

RATIOS AND PERCENTS

In this unit, you will review the idea of ratios, proportions, and percents. You will calculate the connections between fractions, decimals, and percents. You will review ratio models and apply ratios to solve problems with proportions. You will then be given a variety of ways to gain a good understanding of the idea of percent. In the last part of the unit, you will solve problems using the basic three types of percent problems: finding the part, find percent, and find the base.

Fractions to Decimals and Vice Versa

Ratios and Proportions

Introduction to Percent

Percents, Fractions, and Decimals

The Three Types of Percent Problems

Fractions to Decimals and Vice Versa

Fractions to Decimals

To write a fraction as a decimal, recall that the fraction bar means divide.

Example 1: Find the decimal for $\frac{3}{4}$.

To find the decimal for $\frac{3}{4}$, divide the denominator into the numerator, and then add a decimal point and zeros. In this example, the decimal comes out even after adding two zeros.

$$\begin{array}{r} .75 \\ 4 \overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \end{array} \qquad \frac{3}{4} = 0.75$$

The decimal for $\frac{3}{4}$ is 0.75.

Some fractions do not have decimals that come out even and, when dividing, continue on forever. For this type of fraction, divide and round to the given place.

Example 2: Find the decimal for $\frac{2}{3}$ and round to nearest hundredth.

To find the decimal for $\frac{2}{3}$, divide to get one extra decimal place for rounding, stop, and then round. In this problem, divide through thousandths place (3 decimal places).

$$\begin{array}{r}
 .666 \\
 3 \overline{)2.000} \\
 \underline{18} \\
 20 \\
 \underline{18} \\
 20 \\
 \underline{18} \\
 2
 \end{array}
 \qquad
 \frac{2}{3} \approx 0.67$$

Sometimes repeating decimals are expressed with a bar over the repeating pattern in the decimal.

Two-thirds may be written as $0.\overline{6}$.

Click on the tracks below to play a game.



Decimals to Fractions

Decimals may be written as fractions and simplified, if needed.

Example 3: How is 4.53 read?

4	.	53
four	and	$\frac{53}{100}$

Answer: 4 and 53 hundredths

*Notice that when there are **two decimal places**, there are **two zeros** in the denominator of the fraction.

Example 4: How is 7.5 read?

7	.	5
seven	and	$\frac{5}{10}$

Answer: 7 and 5 tenths

*Notice that when there is **one decimal place**, there is **one zero** in the denominator of the fraction.

Example 5: Write the mixed number for 7.5 in simplest form.

To solve this problem, first write the fraction for the mixed number, and then reduce.

$$\begin{aligned} 7.5 &= 7\frac{5}{10} & \left(\frac{5}{10} \div \frac{5}{5} = \frac{1}{2} \right) \\ &= 7\frac{1}{2} \end{aligned}$$

The mixed number for 7.5 is 7 1/2 in simplest form.

Example 6: Write the mixed number for 6.225 in simplest form.

To solve this problem, first write the fraction for the mixed number, and then reduce.

$$\begin{aligned} 6.225 &= 6\frac{225}{1000} & \left(\frac{225}{1000} \div \frac{25}{25} = \frac{9}{40} \right) \\ &= 6\frac{9}{40} \end{aligned}$$

The mixed number for 6.225 is 6 9/40 in simplest form.

*Notice that when there are **three decimal places**, there are **three zeros** in the denominator of the fraction.

Click on the honeycombs below to play a game.



Ratios and Proportions

Ratio Models

A ratio is a **comparison** of two quantities.

The colors of these leaves may be compared many ways.



There are three ways to write ratios.

To compare the yellow leaves to green leaves, you can write:

4 to 3 *or* 4:3 *or* $\frac{4}{3}$

The ratio of yellow leaves to green leaves is **4 : 3**.

Example 1: Write the ratio of red leaves to brown leaves three ways.

2 to 3 *or* 2:3 *or* $\frac{2}{3}$

The ratio of red leaves to brown leaves is **2 : 3**.

Example 2: Write the ratio of green leaves to all of the leaves as a fraction in simplest form.

3 to 12 *or* 3:12 *or* $\frac{3}{12}$

The ratio of green leaves to all of the leaves is **3 : 12**.

This ratio can be simplified; so, use the fraction form of the ratio and reduce.

$$\frac{3}{12} \div \frac{3}{3} = \frac{1}{4}$$

The ratio of green leaves to all of the leaves is **1 : 4**. This means that one in four leaves is green.

Example 3: Write the ratio of all of the leaves to the red leaves as a fraction in simplest form.

$$12 \text{ to } 2 \qquad \text{or} \qquad 12:2 \qquad \text{or} \qquad \frac{12}{2}$$

The ratio of all of the leaves to the red leaves is **12 to 2**.

This ratio can be simplified; so, use the fraction form of the ratio and reduce.

$$\frac{12}{2} \div \frac{2}{2} = \frac{6}{1}$$

The ratio of all of the leaves to the red leaves is **6 : 1**. This means for every six leaves in the group, one is red.

*Ratios such as 6/1 or 5/3 remain as improper fractions when written in fraction form since a ratio is a comparison of two quantities.

Practice

On paper, write the comparisons as ratios, and then compare your answers with the answer key below.



Comparisons

Black Car to Blue Cars

Yellow Cars to Red Cars

Blue Cars to All Cars

All Cars to Black Car

Solutions

Black car to Blue Cars **1 : 3**

Yellow cars to Red Cars **2 : 1**

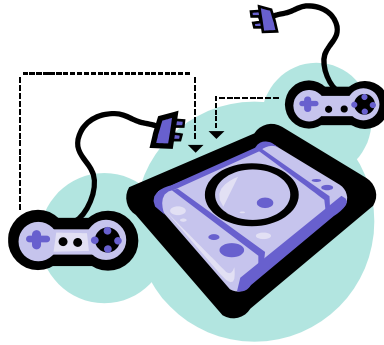
Blue Cars to All Cars **3 : 10**

All Cars to Black Car **10 : 1**

Using Ratios to Solve Proportions

Using ratios to set up a proportion is a very useful problem-solving technique that can be used in a variety of everyday problems.

Example 4: Cindy surveyed her home-room class and found that 11 out of 25 students played video games at home at least once every day of the school week. There are 500 students in the eighth grade at her school. Based on Cindy's survey, predict how many eighth grade students in the school play video games at least once every day of the school week.



To solve, first find the comparison ratio.

$$\frac{11}{25} = \frac{\text{(play a video game at least once every day of the school week)}}{\text{(all students surveyed)}}$$

Then, set up a proportion where both numerators represent the same idea and both denominators represent the same idea.



Thinking through the problem with brief words can be helpful.

11 out of 25 play video games...how many out of 500 play video games...?

The proportion is $\frac{11}{25} = \frac{n}{500}$.

$$\frac{11}{25} = \frac{n}{500} \quad \text{Cross Multiply}$$

$$25 \times n = 11 \times 500 \quad \text{Simplify}$$

$$25n = 5500 \quad \text{Divide}$$

$$n = 220$$

Cindy estimated that, based on her survey, about 220 eighth grade students play a video game at least once every day of the school week.

Introduction to Percent

Models of Percents

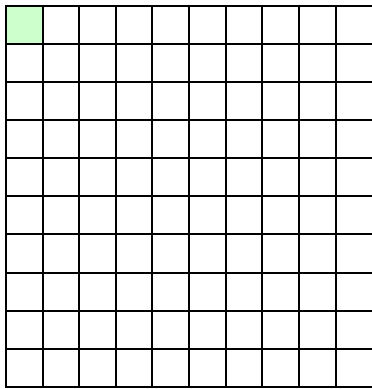
Fractions and decimals can be expressed as percents.

Ten by ten grids may be used to represent percent. The whole grid has 100 squares in it. Each small square represents one percent (%) of the large square.

Let's take a look at some models of percent that are also expressed as decimals and fractions. Each model has a certain percent of the squares shaded.

Example 1: What percent of the squares is shaded green?

1%



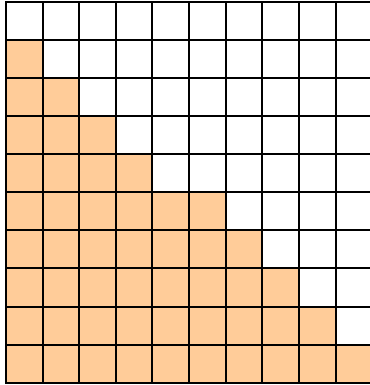
1% means 1 out of 100

$$1\% = \frac{1}{100} = 0.01$$

One percent (1%) of the squares is shaded green.

Example 2: What percent of the squares are shaded orange?

50%



50% means 50 out of 100

$$50\% = \frac{50}{100} = 0.50$$

Fifty percent (50%) of the squares are shaded orange.

Example 3: Write 50% as a fraction and as a decimal in simplest terms.

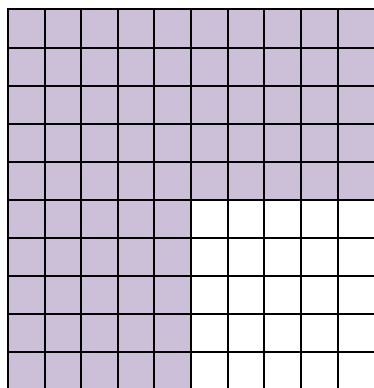
$$50\% = \frac{50}{100} = \frac{5\cancel{0}}{10\cancel{0}} = \frac{5}{10} \div \frac{5}{5} = \frac{1}{2}$$

$$50\% = 0.50 = 0.5\cancel{0} = 0.5$$

Fifty percent (50%) equals 1/2. Fifty percent (50%) equals 0.5.

Example 4: What percent of the squares are shaded purple?

75%



75% means 75 out of 100

$$75\% = \frac{75}{100} = 0.75$$

Seventy-five percent (75%) of the squares are shaded purple.

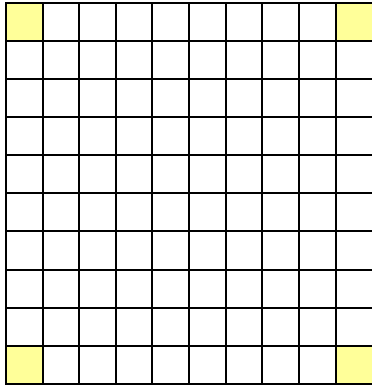
Example 5: Write 75% as a fraction in simplest terms.

$$75\% = \frac{75}{100} \div \frac{25}{25} = \frac{3}{4}$$

Seventy-five percent (75%) equals $\frac{3}{4}$.

Example 6: What percent of the squares are shaded yellow?

4%



4% means 4 out of 100

$$4\% = \frac{4}{100} = 0.04$$

Four percent (4%) of the squares are shaded yellow.

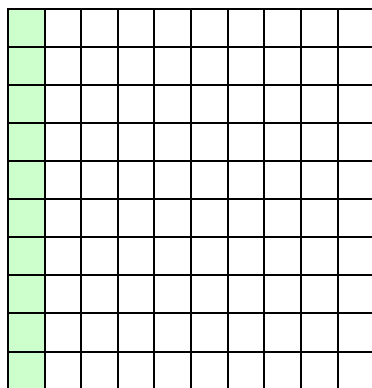
Example 7: Write 4% as a fraction in simplest terms.

$$4\% = \frac{4}{100} \div \frac{4}{4} = \frac{1}{25}$$

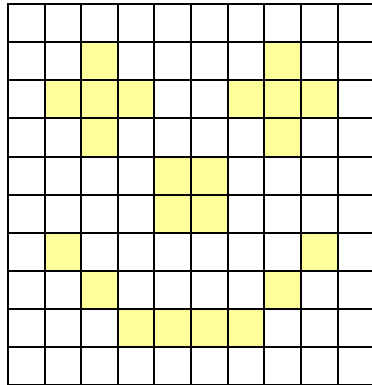
Four percent (4%) equals $\frac{1}{25}$.

Example 8: Study the following graphs. Mentally determine the answer. Look below for answers to see if you are correct.

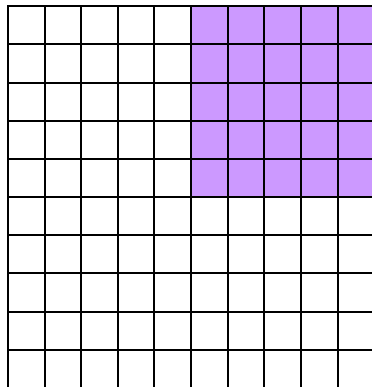
a.) What percent of the squares are shaded green?



b.) What percent of the squares are shaded yellow?



c.) What percent of the squares are shaded purple?

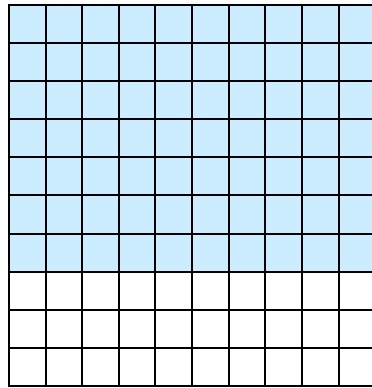


Solutions: a) 10%, b) 22%, c) 25%

Percents, Fractions, and Decimals

Writing Percents as Fractions and Decimals

In the decimal square below, 70% of the squares are shaded blue. Write 70% as a fraction and a decimal in simplest terms.



$$70\% = \frac{70}{100} = \frac{7\cancel{0}}{10\cancel{0}} = \frac{7}{10}$$

$$70\% = 0.70 = 0.7\cancel{0} = 0.7$$

Seventy percent (70%) equals 7/10. Seventy percent (70%) equals 0.7.

Example 1: Write 30 percent as a decimal and a fraction in simplest terms.

30%	
Decimal	$0.30 = .3\cancel{0} = 0.3$
Fraction	$\frac{30}{100} = \frac{3\cancel{0}}{10\cancel{0}} = \frac{3}{10}$

$$30\% = 0.3 \quad 30\% = 3/10$$

Example 2: Write $11/20$ as a percent and a decimal.

$\frac{11}{20}$	
Percent	$\frac{11}{20} \times \frac{5}{5} = \frac{55}{100} = 55\%$
Decimal	$20 \overline{)11.00} = 0.55$

$$11/20 = 55\% \quad 11/20 = 0.55$$

Example 3: Write 0.05 as a fraction and a percent.

0.05	
Fraction	$0.05 = \frac{5}{100} \div \frac{5}{5} = \frac{1}{20}$
Percent	$0.05 = \frac{5}{100} = 5\%$

$$0.05 = 1/20 \quad 0.05 = 5\%$$

Example 4: Write 0.8 as a fraction and a percent.

0.8	
Fraction	$0.8 = \frac{8}{10} \div \frac{2}{2} = \frac{4}{5}$
Percent	$0.8 = \frac{8}{10} \times \frac{10}{10} = \frac{80}{100} = 80\%$

$$0.8 = 4/5 \quad 0.8 = 80\%$$

Example 5: Write 100% as a fraction in simplified form and as a decimal.

100%	
Fraction	$100\% = \frac{100}{100} = 1$
Decimal	$100\% = \frac{100}{100} = 1 = 1.00$

$$100\% = 1 \quad 100\% = 1.00$$

Express Percents as Decimals

Percent to Decimal (Shortcut)

Move the decimal point **two places** to the **left**.

Example 6: Write 89% as a decimal.

The decimal point's starting position is at the right end of the percent. Then, move the decimal point **two places** to the **left**. (This is a shortcut for dividing by 100.)

$$89.\% = 0.89 \qquad \left(\begin{array}{r} .89 \\ 100 \overline{)89.00} \end{array} \right)$$

The decimal for 89% is 0.89.

Example 7: Write 6.25% as a decimal.

Move the decimal point two places to the left or divide by 100.

$$6.25\% = 0.0625 \qquad \left(\begin{array}{r} .0625 \\ 100 \overline{)6.2500} \end{array} \right)$$

(Use a zero as a place holder.)

The decimal for 6.25% is 0.0625.

Example 8: Write 275% as a decimal.

The decimal point's starting position is at the right end of the percent. Then, move the decimal point **two places** to the **left**. (This is a shortcut for dividing by 100.)

$$275. \% = 2.75 \qquad \left(\begin{array}{r} 2.75 \\ 100 \overline{)275.00} \end{array} \right)$$

The decimal for 275% is 2.75.

Express Decimals as Percents

Decimal to Percent (Shortcut)

Move the decimal point **two places** to the **right**.

Example 9: Write 0.34 as a percent.

To express a decimal as percent, move the decimal point **two places** to the **right**. (This is the shortcut for multiplying by 100.)

$$0.34 = 34\% \qquad \left(\begin{array}{r} 0.34 \\ \times 100 \\ \hline 34.00 = 34 \end{array} \right)$$

The percent for 0.34 is 34%.

Example 10: Write 8.5 as a percent.

To express a decimal as percent, move the decimal point **two places** to the **right**. (This is the shortcut for multiplying by 100.)

$$8.5 \xrightarrow{\text{right}} = 850\% \qquad \left(\begin{array}{l} 8.5 \\ \times 100 \\ \hline 850.0 = 850 \end{array} \right)$$

(Use a zero as a place holder.)

The Three Basic Types of Percent Problems

Percent means per hundred; thus, when we say 27% we mean 27 out of 100. Percents can be written as equivalent decimals and fractions.

$27\% = 0.27$ Move the decimal point two (2) places to the left.

$27\% = \frac{27}{100}$ Put 27 over 100 since percent means per hundred.

Percents greater than 100% represent whole numbers or mixed numbers.

$200\% = 2$ $350\% = 3.5 = 3\frac{5}{10} = 3\frac{1}{2}$

The three basic types of percent problems are:

- (1) finding the part (finding a percent of a number)
- (2) finding percent
- (3) finding the base

There are several ways to solve these three types of problems; however, we will focus on the “ratio-proportion” method.

When finding the part or the base, use the ratio $\frac{\text{is}}{\text{of}}$ to set up a proportion.

The **part** is near the word “**is**” and the **base** will follow the word “**of**”. The **percent** is identified by the word percent or the percent symbol (%).

Finding Part

To find **what part a percent is of the base**, use the following proportion.

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}} \rightarrow \frac{\%}{100} = \frac{\text{part}}{\text{base}}$$

Example 1: What is 32% of 350?

What is 32% of 350?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}} \rightarrow \frac{32}{100} = \frac{n}{350}$$

The percent (32%) is placed over 100.

The base follows “of” and is 350.

The part is near “is” and is unknown (n).

Now solve:

$$\frac{32}{100} = \frac{n}{350}$$

$$100n = 32(350)$$

$$100n = 11200$$

$$n = 112$$

Cross Multiply

Simplify

Divide

Thirty-two percent of 350 is 112.

Other ways this problem may be stated are:

Thirty-two percent of 350 is what number?

Find 32% of 350.

Since this type of percent problem is used often, we will discuss another method which is quicker. To find part, the **percent of a number**, first write

the percent as a decimal by moving the decimal point two places to the left, and then multiply the given numbers together.

To find 32% of 350 using the quicker method, simply change 32% to a decimal, interpret the “of” as multiplication, and then multiply.

$$\begin{aligned} 32\% \text{ of } 350 &= \\ &= 0.32 \times 350 \\ &= 112.00 \\ &= 112 \end{aligned}$$

Finding Percent

To find **what percent a number is of another number**, let's go back to the percent proportion.

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}} \rightarrow \frac{\%}{100} = \frac{\text{part}}{\text{base}}$$

Example 2: Twenty-six is what percent of 50?

Twenty-six is **what percent** of 50?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}} \rightarrow \frac{n}{100} = \frac{26}{50}$$

The unknown percent (n) is placed over 100.

The base follows “of” and is 50.

The part is near “is” and is 26.

Now solve:

$$\frac{n}{100} = \frac{26}{50}$$

$$50n = 26(100)$$

$$50n = 2600$$

$$n = 52$$

Cross Multiply

Simplify

Divide

$$\frac{52}{100} = 52\%$$

Fifty-two percent of 50 is 26.

Other ways this problem may be stated are:

What percent of 50 is 26? Twenty-six out of 50 is what percent?

Finding Base

The third type of basic percent problems is **finding the base** when given the percent and part.

To find the base, we'll revisit the percent proportion one more time.

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}} \rightarrow \frac{\%}{100} = \frac{\text{part}}{\text{base}}$$

Example 3: Seventy-five is 15% of what number?

Seventy-five is 15% of what number?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}} \rightarrow \frac{15}{100} = \frac{75}{n}$$

The percent (15%) is placed over 100.

The base follows "of" and is unknown (n).

The part is near "is" and is 75.

Now solve:

$$\frac{15}{100} = \frac{75}{n}$$

$$15n = 75(100)$$

$$15n = 7500$$

$$n = 500$$

Cross Multiply

Simplify

Divide

Seventy-five is 15% of 500.

Other ways this problem may be stated are:

Fifteen percent of what number is 75?

Click on the bricks below to play a game.

