SQUARES AND SQUARE ROOTS

In this unit, you will work through the perfect squares and square roots. Perfect squares and square roots of perfect squares should be numbers that you commit to memory as this memorization will lead to quick answers later. You may wish to create flashcards with the answers on the back to assist in your memorization. You can keep the cards and practice anytime.

You will also apply your knowledge of square roots and estimation to approximate non-perfect square roots. Non-perfect square roots are irrational numbers, decimal numbers that never end and do not develop into a repeating pattern.

Perfect Squares and Square Roots

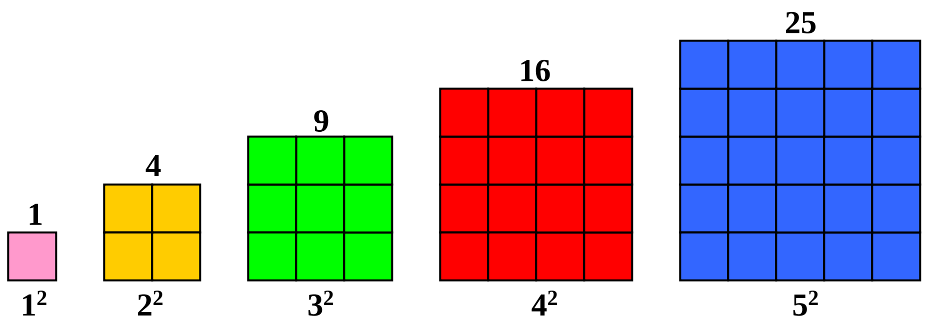
Approximating Non-perfect Square Roots

Perfect Squares and Square Roots

Perfect Squares

Perfect squares are numbers that are squares of integers.

Some examples of perfect squares are shown in the figure below. The first five squares of the counting numbers are shown.



Square Notation	Perfect Square
1^2 (1-squared)	**1**
2^2 (2-squared)	**4**
3^2 (3-squared)	**9**
4^2 (4-squared)	**16**
5^2 (5-squared)	**25**

Example 1: Find the first 12 perfect squares of the counting numbers.

$$1^2 = 1 \times 1 = 1 \qquad 7^2 = 7 \times 7 = 49$$
$$2^2 = 2 \times 2 = 4 \qquad 8^2 = 8 \times 8 = 64$$
$$3^2 = 3 \times 3 = 9 \qquad 9^2 = 9 \times 9 = 81$$
$$4^2 = 4 \times 4 = 16 \qquad 10^2 = 10 \times 10 = 100$$
$$5^2 = 5 \times 5 = 25 \qquad 11^2 = 11 \times 11 = 121$$
$$6^2 = 6 \times 6 = 36 \qquad 12^2 = 12 \times 12 = 144$$

The first 12 perfect squares are:

$\{1, 4, 9, 25, 36, 49, 64, 81, 100, 121, 144...\}$

Perfect squares are used often in math. Try to memorize these familiar numbers so that you can recognize them as they are used in many math problems.

The first five squares of the negative integers are shown below. Remember that a negative integer times a negative integer equals a positive integer.

Square Notation	**Perfect Square**
$\mathbf{(-1)^2 = (-1 \times -1)}$	**1**
$\mathbf{(-2)^2 = (-2 \times -2)}$	**4**
$\mathbf{(-3)^2 = (-3 \times -3)}$	**9**
$\mathbf{(-4)^2 = (-4 \times -4)}$	**16**
$\mathbf{(-5)^2 = (-5 \times -5)}$	**25**

Square Roots of Perfect Squares

The **square root** operation is the reverse operation of squaring a number. In other words, to find the square root of a number, determine what number times itself equals the given number.

Finding a square root of a perfect square can be as easy as guessing the solution to the following algebraic equation:

$$x^2 = 49$$

If we understand the meaning of the exponent "2", we know that a solution for *x* is 7 because we are finding a number that when multiplied times itself equals 49.

$$7(7) = 49$$

We must also remember that if we include negative values, there is another solution, –7.

$$-7(-7) = 49$$

This guess and check system for finding values of this type is fine and will work for perfect square numbers like 49, 64 , 81, 144, or even the value 1.

Actually, the set of values $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144...\}$ all have relatively easy "guessable" square root solutions. This set of values is called the "perfect squares" because the numbers that are used as double factors are **integral** values…**perfect**.

The square root of 25 is 5 or –5.

The symbol for the square root operation is $\sqrt{\ }$.

To indicate which root is desired, we will use the following notation:

$$\sqrt{25} = 5 \qquad -\sqrt{25} = -5$$

Examples:

a. $\sqrt{81} = 9$

b. $-\sqrt{81} = -9$

c. $\sqrt{0.49} = 0.7$

d. $\sqrt{\frac{25}{64}} = \frac{\sqrt{25}}{\sqrt{64}} = \frac{5}{8}$

Try these!

On paper list the answers to the following problems. Look below for the correct answers.

Perfect Squares
1. List the perfect squares of the counting numbers 13 through 20.

2. What is the square of 30?

3. What is the square of 0.09?

Square Roots
4. What is $-\sqrt{144}$ **?**

5. What is $\sqrt{1.21}$?

6. What is $\sqrt{\frac{9}{100}}$?

Solutions

1. 169, 196, 225, 256, 289, 324, 361, 400

2. 900

3. 0.0081

4. –12

5. 1.1

6. $\frac{3}{10}$

$\sqrt{}$ Approximating Square Roots of Non-Perfect Squares

Consider solving this equation: $x^2 = 55$

Keep in mind there is no integer that will give us a solution. However, the value for x will be between 7 and 8 because $7^2 = 49$ and $8^2 = 64$.

We will guess and check until we get an approximate answer…

Try 7.5 $\rightarrow$ $7.5^2 = 56.25$ Close, but greater than 55.

We can get closer…

Try 7.4 $\rightarrow$ $7.4^2 = 54.76$ Close, but less than 55.

We will try to get just a little closer…

Try 7.45 $\rightarrow$ $7.45^2 = 55.5025$ Closer, but greater than 55.

Try a little lower…

Try 7.43 $\rightarrow$ $7.43^2 = 55.2049$ Getting closer, but still greater than 55.

Try a little lower…

Try 7.41 $\rightarrow$ $7.41^2 = 54.9081$ Getting closer, but lower than 55.

Try a little higher…

Try 7.42 $\rightarrow$ $7.42^2 = 55.0564$ **Close enough!**

Solution: $\sqrt{55} \approx 7.42$

The square root of 55 is approximately equal to 7.42.

*We use the "approximately equal" symbol $(\approx)$ since the square root of 55 is not exactly equal to 7.42.

In the previous estimation, we first guessed values of numbers in tenths that were close to the answer, and then we guessed values in hundredths. We kept guessing closer until we were able to be accurate with the best hundredth value. Determining which place value for estimation will depend on the problem at hand.

Example: Evaluate $\sqrt{115}$ to the nearest tenth.

Guess between 10 and 11, and since $10^2 = 100$ and $11^2 = 121$.

Guess closer to 11 since 115 is closer to 121.

Try 10.8 $\rightarrow$ $10.8^2 = 116.64$ Close, but greater than 115.

Try 10.6 $\rightarrow$ $10.6^2 = 112.36$ Close, but less than 115.

Try 10.7 $\rightarrow$ $10.7^2 = 114.49$ Closest answer to nearest tenth.

Solution: $\sqrt{115} \approx 10.7$

To the nearest tenth, the square root of 115 is approximately equal to 10.7.