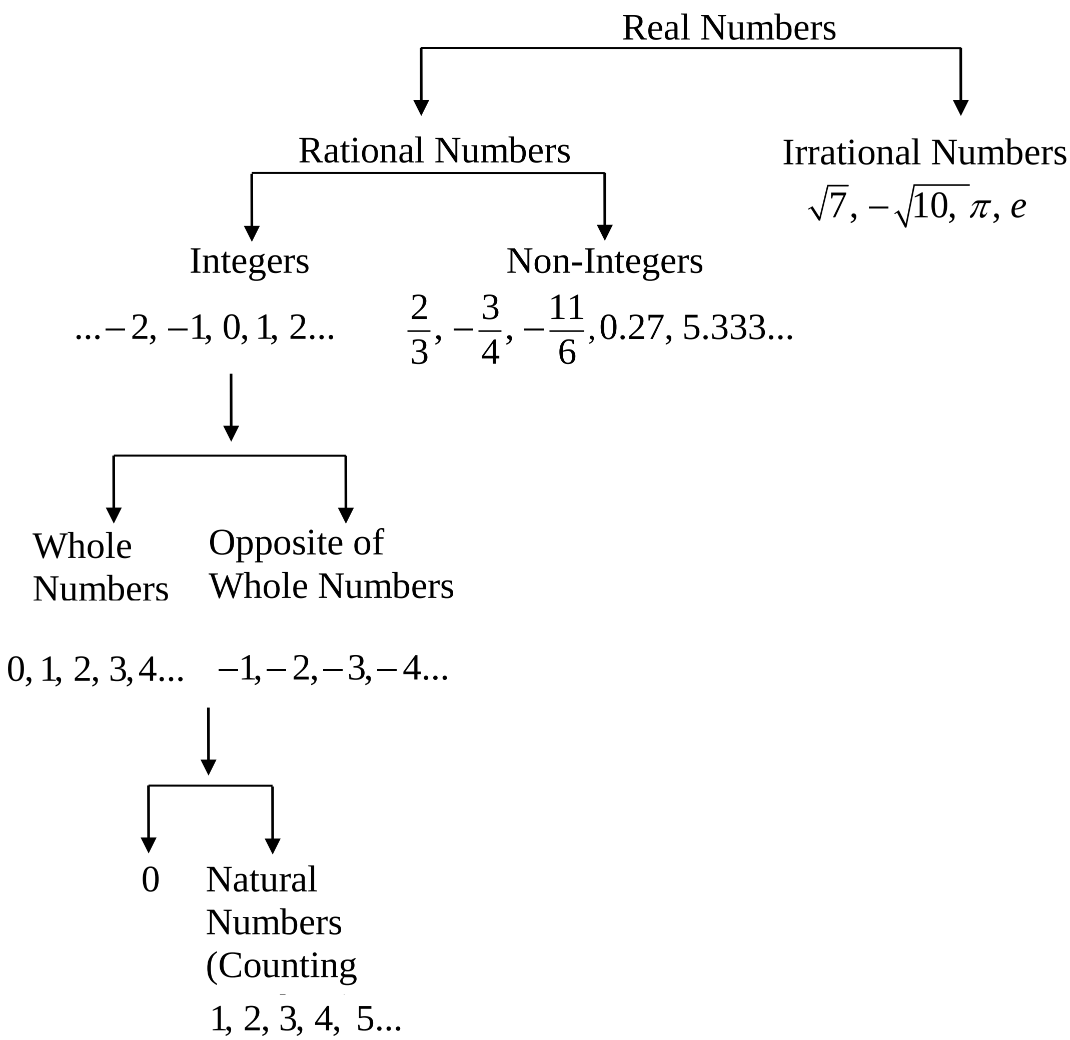
(a) $\sqrt{94}$
(b) $\pi \approx 3.14159265358979323846264338327 95...$
(c) 9.695359714832658028148881150845 3…

real numbers = rational numbers + irrational numbers

Study the two sketches below to better understand the subsets of the real numbers.

Sketch 1

This sketch shows how each subset can be divided into further subsets. For example, zero is a whole number, but is also an integer and a rational number because it is a member of all those subsets. The only subset of the rational numbers that zero is not a part of is the natural numbers.

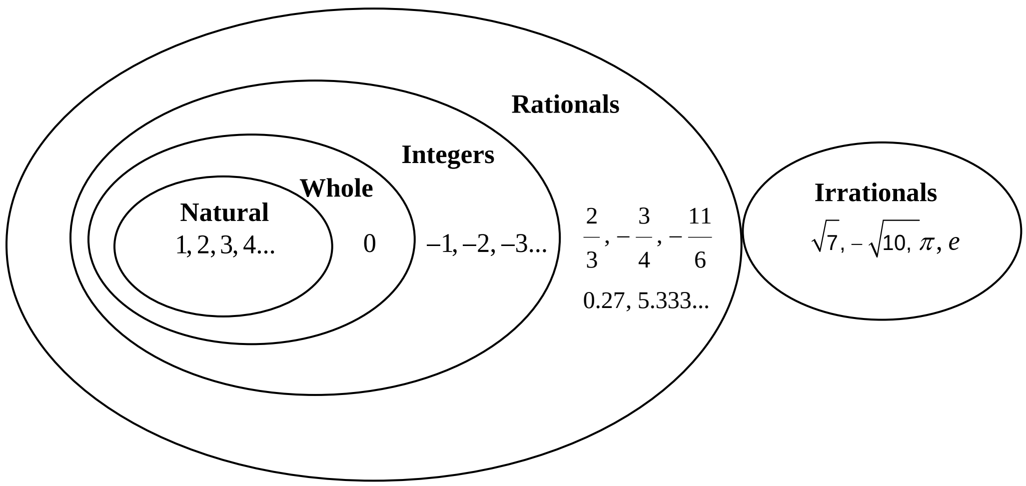


*The numbers listed in the subsets are just a few examples of the infinite possibilities.

Sketch 2

This sketch shows the connection between the subsets using a Venn diagram. Again, looking at zero, it is in all of the ovals that represent the rational numbers with the exception of the natural numbers.

Real Numbers



*Notice that the irrationals are in a league of their own.

The **rational numbers** are the types of numbers shown within the "rational circle". The "rational circle" includes all of the numbers within the circles inside it.

Some examples of rational numbers are –11/6, –4, 0, and 4. Rational numbers can be expression as a ratio of two integers.

$$\left(\frac{-11}{6}\right),\ \left(-4=\frac{-4}{1}\right),\ \left(0=\frac{0}{1}\right),\ \text{and}\ \left(4=\frac{4}{1}\right).$$

The **integers** are the types of numbers shown within the "integer circle". The "integer circle" includes all of the numbers within the circles inside it. Some examples of integers are $-4,\ 0,\ 4$.

The **whole numbers** are the types of numbers shown within the “whole number circle”. The “whole number circle” includes all of the numbers within the circle inside it. Some examples of whole numbers are 0 and 4.

The **natural numbers** are the types of numbers shown within the “natural number circle”. Some examples of whole numbers are 3 and 4.

The **irrational numbers** are in a “league of their own”. Some examples of rational numbers are $\sqrt{26}$, $-\sqrt{43}$, π, and e.

The **real numbers** are the types of numbers shown within all of the circles. The real numbers are the union of the rational and the irrational numbers.

Order of Operations

In order to find the numerical value (**evaluate**) of any combination of numbers and operations (**expression**) correctly, mathematicians have established the order of operations which tells us which operations to do first in any mathematical problem.

P (parentheses)

E (exponents – powers)

M (multiply)
D (divide) } **work left to right**

A (add)
S (subtract) } **work left to right**

This "saying" may be used to help remember the order of operations.

Please
Excuse
My **D**ear
Aunt **S**ally

*Note: Multiplication and division are at the same level, meaning multiplication does NOT take priority over division. Work these two operations as they occur, left to right. The same is true about addition and subtraction. Work the two operations as they occur, left to right.

Thus, if "multiplication and division" or "addition and subtraction" are the only two operations in the expression, work the problem from left to right!

Example 1: Evaluate $6 \times 4 + 2$.

$6 \times 4 + 2$	Multiply 6×4
$24 + 2$	Add $24 + 2$
26	

Example 2: Evaluate $4(6+3)-5\cdot 2$.

$4(6+3)-5\cdot 2$	Parentheses (6 + 3)
$4(9)-5\cdot 2$	Multiply 4(9)
$36-5\cdot 2$	Multiply $5\cdot 2$
$36-10$	Subtract
26	

Example 3: Evaluate $5[(3+12)-2(4)]$.

$5[(3+12)-2(4)]$	Work within brackets [ ]
$5[(3+12)-2(4)]$	Parenthesis $(3+12)$
$5[15-2(4)]$	Multiply 2(4)
$5[15-8]$	Subtract $15-8$
$5(7)$	Multiply 5(7)
35	

Example 4: Evaluate $4[3(3+2)^2]$.

$4[3(3+2)^2]$	Work within brackets [ ]
$4[3(3+2)^2]$	Parenthesis $(3+2)$
$4[3(5)^2]$	Powers $(5)^2$
$4[3(25)]$	Multiply 3(25)
$4(75)$	Multiply 4(75)
300	

Example 5: Evaluate $4[3(3+2)^2]$.

$4[3(3+2)^2]$	Work within brackets [ ]
$4[3(3+2)^2]$	Parenthesis $(3+2)$
$4[3(5)^2]$	Powers $(5)^2$
$4[3(25)]$	Multiply 3(25)
$4(75)$	Multiply 4(75)
300	

Example 6: Evaluate $4\cdot 5-18\div 6+2\cdot 3$.

$4\cdot 5-18\div 6+2\cdot 3$	Multiply and divide left to right
$20-3+6$	Add and subtract left to right
$17+6$	Add
23	

You will use the **order of operations** throughout many mathematics courses that you continue to study.